We introduce \( \mu \)-fuzzy open and \( r \)-fuzzy \( \mu \)-closed sets in fuzzy topological spaces in the sense of \( \hat{\alpha} \)-ostak's. We investigate some of their properties.

AMS Subject Classification: \( r \)-fuzzy \( \mu \)-semiopen, \( r \)-fuzzy \( \mu \)-preopen, \( r \)-fuzzy semi-preopen, \( \mu \)-fuzzy \( \mu \)-open and \( \mu \)-fuzzy \( \mu \)-open.

Key Words and Phrases: 54A40.

1 Introduction and preliminaries
\( \hat{\alpha} \)-ostak [14] introduced the fuzzy topology as an extension of C hang's fuzzy topology [2]. It has been developed in many directions [4, 5, 13]. Ganguly and Saha [3] introduced the notions of fuzzy \( \mu \)-cluster points in fuzzy topological spaces in the sense of Chang [2]. Kim and Park [7] introduced \( r \)-\( \mu \)-cluster points and \( \mu \)-closure operators in fuzzy topological spaces in view of the definition of \( \hat{\alpha} \)-ostak. It is a good extension of the notions of Ganguly and Saha [3]. Park et al. [10] introduced the concept of fuzzy semi-preopen sets which is weaker than any of the concepts of fuzzy semi-open or fuzzy preopen sets. In this paper, we define \( r \)-fuzzy \( e \)-open and \( r \)-fuzzy \( e \)-closed sets in a fuzzy topological space in the sense of \( \hat{\alpha} \)-ostak [14]. Throughout this paper, nonempty sets will be denoted by \( \mathbb{A}, \mathbb{B} \) etc., \( \mathbb{C} = [0, 1] \) and \( \mathbb{C}_0 = (0, 1] \). For \( \mu \in \mathbb{C} \), \( \mu(\mathbb{A}) = \mu \) for all \( \mathbb{A} \in \mathbb{B} \). A fuzzy point \( \mathbb{A}/\mathbb{B} \) for \( \mathbb{B} \in \mathbb{C}_0 \) is an element of \( \mathbb{C}/\mathbb{B} \) such that

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Remark 3. Let \((\mathcal{L}, \mathcal{F})\) be a fuzzy topology on \(\mathcal{X}\). Then for each \(A \subseteq \mathcal{X}\), \(A^0\) is called r-open (resp. r-fuzzy preopen) if for every \(B \subseteq \mathcal{X}\) with \(B \subseteq A \), \(B \subseteq A^0\) (resp. \(B \subseteq A^0\)).

The set of all fuzzy points in \(\mathcal{X}\) is called \(\mathcal{X}\) as a fuzzy topology.

Definition 2. Let \((\mathcal{L}, \mathcal{F})\) be a fuzzy topology on \(\mathcal{X}\). Then for each \(A \subseteq \mathcal{X}\), \(A\) is called r-cluster point of \(A\) if for every \(B \subseteq \mathcal{X}\) with \(A \subseteq B\), \(A \cap B \neq \emptyset\).
is called an r-fuzzy semi-preopen (resp. r-fuzzy semi-preclosed) \([11]\) set if \(A \leq B\), \(A \in C_0\), \(C_1\), \(C_2\) and \(C_3\).

From the above definitions it is clear that the following implications are true for \(A \in C_0\).

3. Let \((A, B, C_0, C_1, C_2, C_3)\) be a fuzzy set. Then \(A\) is called an r-fuzzy strongly semi-open (resp. r-fuzzy strongly semi-closed) set if \(A \leq B\) is r-fuzzy strongly semi-preopen (resp. r-fuzzy strongly semi-preclosed).

4. Let \((A, B, C_0, C_1, C_2, C_3)\) be a fuzzy set. Then \(A\) is called r-fuzzy strongly semi-preopen (resp. r-fuzzy strongly semi-preclosed) set if \(A \leq B\) is r-fuzzy semi-preopen (resp. r-fuzzy semi-preclosed).

5. Let \((A, B, C_0, C_1, C_2, C_3)\) be a fuzzy set. Then \(A\) is called r-fuzzy strongly semi-open (resp. r-fuzzy strongly semi-closed) set if \(A \leq B\) is r-fuzzy strongly semi-open (resp. r-fuzzy strongly semi-closed).

6. Let \((A, B, C_0, C_1, C_2, C_3)\) be a fuzzy set. Then \(A\) is called r-fuzzy strongly semi-preopen (resp. r-fuzzy strongly semi-preclosed) set if \(A \leq B\) is r-fuzzy strongly semi-preopen (resp. r-fuzzy strongly semi-preclosed).

7. Let \((A, B, C_0, C_1, C_2, C_3)\) be a fuzzy set. Then \(A\) is called r-fuzzy strongly semi-open (resp. r-fuzzy strongly semi-closed) set if \(A \leq B\) is r-fuzzy strongly semi-open (resp. r-fuzzy strongly semi-closed).
Define fuzzy topology follows:

Example 9. Let \( r-f \) open, \( r-f \) semiopen, \( r-f \) preclosed, respectively.

Remark 8.

The converses of these implications are not true as the following:

- \( r-f \) open but not \( r-f \) semiopen, \( r-f \) preclosed, respectively.
- \( r-f \) semiopen but not \( r-f \) open, \( r-f \) preclosed, respectively.
- \( r-f \) preclosed but not \( r-f \) open, \( r-f \) semiopen, respectively.
- \( r-f \) open, \( r-f \) semiopen, \( r-f \) preclosed, respectively.

For example, if \( r-f \) open, \( r-f \) semiopen, \( r-f \) preclosed, respectively.

Then \( r-f \) open, \( r-f \) semiopen, \( r-f \) preclosed, respectively.
Theorem 11. Let $(\mu, \nu)$ be a fts and $\mu \in C$. (i) Any union of $r$-feo sets is an $r$-feo set. If $\nu(1 - \nu) \geq \mu$, then $\nu$ is an $r$-feco.

(ii) If $\nu$ is $r$-feo with $\nu(1 - \nu) \geq \mu$, then $\nu$ is an $r$-feco.

Proof. (i) Let $\nu$ be $r$-feo with $\nu(1 - \nu) \geq \mu$, and $\nu(1 - \nu) \geq \mu$. Then $\nu$ is an $r$-feco.

Proof. (ii) If $\nu$ is $r$-feo with $\nu(1 - \nu) \geq \mu$, then $\nu$ is an $r$-feco.
If $u_1D706$ is r-fs $u_1D6FF$ po with $(1 - u_1D706) \geq u_1D45F$, then $u_1D706$ is an r-feo set.

(v) It is trivial from Theorem 17. (vi) From Theorem 18, we only show $u_1D706$ is r-feo iff $u_1D706$ is r-feo just for $u_1D706, u_1D45F$. Other results have similar proofs. (i) 

Proof. Let $u_1D436$ be an r-feo set and $u_1D707, u_1D45F$ such that $(1 - u_1D706) \geq u_1D45F$ and $u_1D70F \leq u_1D43C(u_1D706, u_1D45F) = u_1D43C(u_1D706, u_1D45F) = u_1D70F(u_1D707, u_1D45F) = u_1D70F(u_1D706, u_1D45F)$. We prove (i) and (iii). Other results have similar proofs. (i) 

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Thus \( f(\bar{x}, \bar{y}) = \bar{z} \).

This is a contradiction.

Hence \( f(\bar{x}, \bar{y}) \leq \bar{z} \).

\[ \text{Theorem 19.} \]

Let \((\mathcal{B}, \mathcal{C})\) be a fts. For \( \bar{x} \in \mathcal{B} \) and \( \bar{y} \in \mathcal{C} \) we have

\[ (i) \quad \bar{x} \in \mathcal{C} \implies (1 - \bar{x}, \bar{y}) = 1 - (\bar{x}, \bar{y}) \]

\[ (ii) \quad (1 - \bar{x}, \bar{y}) = 1 - (\bar{x}, \bar{y}) \]

\[ \text{Proof.} \]

\[ (i) \quad \text{For all} \quad \bar{x} \in \mathcal{B}, \bar{y} \in \mathcal{C} \] we have the following:

\[ 1 - (\bar{x}, \bar{y}) = 1 - (\bar{x}, \bar{y}) \]

\[ \text{REFERENCES} \]


