Solution to a Multi-objective Fuzzy Transportation Problem - A New Approach

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Abstract

The main objective of this paper is to obtain a properly efficient solution for the fuzzy multi-objective transportation problem. A bi-objective fuzzy transportation problem is illustrated and solved based on weighted average without converting to a classical one.

AMS Subject Classification: 94D, 90B.

Key Words and Phrases: Fuzzy multi-objective transportation problem, Triangular fuzzy number, Ranking function.

1 Introduction

The transportation problem is one of the earliest applications of linear programming problems. The objective function is to minimize total transportation costs and satisfy the destination requirements within the source availability [8]. Within a given time period each shipping source has a certain capacity and each destination has certain requirements with a given cost of shipping from the source to the destination. In order to solve a transportation
problem, the decision parameters of the problem must be precise. However, in real life situations, the information available is of imprecise nature and there is an inherent degree of vagueness or uncertainty present in the problem under consideration. In order to tackle this uncertainty the concept of fuzzy sets can be used as an important decision making tool. These imprecise data may be represented by fuzzy numbers.


Here, we solve the multi-objective fuzzy transportation problem based on weighted average without converting it to a classical one. In the next section, we give the basic concepts of triangular fuzzy number and their arithmetic operations. In section 3, we introduce the fuzzy multi-objective transportation problem with cost co-efficients, time co-efficients, supplies and demands as triangular fuzzy numbers. In section 4, the algorithm for obtaining the properly efficient solution for the bi-objective fuzzy transportation problem is proposed. The triangular fuzzy numbers are represented in its parametric form and the bi-objective problem is solved without converting to a classical one using our arithmetic operations and ranking function. A numerical example is also given to illustrate the same.
2 Preliminaries

Definition 2.1. A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number, if its membership function $\mu_{\tilde{A}} : \mathbb{R} \to [0, 1]$ has the following characteristics:

(i) $\mu_{\tilde{A}}$ is convex.

(ii) $\mu_{\tilde{A}}$ is normal, (i.e.) there exists an $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$

(iii) $\tilde{A}$ is upper semi-continuous.

(iv) $\sup(\tilde{A})$ is bounded in $\mathbb{R}$.

Definition 2.2. A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$ where $a_1, a_2, a_3$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below:

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
$$

We use $F(\mathbb{R})$ to denote the set of all fuzzy numbers defined on $\mathbb{R}$.

Definition 2.3. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) \in F(\mathbb{R})$ can also be represented as a pair $\tilde{A} = (\varrho, \varpi)$ of function $\varrho(r), \varpi(r)$ for $0 \leq r \leq 1$ which satisfies the following requirements:

(i). $\varrho(r)$ is a bounded monotonic increasing left continuous function.

(ii). $\varpi(r)$ is a bounded monotonic decreasing left continuous function.

(iii). $\varrho(r) \leq \varpi(r), 0 \leq r \leq 1$. 

Definition 2.4. For an arbitrary triangular fuzzy number \( \tilde{A} = (a, a) \), the number \( a_0 = \frac{(a(1) + \pi(1))}{2} \) is said to be a location index number of \( \tilde{A} \).

The two non-decreasing left continuous functions \( a_* = (a_0 - a), a^* = (\pi - a_0) \) are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number \( \tilde{A} = (a_1, a_2, a_3) \) can also be represented by \( \tilde{A} = (a_0, a_*, a^*) \).

2.1 Ranking of Triangular Fuzzy Numbers

For every \( \tilde{A} = (a_1, a_2, a_3) \in F(R) \), the ranking function \( R : F(R) \rightarrow R \) by graded mean is defined as \( R(\tilde{A}) = \frac{a^* + 4a_0 + a_*}{6} \).

For any two triangular fuzzy numbers \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \) in \( F(R) \), we have the following comparison:

(i). \( \tilde{A} \succ \tilde{B} \) if and only if \( R(\tilde{A}) \succ R(\tilde{B}) \)
(ii). \( \tilde{A} \prec \tilde{B} \) if and only if \( R(\tilde{A}) \prec R(\tilde{B}) \)
(iii). \( \tilde{A} \approx \tilde{B} \) if and only if \( R(\tilde{A}) = R(\tilde{B}) \)
(iv). \( \tilde{A} - \tilde{B} \approx \tilde{0} \) if and only if \( R(\tilde{A}) - R(\tilde{B}) = 0 \)

2.2 Arithmetic Operations

In particular for any two fuzzy numbers \( \tilde{A} = (a_0, a_*, a^*) \) and \( \tilde{B} = (b_0, b_*, b^*) \), we define

(i). Addition: \( \tilde{A} + \tilde{B} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)
(ii). Subtraction: \( \tilde{A} - \tilde{B} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \min\{a_*, b_*\}, \min\{a^*, b^*\}) \)
(iii). Multiplication: \( \tilde{A} \times \tilde{B} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}) \)
(iv). Division: \( \tilde{A} \div \tilde{B} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \min\{a_*, b_*\}, \min\{a^*, b^*\}) \)
3 Fuzzy Multi-objective Transportation Problem

3.1 Mathematical formulation of Fuzzy Transportation Problem

(P) Minimize \( \tilde{Z}_k = \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{p}_{ij}) \tilde{x}_{ij} \)
subject to \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \ i = 1, 2, \ldots, m \)
\( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \ j = 1, 2, \ldots, n \)

Consider a fuzzy transportation with \( m \) sources and \( n \) destinations in which all the decision parameters are trapezoidal fuzzy numbers. The cost of shipping of an item from each of \( m \) sources to each of \( n \) destinations \([n \text{ may not be equal to } m]\) are known either directly or indirectly. If the objective of the transportation problem is to minimize fuzzy cost and fuzzy time then it treated as a fuzzy multi-objective transportation problem.

Table 1: Fuzzy multi-objective transportation problem model with fuzzy cost \( \tilde{c}_{ij} \) and fuzzy time \( \tilde{t}_{ij} \).  

<table>
<thead>
<tr>
<th>Sources</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>\cdots</th>
<th>( D_n )</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>( \tilde{c}<em>{11} \tilde{t}</em>{11} )</td>
<td>( \tilde{c}<em>{12} \tilde{t}</em>{12} )</td>
<td>\cdots</td>
<td>( \tilde{c}<em>{1n} \tilde{t}</em>{1n} )</td>
<td>( \tilde{a}_1 )</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>( \tilde{c}<em>{21} \tilde{t}</em>{21} )</td>
<td>( \tilde{c}<em>{22} \tilde{t}</em>{22} )</td>
<td>\cdots</td>
<td>( \tilde{c}<em>{2n} \tilde{t}</em>{2n} )</td>
<td>( \tilde{a}_2 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\cdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>( S_m )</td>
<td>( \tilde{c}<em>{m1} \tilde{t}</em>{m1} )</td>
<td>( \tilde{c}<em>{m2} \tilde{t}</em>{m2} )</td>
<td>\cdots</td>
<td>( \tilde{c}<em>{mn} \tilde{t}</em>{mn} )</td>
<td>( \tilde{a}_m )</td>
</tr>
<tr>
<td>Demand</td>
<td>( b_1 )</td>
<td>( b_2 )</td>
<td>\cdots</td>
<td>( b_n )</td>
<td></td>
</tr>
</tbody>
</table>

Definition 3.1. A feasible point \( \tilde{x}^o \) is said to be efficient for (P) if there exists no other feasible point \( \tilde{x} \) in (P) \( \ni \)
\( \tilde{Z}_i(\tilde{x}) \leq \tilde{Z}_i(\tilde{x}^o), \ i = 1 \text{ to } k \) and \( \tilde{Z}_r < \tilde{Z}_r(\tilde{x}^o), \) for some \( r \in \{1, 2, 3, \cdots, k\} \)

Definition 3.2. An efficient solution \( \tilde{x}^o \) is said to be a fair (properly efficient) solution if \( \ni \) a scalar \( M > 0 \) \( \ni \) for each \( i \in \{1, 2, 3, \cdots, k\} \) and
for all feasible $\tilde{x}$ of (P) satisfying $\tilde{Z}_i(\tilde{x}) < \tilde{Z}_i(\tilde{x}^\circ)$, we have $\tilde{Z}_i(\tilde{x}^\circ) - \tilde{Z}_i(\tilde{x}) \leq M(\tilde{Z}_r(\tilde{x}) - \tilde{Z}_r(\tilde{x}^\circ))$ for some $r \ni \tilde{Z}_r(\tilde{x}) > \tilde{Z}_r(\tilde{x}^\circ)$

**Theorem 3.2.1.** If $\tilde{x}^\circ$ is an optimal solution for the scalar programming problem $(P_1)$, $\min \sum_{i=1}^{k} \tilde{Z}_k(\tilde{x}), \tilde{x} \in X$ then $\tilde{x}^\circ$ is the properly efficient solution of (P) [7].

### 3.2 Proposed Algorithm

**Step 1** Construct the fuzzy transportation table for the given fuzzy transportation problem (P) and balance it if not.

**Step 2** Represent all the triangular fuzzy numbers in its parametric form.

**Step 3** We assign weights $w_1$ and $w_2$ for the objectives and these weights determine the relative importance of each quantity on the average.

**Step 4** Form the single objective optimization problem $(P_1)$ for the given multi-objective fuzzy transportation problem using weighted average method.

**Step 5** Solve the problem $(P_1)$ using the fuzzy version of modified Vogel’s approximation method to get the optimal solution $\tilde{x}^\circ$ for $P_1$ [5].

**Step 6** Obtain the properly efficient solution $\tilde{x}^\circ$ for the problem (P).

### 4 Numerical Example

Consider the following bi-objective fuzzy transportation problem where the cost coefficients, time coefficients, demand and supply are triangular fuzzy numbers. We assign $w_1 = 3$ and $w_2 = 2$ for the objectives $\tilde{Z}_1$ and $\tilde{Z}_2$ respectively.
Table 2- Bi-objective fuzzy transportation problem

<table>
<thead>
<tr>
<th>Sources</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>(0.6,0.8,0.9), (1,1.2,1.4)</td>
<td>(2.4,3.5,4.6), (1.5,2.1,2.8)</td>
<td>(1.8,2.1,2.3), (1,1.4,1.6)</td>
<td>(1.5, 1.8, 2), (0.7, 0.9, 1)</td>
<td>(17,2, 18, 19.2)</td>
</tr>
<tr>
<td>$S_2$</td>
<td>(1.1, 1.3, 1.4), (0.4, 0.6, 0.7)</td>
<td>(3.1, 3.6, 4.1), (1.5, 1.8, 2.1)</td>
<td>(1.4, 1.6, 1.7), (0.8, 0.9, 1.1)</td>
<td>(2.2, 2.5, 2.7), (1, 1.4, 1.8)</td>
<td>(22.6, 24, 25.8)</td>
</tr>
<tr>
<td>$S_3$</td>
<td>(1.5, 1.8, 2), (0.6, 0.9, 1.1)</td>
<td>(3.1, 3.5, 3.9), (1.4, 1.8, 2.2)</td>
<td>(21, 2.4, 2.7), (13, 1.5, 1.7)</td>
<td>(0.8, 1.1, 1), (0.2, 0.3, 0.4)</td>
<td>(12.6, 13, 13.6)</td>
</tr>
<tr>
<td>Demand</td>
<td>(11.2, 12, 12.6)</td>
<td>(11.2, 12, 12.6)</td>
<td>(5.6, 6, 6.4)</td>
<td>(14.8, 16, 16.8)</td>
<td>(19, 20, 20.8)</td>
</tr>
</tbody>
</table>

Table 3- Single objective fuzzy transportation problem ($P_1$)

<table>
<thead>
<tr>
<th>Sources</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>(0.6,0.12-0.12,0.06-0.06)</td>
<td>(2.48-3.6-3.6,0.3-0.3)</td>
<td>(1.56.0-18.0-18.0,0.12-0.12)</td>
<td>(1.44-0.18-0.18,0.12-0.12)</td>
<td>(18,9.6-0.8-0.8,1.2-1.2)</td>
</tr>
<tr>
<td>$S_2$</td>
<td>(1.92-0.12-0.12,0.06-0.06)</td>
<td>(2.88-3.3-3.3,0.3-0.3)</td>
<td>(1.32-0.12-0.12,0.08-0.08)</td>
<td>(2.08-0.18-0.18,0.16-0.16)</td>
<td>(24.1-1.4-1.4,1.8-1.8)</td>
</tr>
<tr>
<td>$S_3$</td>
<td>(1.44-0.18-0.18,0.12-0.12)</td>
<td>(2.82-2.4-2.4,0.24-0.24)</td>
<td>(2.04-0.18-0.18,0.18-0.18)</td>
<td>(0.72-0.12-0.12,0.06-0.06)</td>
<td>(12,0-0.4-0.4,0.6-0.6)</td>
</tr>
<tr>
<td>Demand</td>
<td>(12,0.8-0.8,0.6-0.6)</td>
<td>(0.6-0.6,0.4-0.4,0.4-0.4)</td>
<td>(16.1-2.1,2.1,2.1,0.8-0.8)</td>
<td>(20,1-1,0.8-0.8,0.8)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 The fuzzy transportation table after giving the allocations $z_{ij}$

<table>
<thead>
<tr>
<th>Sources</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>(1.5, 1.5, 1.5), (0.5, 0.5, 0.5), (0.6, 0.6, 0.6)</td>
<td>(1.5, 1.5, 1.5), (0.5, 0.5, 0.5), (0.6, 0.6, 0.6)</td>
<td>(2.0, 2.0, 2.0), (1.1, 1.1, 1.1)</td>
<td>(0.9, 0.9, 0.9), (0.4, 0.4, 0.4)</td>
<td>0</td>
</tr>
<tr>
<td>$S_2$</td>
<td>(1.3, 1.3, 1.3), (0.6, 0.6, 0.6), (0.1, 0.1, 0.1)</td>
<td>(1.6, 1.6, 1.6), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)</td>
<td>(2.0, 2.0, 2.0), (1.1, 1.1, 1.1)</td>
<td>(0.9, 0.9, 0.9), (0.4, 0.4, 0.4)</td>
<td>0</td>
</tr>
<tr>
<td>$S_3$</td>
<td>(1.8, 1.8, 1.8), (0.9, 0.9, 0.9), (0.2, 0.2, 0.2)</td>
<td>(1.5, 1.5, 1.5), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4)</td>
<td>(2.0, 2.0, 2.0), (1.1, 1.1, 1.1)</td>
<td>(0.9, 0.9, 0.9), (0.4, 0.4, 0.4)</td>
<td>0</td>
</tr>
</tbody>
</table>

From the above table, the objective values $\tilde{Z}_1$ and $\tilde{Z}_2$ for the bi-objective fuzzy transportation problem is $(78.54,1.2-1.2a,0.8-0.8a)$; $(39.3,1.2-1.2a,0.8-0.8a)$ where $0 \leq \alpha \leq 1$ can be suitably chosen by the decision maker.

The optimal solution of the single objective fuzzy transportation problem ($P_1$) is the fair solution of the bi-objective fuzzy transportation problem ($P$).

5 Conclusion

In our approach, the triangular fuzzy numbers are represented in terms of location index and fuzziness index functions and a fair solution is obtained in its parametric form. The bi-objective problem is solved in an easier way and the decision maker can suitably choose the value of $\alpha$. 7
References


