Extending Linear Approaches to Solve Fuzzy Quadratic Programming Problem

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Abstract

Quadratic programming problem has been widely applied for solving a real-world problems. Although quadratic programming problems are a special class of nonlinear programming, they can also be seen as general linear programming problems. This work proposes a new methods to solve a particular class of Fuzzy Quadratic Programming problems (FQPP) which have vagueness coefficients in the objective function and constraints coefficients. Moreover, linear approaches are extended to solve the quadratic case. Finally, it is shown that the solutions reached from the extended approaches may be obtained from proposed parametric approaches.

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1 Introduction

The optimization methods are algorithms based on mathematical knowledge that theoretically guarantees the convergence to the optimal solution. The concept of fuzzy mathematical programming emerges when fuzzy elements are used to deal with uncertain data in real problems. This area can be divided in some sub-fields and
one of them is the quadratic programming which is characterized by optimizing a quadratic objective function subject to linear constraints. This kind of problems can be formulated as

\[
\text{Optimize } z = cx + \frac{1}{2}x^tQx \\
\text{subject to } Ax \leq b \text{ and } x \geq 0
\]  

(1)

where \(x\) is a vector of decision variables that belongs to \(\mathbb{R}^n\), \(c\) is a vector that belongs to \(\mathbb{R}^n\). \(A\) is an \(m \times n\) matrix with entries in real numbers and \(Q\) is an \(n \times n\) symmetric matrix with entries in real numbers. As it is known, Quadratic Programming (QP), can be viewed as a generalization of linear programming problems. However in most real practical applications (portfolio, game theory, engineering modeling, design and control, logistics, etc.) one lacks this kind of exact knowledge, and only approximate, vague and imprecise values of the parameters are known. The application of fuzzy logic in this context of fuzzy quadratic programming problems also a way of describing this vagueness mathematically as described in Bector and Chandra (2005). Despite the uncertainty can be presented in any part of the formulation, the focus of this work is to show it in the costs of the objective function, which can be dealt with by fuzzy numbers. Nowadays there is a necessity for solving some optimization problems such as energy management, portfolio selection and investment risk, control systems, production problem and scheduling, among other problem, which are formulated as a quadratic programming problem. These real problems have uncertain and vague data that can be dealt with using fuzzy logic. Thus, the development of methods for solving quadratic programming problem under fuzzy environment emerges as a way of solving these kinds of problems. With this focus, solve fuzzy linear programming in the case of quadratic programming problems with fuzzy costs in the linear term of the objective function are extended to the quadratic case. The main objective is to propose an approach that achieves a set of optimal solutions to quadratic programming problems with fuzzy costs in both linear and quadratic terms of the objective function. We present a new approach to obtain the optimal solution for the FQPP considered.

The paper is organized as follows: Sect. 2 shows some basic concept of triangular fuzzy numbers and related results. Sect. 3 we intro-
duced fuzzy approaches that solve quadratic programming problems with fuzzy costs in the non linear term of the objective function; Sect. 4 proposes parametric approaches to solve quadratic programming problems with fuzzy costs in both linear and quadratic terms of the objective function; Sect. 5 to clarify the above developments numerical examples are analyzed, Finally, conclusions are presented in Sect. 6.

2 Preliminaries

We review the fundamental notation of fuzzy set theory, initiated by Zadeh [1].

Definition 2.1. A fuzzy set \( \tilde{A} \) defined on the set of real numbers \( R \) is said to be a fuzzy number, if its membership function \( \mu_{\tilde{A}} : R \rightarrow [0, 1] \), has the following characteristics:

1. \( \mu_{\tilde{A}}(x) \) is convex, i.e., \( \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0, 1], \forall x_1, x_2 \in R \)

2. \( \mu_{\tilde{A}} \) is normal, i.e., there exists an \( x \in R \) such that \( \mu_{\tilde{A}}(x) = 1 \)

3. \( \tilde{A} \) is upper semi-continuous.

4. \( \text{sup} (\tilde{A}) \) is bounded in \( R \).

Definition 2.2. A fuzzy number \( \tilde{A} \) on \( R \) is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function \( \tilde{A} : R \rightarrow [0, 1] \) has the following characteristics:

\[
\tilde{A}(x) = \begin{cases} 
    x - a_1 & \text{for } a_1 \leq x \leq a_2 \\
    a_2 - a_1 & \text{for } a_2 < x \leq a_3 \\
    a_3 - a_2 & \text{for } a_3 - a_2 \end{cases}
\]

We denote this triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \) and use \( F(R) \) to denote the set of all triangular fuzzy numbers.

Definition 2.3. Let \( \tilde{A} = (a_1, a_2, a_3) \) be a triangular fuzzy numbers then the parametric form of the TFN is defined as \( \tilde{A} = (a_0, a_*, a^*) \) where \( a_* = (a_0 - a) \) and \( a^* = (\overline{a} - a_0) \).

\[
\overline{a}(r) = (a_3 - (a_3 - a_2)r), a(r) = (a_2 - a_1)r + a_1, a_0 = \frac{a(r) + \overline{a}(r)}{2}
\]

here \( r = 1 \), we get \( a_0 = a_2 \) where \( r \in [0, 1] \)
2.1 Ranking Function

One of the ways for solving mathematical programming problems in a fuzzy environment is to compare fuzzy numbers. The comparison between fuzzy numbers is done by using a ranking function that attends some conditions described in [10]. An appropriate approach for ordering the elements of F(R) is to define a ranking function \( \tilde{R} : F(R) \rightarrow R \), which maps each fuzzy number into the real line, where a natural order exists. Let \( \tilde{A} = (a_1, a_2, a_3) \) be a triangular fuzzy number, then a special form of the ranking function is:

\[
R(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}
\]

Some order on F(R) is defined as follows: \( a_1 \leq_f a_2 \) if and only if \( R(\tilde{a}_1) \leq R(\tilde{a}_2) \) where \( \tilde{a}_1 \) and \( \tilde{a}_2 \) belong to \( F(R) \). R is a ranking function, and the symbol \( \leq_f \) represents the fuzzy order relation.

2.2 Arithmetic Operations:

According to Ming Ma et al. have proposed a new fuzzy arithmetic operation based upon the both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, where as the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \lor b = \max\{a, b\} \) and \( a \land b = \min\{a, b\} \). For arbitrary fuzzy numbers \( \tilde{A} = (a_0, a_*, a^*) \), \( \tilde{B} = (b_0, b_*, b^*) \) and \( * = \{+, -, \times, \div\} \), the arithmetic operations on the fuzzy numbers are defined by

(i) Addition: \( \tilde{A} + \tilde{B} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

(ii) Subtraction: \( \tilde{A} - \tilde{B} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)

(iii) Multiplication: \( \tilde{A} \times \tilde{B} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)
Division: $\tilde{A} \div \tilde{B} = (a_0, a_*, a_*) \div (b_0, b_*, b_*)$

= $(a_0 \div b_0, \max\{a_*, b_*\}, \max\{a_*, b_*\})$

**Definition 2.4.** Let $V$ be the real vector space and $F(R)$ be the number of space. Then a function $\tilde{f}: V \to F(R)$ is called fuzzy-valued function defined on $V$. Corresponding to such function $\tilde{f}$ and $\alpha \in [0, 1]$, we define two real valued functions $\tilde{f}_L$ and $\tilde{f}_U$ on $V$ as $\tilde{f}_L(x) = (\tilde{f}(x))^L$ and $\tilde{f}_U(x) = (\tilde{f}(x))^U$, for all $x \in V$.

**Definition 2.5.** Let $\tilde{f}: R^n \to F(R)$ be a fuzzy-valued function. We say that $\tilde{f}$ is continuous at $c \in R^n$ if for every $\varepsilon > 0$ there exists a $\delta = \delta(c, \varepsilon) > 0$ such that $d_F(\tilde{f}(x), \tilde{f}(c)) < \varepsilon$.

For all $x \in R^n$ with $\|x - c\| < \delta$. That is, $\lim_{x \to c} \tilde{f}(x) = \tilde{f}(c)$.

**Proposition 1.** Let $\tilde{f}: R^n \to F(R)$ be a fuzzy-valued function. If $\tilde{f}$ is continuous at $c \in R^n$, then functions $\tilde{f}_L$ and $\tilde{f}_U$ are continuous at $c$, for all $\alpha \in [0, 1]$.

**Definition 2.6.** Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers, if there exists a fuzzy number $\tilde{c}$ such that $\tilde{c} \oplus \tilde{b} = \tilde{a}$. Then $\tilde{c}$ is called Hukuhara difference of $\tilde{a}$ and $\tilde{b}$ and is denoted by $\tilde{a} \oplus_H \tilde{b}$.

**Definition 2.7.** Let $X$ be a subset of $R$. A fuzzy-valued function $\tilde{f}: X \to F(R)$ is said to be H-differentiability at $x^0 \in X$ if there exists a fuzzy number $D\tilde{f}(x^0)$ such that the limits (with respect to metric $d_F$) $\lim_{h \to 0^+} \frac{1}{h} \otimes [\tilde{f}(x^0 + h) \ominus_H \tilde{f}(x^0)]$ and $\lim_{h \to 0^+} \frac{1}{h} \otimes [\tilde{f}(x^0) \ominus_H \tilde{f}(x^0 - h)]$ both exists and are equal to $D\tilde{f}(x^0)$. In this case, $D\tilde{f}(x^0)$ is called the H-derivative of $\tilde{f}$ at $x^0$. If $\tilde{f}$ is H-differentiability at any $x^0 \in X$, we call $\tilde{f}$ is H-differentiability over-$X$.

3 Fully fuzzy quadratic programming problem

Quadratic programming is one of the most important optimization techniques in operations research. In the real life problems there may exists uncertainly about the parameters. In such a situation
the parameters of quadratic programming problems may be represented as fuzzy numbers. We consider a FQPP with fuzzy numbers and fuzzy variables as follows:

\begin{align*}
\text{Optimize } & f(z) = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \tilde{x}_k \otimes \tilde{q}_{kj} \otimes \tilde{x}_j \\
\text{S. to } & \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \leq \tilde{b}_i, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \quad (2)
\end{align*}

Here $\tilde{x}_j$ is a non-negative triangular fuzzy number and $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j$ and $\tilde{x}_j \in F(R)$.

**Definition 3.1.** A non-negative triangular fuzzy vector $\tilde{x}$ is said to be a fuzzy feasible solution for (2) if $\tilde{x}$ satisfies the constraints

\[ \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \leq \tilde{b}_i, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \]

**Definition 3.2.** A fuzzy feasible solution $\tilde{x}^*$ called a fuzzy optimal solution for (2) if for all fuzzy feasible solutions $\tilde{x}$, we have

\[ \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \tilde{x}_k \otimes \tilde{q}_{kj} \otimes \tilde{x}_j \leq \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \tilde{x}_k \otimes \tilde{q}_{kj} \otimes \tilde{x}_j \]

**Definition 3.3.** If a quadratic programming problem $f(z)$ cannot have an unbounded minimum then $\tilde{x}_k \otimes \tilde{q}_{kj} \otimes \tilde{x}_j$ is positive definite and $\tilde{c}_j = 0$. If $\tilde{c}_j \neq 0$ and $\tilde{x}_k \otimes \tilde{q}_{kj} \otimes \tilde{x}_j$ is positive semi-definite, then $f(z)$ may have an unbounded minimum.

### 4 The solution procedure

To find optimal solution of any QPP by an alternative method for modified simplex method, the algorithm is given as follows:

**Step 1:** First, convert the inequality constraints into equations by introducing slack-variables $\tilde{s}_i (i = 1, 2, \ldots, m) \geq 0$ in the $i^{th}$ constraints and the non-negative restrictions by introducing slack variables in the restrictions by introducing slack variables $\tilde{s}_j (j = 1, 2, \ldots, n) \geq 0$ in the $j^{th}$ restrictions.

**Step 2:** Construct the Lagrangian function

\[ L(\tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu}) = \tilde{f}(z) - \sum_{i=1}^{m} \tilde{\lambda}_i [\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j - \tilde{b}_i + \tilde{q}_i^2] - \sum_{j=1}^{n} \tilde{\mu}_j [-\tilde{x}_j + \tilde{s}_j] \]
Differentiating this Lagrangian function \( L(\tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu}) \) with respect to the components of \( \tilde{x}, \tilde{s}, \tilde{\lambda}, \tilde{\mu} \) and equating the first order partial derivatives to zero, derive Kuhn-Tucker Conditions from the resulting equations.

Step 3: Introduce non-negative variables \( \tilde{a}_j (j = 1, 2, \cdots, n) \) in the Kuhn-Tucker Conditions \( \tilde{c}_j + \sum_{k=1}^{n} \tilde{c}_{jk} \tilde{x}_k - \sum_{i=1}^{m} \tilde{\lambda}_i \tilde{a}_{ij} + \tilde{\mu}_j = 0 \) for \( j = 1, 2, \cdots, n \) and construct an objective function 
\[ z = \tilde{a}_1 + \tilde{a}_2 + \cdots + \tilde{a}_n \]

Step 4: Obtain the initial basic feasible solution to the LPP
\[
\text{min } z = \tilde{a}_1 + \tilde{a}_2 + \cdots + \tilde{a}_n \\
\text{Subject to the constraints:} \\
\sum_{k=1}^{n} \tilde{c}_{jk} \tilde{x}_k - \sum_{i=1}^{m} \tilde{\lambda}_i \tilde{a}_{ij} + \tilde{\mu}_j + \tilde{a}_j = -\tilde{c}_j; \ (j = 1, 2, \cdots, n) \\
\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j + \tilde{s}_i = \tilde{b}_i; \ (i = 1, 2, \cdots, m) \\
\text{where } \tilde{x}_j, \tilde{s}_i, \tilde{\lambda}_i, \tilde{\mu}_j, \tilde{a}_j \geq 0 \text{ and satisfying the slackness condition } \tilde{\lambda}_i \tilde{s}_i = 0 \text{ and } \tilde{\mu}_j \tilde{x}_j = 0. \]

Step 5: Solve this LPP by an alternative two-phase method. Choose greatest coefficient of decision variables.
(i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.
(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 6: Compute the ratio with \( \tilde{X}_B \). Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step 7: Use usual simplex method for this table and go to next step.

Step 8: Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure either an optimal solution is obtain or there is an indication of an unbounded solution.
Step 9: If all rows and columns are ignored, current solution is an optimal solution. Thus optimum solution is obtained and which is optimum solution of given FQPP also.

5 Numerical Example

In this section, examples are presented to illustrate the proposed method.

Example:

Consider the fully fuzzy quadratic programming problem as follows

\[
\begin{align*}
\min & \quad (0, 1, 2) \otimes (x_1) + (-1, 1, 3) \otimes (x_2) + (1, 1, 1) \otimes (s_1) = (-2, 3, 4) \\
\text{subject to } & \quad (0, 1, 2) \otimes (x_1) + (0, 1, 2) \otimes (x_1) + (1, 2, 3) \otimes (x_2) + (1, 1, 1) \otimes (s_2) = (-2, 3, 4), \\
& \quad (-4, -2, 4) \otimes (x_1) + (1, 2, 3) \otimes (x_2) + (1, 1, 1) \otimes (s_2) = (-2, 3, 4), \\
& \quad (0, 2, 4) \otimes (x_1) + (1, 1, 1) \otimes (x_2) \leq f (1, 3, 5), \\
& \quad \lambda_1, \lambda_2, \lambda_3 \geq 0, i = 1, 2, 3 \text{ satisfying the complementary slackness conditions are } \hat{\lambda}_1, \hat{s}_1 = \lambda_1, \hat{s}_2 = \lambda_2, \hat{s}_3 = \lambda_3, \hat{s}_4 = 0.
\end{align*}
\]

We want to solve the problem by using the proposed method.

Step 1: Now apply Kuhn-Tucker Conditions for the given FQPP, We get,

\[
\begin{align*}
& \quad \begin{pmatrix}
(0, 1, 2) \\
(-4, -2, 4)
\end{pmatrix} \otimes \begin{pmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{pmatrix} = \begin{pmatrix}
(0, 1, 2) \\
(-4, -2, 4)
\end{pmatrix}, \\
& \quad \begin{pmatrix}
(1, 2, 3) \\
(3, 6, 9)
\end{pmatrix} + \begin{pmatrix}
\hat{\mu}_1 \\
\hat{\mu}_2
\end{pmatrix} = \begin{pmatrix}
(1, 1, 2) \\
(-1, 1, 3)
\end{pmatrix}, \\
& \quad \begin{pmatrix}
(0, 1, 2) \\
(-4, -2, 4)
\end{pmatrix} + \begin{pmatrix}
\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3
\end{pmatrix} = \begin{pmatrix}
(0, 1, 2) \\
(-4, -2, 4)
\end{pmatrix}
\end{align*}
\]

Step 2: Now introducing the artificial variables \(\tilde{a}_1, \tilde{a}_2 \leq 0\) then the given FQPP is equivalent to:

Minimize \(Z = -(1, 1, 1)\tilde{a}_1 - (1, 1, 1)\tilde{a}_2\)

Subject to: \(0, 1, 2)\tilde{x}_1 + (-4, -2, 4)\tilde{x}_2 + (0, 1, 2)\hat{\lambda}_1 + (-4, -2, 4)\hat{\lambda}_2 + (0, 2, 4)\hat{\lambda}_3 - (1, 1, 1)\tilde{a}_1 = (1, 2, 3)

\((-4, -2, 4)\tilde{x}_1 + (-4, 3, 6)\tilde{x}_2 + (-1, 1, 3)\hat{\lambda}_1 + (1, 2, 3)\hat{\lambda}_2 + (1, 1, 1)\hat{\lambda}_3 - (1, 1, 1)\tilde{a}_2 = (3, 6, 9)

\((0, 1, 2)\tilde{x}_1 + (-1, 1, 3)\tilde{x}_2 + (1, 1, 1)\tilde{s}_1 = (-2, 3, 4)

\((-4, -2, 4)\tilde{x}_1 + (1, 2, 3)\tilde{x}_2 + (1, 1, 1)\tilde{s}_2 = (-2, 3, 4)

\]
(0, 2, 4)\ddot{x}_1 + (1, 1, 1)\ddot{x}_2 + (1, 1, 1)\ddot{s}_3 = (1, 3, 5)
where \ddot{x}_j, \ddot{\mu}_j, \ddot{a}_j, \ddot{\lambda}_i, \ddot{s}_i \geq 0, i = 1, 2, 3 and j = 1, 2.

Step 3: Now using definition (2.3) convert the above equations in to the parametric form we have,
Minimize \( Z = -(1, 0r, 0r)\ddot{a}_1 - (1, 0r, 0r)\ddot{a}_2 \)
Subject to: \( (1, 1 - r, 1 - r)\ddot{x}_1 + (-2, 2 - 2r, 6 - 6r)\ddot{x}_2 \)
\(+ (1, 1 - r, 1 - r)\ddot{\lambda}_1 + (-2, 2 - 2r, 6 - 6r)\ddot{\lambda}_2 \)
\(+ (2, 2 - 2r, 6 - 6r)\ddot{\lambda}_3 - (1, 0r, 0r)\ddot{\mu}_1 + (1, 0r, 0r)\ddot{a}_1 \)
\= (2, 1 - r, 1 - r) \)
\(- (2, 2 - 2r, 6 - 6r)\ddot{x}_1 + (3, 1 - 7r, 3 - 3r)\ddot{x}_2 + (1, 2 - 2r, 2 - 2r)\ddot{\lambda}_1 \)
\(+ (2, 1 - r, 1 - r)\ddot{\lambda}_2 + (1, 0r, 0r)\ddot{\lambda}_3 - (1, 0r, 0r)\ddot{\mu}_2 \)
\(+ (1, 0r, 0r)\ddot{a}_2 = (6, 3 - 3r, 3 - 3r) \)
\( (1, 1 - r, 1 - r)\ddot{x}_1 + (1, 2 - 2r, 2 - 2r)\ddot{x}_2 \)
\(+ (1, 0r, 0r)\ddot{s}_1 = (3, 5 - 5r, 1 - r) \)
\(- (2, 2 - 2r, 6 - 6r)\ddot{x}_1 + (2, 1 - r, 1 - r)\ddot{x}_2 + (1, 0r, 0r)\ddot{s}_2 \)
\= (3, 5 - 5r, 1 - r) \)
\( (2, 2 - 2r, 6 - 6r)\ddot{x}_1 + (1, 0r, 0r)\ddot{x}_2 + (1, 0r, 0r)\ddot{s}_3 \)
\= (3, 2 - 2r, 2 - 2r) \)
where \( \ddot{x}_j, \ddot{\mu}_j, \ddot{a}_j, \ddot{\lambda}_i, \ddot{s}_i \geq 0, i = 1, 2, 3 and j = 1, 2. \)

Solve the above equation by using Two-Phase simplex method we get the optimal solution.

Therefore, the optimal solution of the above problem is
\( \ddot{x}_1 = \begin{pmatrix} -161 \\ 36 \\ + 5r \\ 19 \\ 36 \\ 235 \\ 36 \\ 6r \end{pmatrix}, \ddot{x}_2 = \begin{pmatrix} -37 \\ 12 \\ + 5r \\ 23 \\ 12 \\ 95 \\ 12 \\ 6r \end{pmatrix}, \)
\min z = \begin{pmatrix} -431 \\ 81 \\ + 6r \\ 53 \\ 81 \\ 458 \\ 81 \\ 5r \end{pmatrix} \).

Therefore the fuzzy optimal solution of the given fuzzy triangular problem for different values of \( r \) is

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \ddot{x}_1 )</th>
<th>( \ddot{x}_2 )</th>
<th>( \ddot{z} )</th>
</tr>
</thead>
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<td>0</td>
<td>\begin{pmatrix} -161 \ 36 \ 19 \ 36 \ 235 \ 36 \ 6r \end{pmatrix}</td>
<td>\begin{pmatrix} -37 \ 12 \ 23 \ 12 \ 95 \ 12 \ 6r \end{pmatrix}</td>
<td>\begin{pmatrix} -431 \ 81 \ 53 \ 458 \ 81 \ 5r \end{pmatrix}</td>
</tr>
<tr>
<td>0.5</td>
<td>\begin{pmatrix} -71 \ 36 \ 19 \ 36 \ 127 \ 36 \ 6r \end{pmatrix}</td>
<td>\begin{pmatrix} -7 \ 12 \ 23 \ 12 \ 59 \ 12 \ 6r \end{pmatrix}</td>
<td>\begin{pmatrix} -188 \ 81 \ 53 \ 411 \ 81 \ 5r \end{pmatrix}</td>
</tr>
<tr>
<td>1</td>
<td>\begin{pmatrix} 19 \ 36 \ 19 \ 36 \ 19 \ 36 \end{pmatrix}</td>
<td>\begin{pmatrix} 23 \ 12 \ 23 \ 12 \ 23 \ 12 \end{pmatrix}</td>
<td>\begin{pmatrix} 53 \ 81 \ 53 \ 81 \end{pmatrix}</td>
</tr>
</tbody>
</table>
6 Conclusion

In this paper, we investigated the solution method of fully fuzzy quadratic programming problem with triangular fuzzy number. We applied the fuzzy version of modified simplex method for the fuzzy optimal solution of the fuzzy quadratic programming problem. The numerical example discussed by Behrouz et al [?] is solved using by the proposed method without converting the given problem to crisp equivalent problem. It is to be noted that the decision maker have the flexibility of choosing $r \in [0, 1]$ depending upon the situation and his wish by the applying the proposed method.

References


