SOLUTION OF SOLID TRANSPORTATION PROBLEM IN FUZZY APPROACH

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Abstract

This paper develops a new algorithm for finding an optimal solution for solid transportation problem using α-cut under imprecise environment. The procedure of the proposed method is illustrated by numerical example.

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Keywords: Fuzzy solid transportation problem, Triangular fuzzy numbers, α-cut.

1 Introduction

The STP is a three dimensional problem in which the parameters are supply, demand and conveyance (mode of transport). In many industrial problems, a homogeneous commodity is delivered from a transportation origin to a transportation destination by means of conveyances, such as trucks, cargo flights, goods trains, ships, etc. The STP was introduced by Shell in[1]. Haley [2] proposed a
new algorithm for finding an STP which is based on modify distribution method it needs $m+n+l-2$ nonzero values of the decision variables to start with a basic feasible solution. Pandian and Anuradha [3] and Anuradha [4] proposed a new algorithm namely, min zero-min cost method for finding an optimal solution of STP. Radhakrishnan and Anukokila [5] investigated a FSTP with interval cost by adopting fractional goal programming technique. Nithishkumar and Dutta [6] proposed and solved the multi-objective STP with fuzzy coefficients for the objectives and constraints. Dutta and Nithish Kumar [7] presented a rough interval approach and sensitivity analysis. The primary focus of this paper is to find an optimal solution for solid transportation problem under imprecise environment. Section 2 and 3 projects the basics and proposed algorithm. In section 4 numerical example is chosen to illustrate the proposed algorithm. Finally the paper is concluded in section 5.

2 Preliminaries

We need the following basic definitions which can be found in [[8],[9]]

Definition 1. Let $A$ be a classical set and $\mu_A(x)$ be a membership function from $A$ to $[0, 1]$. A fuzzy set $A$ with the membership function $\mu_A(x)$ is defined by

$A = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0, 1]\}$

Definition 2. A Fuzzy set $A$ defined on the set of real numbers $R$ is said to be a fuzzy number if its membership function has the following conditions:

(i) $\mu_A(x) : R \to [0, 1]$ is continuous

(ii) $\mu_A(x) = 0$ for all $(-\infty, a] \cup (c, \infty)$

(iii) $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[a, b]$.

(iv) $\mu_A(x) = 1$ for all $x \in b$ where $a \preceq b \preceq c$.

Definition 3. A linear membership function can be defined
as

$$\mu_{x_i}(X) = \begin{cases} 
0 & \text{if } x_{ijk} \prec x_{ijk} \\
(\overline{x}_{ijk} - x_{ijk})/(\overline{x}_{ijk} - \underline{x}_{ijk}) & \text{if } x_{ijk} \prec x_{ijk} \prec \overline{x}_{ijk} \\
1 & \text{if } x_{ijk} \succ \overline{x}_{ijk} 
\end{cases}$$

To transform the fuzzy system to crisp set, $\alpha$-cut for the linear membership function is written below.

For all $\alpha \in [0, 1]$, $$(\overline{x}_{ijk} - x_{ijk})/(\overline{x}_{ijk} - \underline{x}_{ijk}) = \alpha$$

such that $x_{ijk} = (1 - \alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}$

3 Fuzzy Solid Transportation Problem (FSTP)

A Mathematical model of FSTP is

(P) Minimize $\hat{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \hat{c}_{ijk} \hat{x}_{ijk}$

Subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{l} \hat{x}_{ijk} = \overline{a}_i, \quad i = 1, 2, ..., m$$
$$\sum_{i=1}^{m} \sum_{k=1}^{l} \hat{x}_{ijk} = \overline{b}_j, \quad j = 1, 2, ..., n$$
$$\sum_{i=1}^{m} \sum_{j=1}^{n} \hat{x}_{ijk} = \overline{e}_k, \quad k = 1, 2, ..., l$$
$$\hat{x}_{ijk} \geq 0, \text{ for all } i, j \text{ and } k$$

where $\hat{c}_{ijk}$ is the fuzzy transportation cost from origin $i$ to the destination $j$ by means of the conveyance $k$ and $\hat{x}_{ijk}$ is the fuzzy number of units transported from origin $i$ to destination $j$ by means of the $k$th conveyance. $\overline{a}_i$ is the amount of material available at origin, $\overline{b}_j$ is the amount of the material required at destination and $\overline{e}_k$ is the amount of material shipped by conveyance $k$.

3.1 FSTP using $\alpha$-cut

A mathematical model of FSTP using $\alpha$-cut is

(P1) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \hat{c}_{ijk}((1 - \alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk})$, $0 < \alpha < 1$

Subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{l}((1 - \alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}) = (1 - \alpha)\overline{a}_i + \alpha\underline{a}_i, \quad i = 1, 2, ..., m$$
$$\sum_{i=1}^{m} \sum_{k=1}^{l}((1 - \alpha)\overline{x}_{ijk} + \alpha\underline{x}_{ijk}) = (1 - \alpha)\overline{b}_j + \alpha\underline{b}_j, \quad j = 1, 2, ..., n$$

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A heuristic algorithm for finding an optimal solution to a fuzzy solid transportation problem proceeds as follows:

Step 1: Construct the upper bound problem (UBP) by substituting $\alpha = 0$ of the problem (P1).
Step 2: Check the UBP is balanced. If not, convert the problem as a balanced one.
Step 3: Subtract each data of the supply/demand/conveyance of the given table by its least value.
Step 4: Check if there is a possibility to allot each supply/demand/conveyance with the corresponding demands/conveyance/supply (respectively) using the cells having zero cost, then go to step 6. If not, go to step 5.
Step 5: Draw the minimum number of horizontal and vertical line crossing all zeros of the reduced table. Select minimum value from the uncovered reduced table and then subtract that value from the uncovered elements and then add that minimum value on the line which is intersected by two lines. Then, go to step 4.
Step 6: Select a supply/demand/conveyance having least number of zeros of the reduced table. Then, assign maximum possible to the zero cell having least original cost. If more than one occurs, select any one.
Step 7: Reform the reduced table after removing fully used supply and conveyance points and the fully received demand points or modifying not fully used supply and conveyance points, fully received demand points.
Step 8: Repeat step 6 and step 7 until all supply and conveyance points are fully used and all demand points are fully received.
Step 9: This allotment yields an optimal solution to the UB problem ($\bar{x}_{ijk}$).
Step 10: Construct the lower bound problem (LBP) by substituting $\alpha = 1$ of the problem (P1).
Step 11: For finding the optimal solution to LB Problem ($\bar{x}_{ijk}$), repeat the steps 2 to step 9.
Step 12: Obtain the optimal solution of the (P) by average of the ($\bar{x}_{ijk}$) and ($\bar{x}_{ijk}$) of the solution. Also by taking the average of the point we get the point where optimal solution will exist.
4 Illustrative Example

Consider a fuzzy solid transportation problem with three sources, three destinations and three types of conveyances. The values of the corresponding sources, destinations, maximum amount to be transported by a particular conveyance and the unit cost of transportation amount from source $i$ to destination $j$ by conveyance $k$ is given in the following table.

<table>
<thead>
<tr>
<th>Convey</th>
<th>E1</th>
<th>E2</th>
<th>E1</th>
<th>E2</th>
<th>E1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tilde{c}_{11}$</td>
<td>$\tilde{c}_{12}$</td>
<td>$\tilde{c}_{13}$</td>
<td>$\tilde{c}_{21}$</td>
<td>$\tilde{c}_{22}$</td>
</tr>
<tr>
<td>D1</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
<td>$\bar{a}_3$</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
</tr>
<tr>
<td>D2</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
<td>$\bar{a}_3$</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
</tr>
<tr>
<td>D3</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
<td>$\bar{a}_3$</td>
<td>$\bar{a}_1$</td>
<td>$\bar{a}_2$</td>
</tr>
</tbody>
</table>

where $\tilde{c}_{11}=(3, 4, 5); \tilde{c}_{12}=(6, 7, 8); \tilde{c}_{13}=(7, 8, 9); \tilde{c}_{21}=(2, 3, 4); \tilde{c}_{22}=(8, 9, 10); \tilde{c}_{23}=(6, 7, 8); \tilde{c}_{31}=(5, 6, 7); \tilde{c}_{32}=(6, 7, 8); \tilde{c}_{33}=(1, 2, 3); \tilde{c}_{21}=(3, 4, 5); \tilde{c}_{22}=(1, 2, 3); \tilde{c}_{23}=(5, 6, 7); \tilde{c}_{22}=(0, 1, 2); \tilde{c}_{23}=(2, 3, 4); \tilde{c}_{22}=(7, 8, 9); \tilde{c}_{23}=(7, 8, 9); \tilde{c}_{23}=(4, 5, 6); \tilde{c}_{31}=(7, 8, 9); \tilde{c}_{32}=(3, 4, 5); \tilde{c}_{33}=(0, 1, 2); \tilde{c}_{32}=(3, 4, 5); \tilde{c}_{33}=(6, 7, 8); \tilde{c}_{33}=(2, 3, 4); \tilde{c}_{31}=(4, 5, 6); \tilde{c}_{32}=(5, 6, 7); \tilde{c}_{33}=(3, 4, 5); \bar{a}_1=(10, 11, 12); \bar{a}_2=(12, 13, 14); \bar{a}_3=(9, 10, 11); b_1=(5, 6, 7); b_2=(15, 16, 17); b_3=(11, 12, 13); \bar{e}_1=(10, 11, 12); \bar{e}_2=(13, 14, 15); \bar{e}_3=(8, 9, 10). 

Now, using equation (2), $\alpha$-cut for the costs $\tilde{c}_{ijk}$, supply $a_i$, demand $b_j$ and conveyance $\bar{e}_k$ to the given FSTP is given below.

$(3, 4, 5)=(1-\alpha)5\bar{e}_{11}+3\alpha\bar{e}_{111}; (6, 7, 8)=(1-\alpha)8\bar{e}_{12}+6\alpha\bar{e}_{112}; (7, 8, 9)=\alpha\bar{e}_{13}+7\alpha\bar{e}_{113}; (2, 3, 4)=(1-\alpha)4\bar{e}_{21}+2\alpha\bar{e}_{211}; (8, 9, 10)=(1-\alpha)10\bar{e}_{22}+8\alpha\bar{e}_{222}; (6, 7, 8)=(1-\alpha)8\bar{e}_{31}+6\alpha\bar{e}_{312}; (5, 6, 7)=\alpha\bar{e}_{32}+5\alpha\bar{e}_{321}; (6, 7, 8)=(1-\alpha)8\bar{e}_{32}+6\alpha\bar{e}_{322}; (1, 2, 3)=\alpha\bar{e}_{33}+\alpha\bar{e}_{333}; (3, 4, 5)=(1-\alpha)5\bar{e}_{22}+3\alpha\bar{e}_{222}; (1, 2, 3)=\alpha\bar{e}_{31}+2\alpha\bar{e}_{312}; (5, 6, 7)=(1-\alpha)\bar{e}_{31}+5\alpha\bar{e}_{313}; (0, 1, 2)=\alpha\bar{e}_{32}+2\alpha\bar{e}_{321}; (7, 8, 9)=\alpha\bar{e}_{33}+7\alpha\bar{e}_{333}; (7, 8, 9)=(1-\alpha)9\bar{e}_{23}+7\alpha\bar{e}_{233}; (3, 4, 5)=(1-\alpha)5\bar{e}_{22}+3\alpha\bar{e}_{222}; (4, 5, 6)=(1-\alpha)6\bar{e}_{33}+4\alpha\bar{e}_{333}; (7, 8, 9)=\alpha\bar{e}_{33}+7\alpha\bar{e}_{333}; (0, 1, 2)=(1-\alpha)2\bar{e}_{312}+6\alpha\bar{e}_{312}; (2, 3, 4)=\alpha\bar{e}_{33}+7\alpha\bar{e}_{333}; (8, 9, 10).
Now, using step 1 the UB problem to the given FSTP is given below.

\[ (1 - \alpha)4\bar{x}_{313} + 2\alpha\bar{x}_{313},(3, 4, 5) = (1 - \alpha)5\bar{x}_{321} + 3\alpha\bar{x}_{321},(6, 7, 8) = \]
\[ (1 - \alpha)8\bar{x}_{322} + 6\alpha\bar{x}_{322},(2, 3, 4) = (1 - \alpha)4\bar{x}_{323} + 2\alpha\bar{x}_{323},(4, 5, 6) = \]
\[ (1 - \alpha)6\bar{x}_{331} + 4\alpha\bar{x}_{331},(5, 6, 7) = (1 - \alpha)7\bar{x}_{332} + 5\alpha\bar{x}_{332},(3, 4, 5) = \]
\[ (1 - \alpha)5\bar{x}_{333} + 3\alpha\bar{x}_{333},(10, 11, 12) = (1 - \alpha)12\bar{a}_1 + 10\alpha\bar{a}_1,(12, 13, 14) = \]
\[ (1 - \alpha)14\bar{a}_2 + 12\alpha\bar{a}_2,(9, 10, 11) = (1 - \alpha)11\bar{a}_3 + 9\alpha\bar{a}_3,(5, 6, 7) = \]
\[ (1 - \alpha)7\bar{b}_1 + 5\alpha\bar{b}_1,(15, 16, 17) = (1 - \alpha)17\bar{b}_2 + 15\alpha\bar{b}_2,(11, 12, 13) = \]
\[ (1 - \alpha)13\bar{b}_3 + 11\alpha\bar{b}_3,(10, 11, 12) = (1 - \alpha)12\bar{e}_1 + 10\alpha\bar{e}_1,(13, 14, 15) = \]
\[ (1 - \alpha)15\bar{e}_2 + 13\alpha\bar{e}_2,(8, 9, 10) = (1 - \alpha)10\bar{e}_3 + 8\alpha\bar{e}_3. \]

Now, using step 1 the UB problem to the given FSTP is given below.

<table>
<thead>
<tr>
<th>Convey</th>
<th>E1</th>
<th>E2</th>
<th>E1</th>
<th>E3</th>
<th>E2</th>
<th>E3</th>
<th>E2</th>
<th>E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>12</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O1</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>O2</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>O3</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Demand</td>
<td>7</td>
<td>17</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, using the Step 2 to the Step 9, we obtain the following optimum allotment table.

<table>
<thead>
<tr>
<th>Convey</th>
<th>E1</th>
<th>E1</th>
<th>E1</th>
<th>E3</th>
<th>E2</th>
<th>E3</th>
<th>E2</th>
<th>E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>12</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O1</td>
<td>6</td>
<td>7</td>
<td>11</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>O2</td>
<td>8</td>
<td>4</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>O3</td>
<td>9</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Demand</td>
<td>7</td>
<td>17</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, using the Step 2 to the Step 9, we obtain the following optimum allotment table.

The optimal solution to the above reduced UB problem is \( \bar{x}_{121} = 2, \bar{x}_{132} = 10, \bar{x}_{221} = 9, \bar{x}_{222} = 5, \bar{x}_{312} = 7, \bar{x}_{321} = 1, \bar{x}_{322} = 3 \) with the total minimum transportation cost 116.

Now, using step 10 the LB problem to the given FSTP is given below.
Now, using the Step 2 to the Step 9, we obtain the following optimum allottment table.

<table>
<thead>
<tr>
<th>Convey</th>
<th>E1</th>
<th>E1</th>
<th>E1</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E2</td>
<td>E2</td>
<td>E2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>E3</td>
<td>E3</td>
<td>13</td>
</tr>
<tr>
<td>D1</td>
<td>D2</td>
<td>D3</td>
<td>Supply</td>
<td></td>
</tr>
<tr>
<td>O1</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>O2</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>O3</td>
<td>7</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Demand</td>
<td>5</td>
<td>15</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

The optimal solution to the above reduced LB problem is $\mathbf{x_{121}}=2$, $\mathbf{x_{133}}=8$, $\mathbf{x_{221}}=7$, $\mathbf{x_{222}}=5$, $\mathbf{x_{312}}=5$, $\mathbf{x_{321}}=1$, $\mathbf{x_{332}}=3$ with the total minimum transportation cost 40.

Now, using the step 12 the optimal solution of the problem $(P)$ occurs at $(2(2+2)/2, (8+10)/2, (5+5)/2, (5+7)/2, (1+1)/2, (3+3)/2) = (2, 9, 5, 6, 1, 3)$.

### 5 Conclusion

In this paper, we have presented a new algorithm of solid transportation problem under imprecise environments. The necessity of this problem arises when heterogeneous conveyances are accessible for shipment of commodities in public transportation system. This procedure can help the decision makers in logistics related issues real life problems.
References


