

Some New Semi-Normed Triple Entire Sequence Spaces Defined By A Sequence Of Moduli

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Abstract

In this paper we study some properties of the sequence space $\Gamma_f^3(p, q, u)$ and obtain some inclusion relations. **AMS**

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1 Introduction

Throughout w , Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}) , where $m, n, k \in \mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinate

wise addition and scalar multiplication.

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double series is found in *Apostol [1]* and double sequence spaces is found in *Hardy [7]*, *Subramanian et al. [8-14]*, and many others. Later on investigated by some initial work on triple sequence spaces is found in *Sahiner et al. [15]*, *Esi et al. [2-6]*, *Subramanian et al. [16-25]* and many others.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}(m, n, k = 1, 2, 3, \dots).$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3 . A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3 . The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x, y) = \sup_{m,n,k} \left\{ |x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, \dots \right\}, \quad (1)$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3 . Let $\phi = \{\text{finite sequences}\}$.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq} \delta_{ijq}$ for all $m, n, k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots \\ \vdots & & & & & \\ 0 & 0 & \dots & 1 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots \end{pmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero otherwise.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m, n, k]}$ of the sequence is defined by $x^{[m, n, k]} = \sum_{i, j, q=0}^{m, n, k} x_{ijq} \mathfrak{S}_{ijq}$ for all $m, n, k \in \mathbb{N}$; where $\mathfrak{S}_{ijq}$ denotes the triple sequence whose only non zero term is a $\frac{1}{(i+j+k)!}$ in the $(i, j, k)^{th}$ place for each $i, j, k \in \mathbb{N}$.

An FK-space (or a metric space) X is said to have AK property if $(\mathfrak{S}_{mnk})$ is a Schauder basis for X , or equivalently $x^{[m, n, k]} \rightarrow x$.

An FDK-space is a triple sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings are continuous.

If X is a sequence space, we give the following definitions:

- (i) X' is continuous dual of X ;
- (ii) $X^\alpha = \{a = (a_{mnk}) : \sum_{m, n, k=1}^{\infty} |a_{mnk} x_{mnk}| < \infty, \text{ for each } x \in X\}$;
- (iii) $X^\beta = \{a = (a_{mnk}) : \sum_{m, n, k=1}^{\infty} a_{mnk} x_{mnk} \text{ is convergent, for each } x \in X\}$;
- (iv) $X^\gamma = \{a = (a_{mnk}) : \sup_{m, n \geq 1} \left| \sum_{k=1}^{M, N, K} a_{mnk} x_{mnk} \right| < \infty, \text{ for each } x \in X\}$;
- (v) Let X be an FK-space $\supset \phi$; then $X^f = \{f(\mathfrak{S}_{mnk}) : f \in X'\}$;
- (vi) $X^\delta = \{a = (a_{mnk}) : \sup_{m, n, k} |a_{mnk} x_{mnk}|^{1/m+n+k} < \infty, \text{ for each } x \in X\}$;

$X^\alpha, X^\beta, X^\gamma$ are called α - (or Köthe - Toeplitz) dual of X , β - (or generalized - Köthe - Toeplitz) dual of X , γ - dual of X , δ - dual of X respectively. X^α is defined by Gupta and Kamptan [26]. It is clear that $X^\alpha \subset X^\beta$ and $X^\alpha \subset X^\gamma$, but $X^\alpha \subset X^\gamma$ does not hold.

2 Definitions and Preliminaries

A sequence $x = (x_{mnk})$ is said to be triple analytic if $\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty$. The vector space of all triple analytic sequences is usually denoted by Λ^3 . A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0$ as $m, n, k \rightarrow \infty$. The vector space of triple entire sequences is usually denoted by Γ^3 .

Let p, q be semi norms on a vector space X . Then p is said to be stronger than q if whenever (x_{mnk}) is a sequence such that $p(x_{mnk}) \rightarrow 0$, then also $q(x_{mnk}) \rightarrow 0$. If each is stronger than the others, then p and q are said to be equivalent.

A sequence space E is said to be solid or normal if $(\alpha_{mnk} x_{mnk}) \in E$ whenever $(x_{mnk}) \in E$ and for all sequences of scalars (α_{mnk}) with $|\alpha_{mnk}| \leq 1$, for all $m, n, k \in \mathbb{N}$.

A sequence space E is said to be monotone if it contains the canonical pre-images of all its step spaces.

A sequence E is said to be convergence free if $(y_{mnk}) \in E$ whenever $(x_{mnk}) \in E$ and $x_{mnk} = 0$ implies that $y_{mnk} = 0$.

Let $p = (p_{mnk})$ be a sequence of positive real numbers with $0 < p_{mnk} < \sup p_{mnk} = G$ and Let $D = \max(1, 2^{G-1})$.

Then for $a_{mnk}, b_{mnk} \in \mathbb{C}$, the set of complex numbers for all $m, n, k \in \mathbb{N}$ we have

$$|a_{mnk} + b_{mnk}|^{p_{mnk}} \leq D \{ |a_{mnk}|^{p_{mnk}} + |b_{mnk}|^{p_{mnk}} \}, \quad (2)$$

where $D = \max(1, 2^{H-1})$, $H = \sup_{m,n,k} p_{mnk}$

By $S(X)$ we denote the linear space of all sequences $x = (x_{mnk})$ with $(x_{mnk}) \in X$ and the usual coordinate wise operations: $\alpha x = (\alpha x_{mnk})$ and $x + y = (x_{mnk} + y_{mnk})$, for each $\alpha \in \mathbb{C}$. If $\lambda = (\lambda_{mnk})$ is a scalar sequence and $x \in S(X)$ then we shall write $\lambda x = (\lambda_{mnk} x_{mnk})$

Let U be the set of all sequences $u = (u_{mnk})$ such that $u_{mnk} \neq 0$ and complex for all $m, n, k = 1, 2, 3, \dots$.

A function $f : [0, \infty) \rightarrow [0, \infty)$ such that modulus f is (i) $f(x) = 0$ if and only if $x = 0$, (ii) $f(x+y) \leq f(x) + f(y)$, for all $x \geq 0, y \geq 0$, (iii) f is increasing, (iv) f is continuous from the right of 0. It follows from (ii) and (iv) f must be continuous everywhere on $[0, \infty)$. For a sequence of moduli $f = (f_{mnk})$ we give the following conditions: (v) $\sup_{m,n,k} f_{mnk}(t) < \infty$ for all $t \geq 0$, (vi) $\lim_{t \rightarrow 0} f_{mnk}(t) = 0$ uniformly in $m, n, k \geq 1$. We remark that in case $f_{mnk} = f(m, n, k \geq 1)$, where f is a modulus function, the conditions (v) and (vi) are automatically fulfilled.

Let (X, q) be a semi normed space over the field $\mathbb{C}$ of complex numbers with the semi norm q . The symbol $\Gamma_f^3(X)$ denotes the spaces of all triple entire sequences defined over X . We define the following sequence space:

$$\Gamma_f^3(p, q, u) = \{x \in S(X) : u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty\}$$

We get the following sequence spaces from $\Gamma_f^3(p, q, u)$ on giving particular values to p and u . Taking $p_{mnk} = 1$ for all $m, n, k \in \mathbb{N}$ we have

$$\Gamma_f^3(q, u) = \{x \in S(X) : u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right] \rightarrow 0 \text{ as } m, n, k \rightarrow \infty\}$$

If we take $u_{mnk} = 1$, then we have

$$\Gamma_f^3(p, q) = \{x \in S(X) : \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty\}$$

If we take $p_{mnk} = 1$ and $u_{mnk} = 1$ for all $m, n, k \in \mathbb{N}$, then we have

$$\Gamma_f^3(q) = \{x \in S(X) : \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right] \rightarrow 0 \text{ as } m, n, k \rightarrow \infty\}$$

In addition to the above sequence spaces, we have $\Gamma_f^3(p, q, u) = \Gamma_f^3(p)$, on taking $u_{mnk} = 1$ for all $m, n, k \in \mathbb{N}$, $q(x) = |x|$, $(f_{mnk}) = f$

for all $m, n, k \in \mathbb{N}$ and $X = \mathbb{C}$. In this chapter we introduce the sequence spaces $\Gamma_f^3(p, q, u)$, using an modulus function f and defined over a semi normed space (X, q) , semi normed by q . We study some properties of these sequence spaces and obtain some inclusion relations.

2.1 Lemma:

Let p and q be semi norms on a linear space X . Then p is stronger than q if and only if there exists a constant M such that $q(x) \leq Mp(x)$ for all $x \in X$.

2.2 Remark:

From the two above definitions it is clear that a sequence space E is solid implies that E is monotone.

3 Main Results

3.1 Theorem:

If $f = (f_{mnk})$ be a sequence of moduli, then $\Gamma_f^3(p, q, u)$ are linear spaces over the set of complex numbers.

Proof: It is routine verification. Therefore the proof is omitted.

3.2 Theorem:

$\Gamma_f^3(p, q, u)$ are paranormed spaces with the paranorm g defined by

$$g(x) = \sup_{m,n,k} u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right].$$

Proof: Clearly $g(x) = g(-x)$ and $g(\theta) = 0$, where θ is the zero sequence. It can be easily verified that $g(x+y) \leq g(x) + g(y)$. Next $x \rightarrow \theta, \lambda$ fixed implies $g(\lambda x) \rightarrow 0$. Also $x \rightarrow \theta$ and $\lambda \rightarrow 0$ implies $g(\lambda x) \rightarrow 0$. The case $\lambda \rightarrow 0$ and x fixed implies that $g(\lambda x) \rightarrow 0$ follows from the following expressions.

$$g(\lambda x) = \sup_{m,n,k} u_{mnk} \left[f_{mnk} \left(q \left(|\lambda_{mnk} x_{mnk}|^{1/m+n+k} \right) \right) \right].$$

$$g(\lambda x) = \left\{ |\lambda|^{1/m+n+k} : \sup_{m,n,k} u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right] \right\}.$$

Hence $\Gamma_f^3(p, q, u)$ is a paranormed space. This completes the proof.

3.3 Theorem:

Let $f = (f_{mnk})$ and $T = (T_{mnk})$ be a two sequence of moduli. Then $\Gamma_f^3(p, q, u) \cap \Gamma_T^3(p, q, u) \subseteq \Gamma_{f+T}^3(p, q, u)$

Proof: The proof is easy, so omitted.

3.4 Remark:

Let $f = (f_{mnk})$ be a sequence of moduli q_1 and q_2 be two semi norms on X , we have

- (i) $\Gamma_f^3(p, q_1, u) \cap \Gamma_f^3(p, q_2, u) \subseteq \Gamma_f^3(p, q_1 + q_2, u)$
- (ii) If q_1 is stronger than q_2 then $\Gamma_f^3(p, q_1, u) \subseteq \Gamma_f^3(p, q_2, u)$
- (iii) If q_1 is equivalent to q_2 then $\Gamma_f^3(p, q_1, u) = \Gamma_f^3(p, q_2, u)$

3.5 Theorem:

- (i) Let $0 \leq p_{mnk} \leq r_{mnk}$ and $\left\{ \frac{r_{mnk}}{p_{mnk}} \right\}$ be bounded. Then $\Gamma_f^3(r, q, u) \subset \Gamma_f^3(p, q, u)$
- (ii) $u_1 \leq u_2$ implies $\Gamma_f^3(p, q, u_1) \subset \Gamma_f^3(p, q, u_2)$

Proof:

Let

$$x \in \Gamma_f^3(r, q, u) \quad (3)$$

$$u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty \quad (4)$$

Let $t_{mnk} = u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}}$ and $\lambda_{mnk} = \frac{p_{mnk}}{r_{mnk}}$. Since $p_{mnk} \leq r_{mnk}$,

we have $0 \leq \lambda_{mnk} \leq 1$. Take $0 < \lambda < \lambda_{mnk}$.

Define $u_{mnk} = t_{mnk}(t_{mnk} \geq 1); u_{mnk} = 0(t_{mnk} < 1);$ and $v_{mnk} = 0(t_{mnk} \geq 1); v_{mnk} = t_{mnk}(t_{mnk} < 1);$

$t_{mnk} = u_{mnk} + v_{mnk}; t_{mnk}^{\lambda_{mnk}} + v_{mnk}^{\lambda_{mnk}}$. Now it follows that

$$u_{mnk}^{\lambda_{mnk}} \leq t_{mnk} \text{ and } v_{mnk}^{\lambda_{mnk}} \leq v_{mnk}^{\lambda} \quad (5)$$

i.e $t_{mnk}^{\lambda_{mnk}} = u_{mnk}^{\lambda_{mnk}} + v_{mnk}^{\lambda_{mnk}}; t_{mnk}^{\lambda_{mnk}} \leq t_{mnk} + v_{mnk}^{\lambda}$ by (5)

$$u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right)^{r_{mnk}} \right]^{\lambda_{mnk}} \\ \leq u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}}$$

$$u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right)^{r_{mnk}} \right]^{p_{mnk}/r_{mnk}} \\ \leq u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}}$$

$$u_{mnk} \left[f_{mnk} \left(q \left(\frac{|x_{mn}|^{1/m+n+k}}{\rho} \right) \right) \right]^{p_{mnk}} \\ \leq u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}}$$

But $u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{r_{mnk}} \rightarrow 0$ as $m, n, k \rightarrow \infty$. By (4), we have

$$u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty.$$

Hence

$$x \in \Gamma_f^3(p, q, u) \quad (6)$$

From (3) and (6) we get $\Gamma_f^3(r, q, u) \subset \Gamma_f^3(p, q, u)$.

This completes the proof.

Proof (ii): The proof is easy, so omitted.

3.6 Theorem:

The space $\Gamma_f^3(p, q, u)$ is solid, hence is monotone.

Proof: Let $(x_{mnk}) \in \Gamma_{f_{mnk}}^3(p, q, u)$ and (α_{mnk}) be a sequence of scalars such that $|\alpha_{mnk}|^{1/m+n+k} \leq 1$ for all $m, n, k \in \mathbb{N}$. Then

$$u_{mnk} \left[f_{mnk} \left(q \left(|\alpha_{mnk} x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \\ \leq u_{mnk} \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \text{ for all } m, n, k \in \mathbb{N}$$

$$\left[f_{mnk} \left(q \left(|\alpha_{mnk} x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \\ \leq \left[f_{mnk} \left(q \left(|x_{mnk}|^{1/m+n+k} \right) \right) \right]^{p_{mnk}} \text{ for all } m, n, k \in \mathbb{N}.$$

This completes the proof.

3.7 Result:

The space $\Gamma_f^3(p, q, u)$ are not convergence free in general.

Proof: The proof follows from the following example.

Example: Consider the sequences $(x_{mnk}), (y_{mnk}) \in \Gamma_f^3(p, q, u)$.

Defined as $(x_{mnk}) = \left(\frac{1}{m+n+k}\right)^{m+n+k}$ and $(y_{mnk}) = \left(\frac{m-n-k}{m+n+k}\right)^{m+n+k}$.

Hence

$u_{mnk} \left[f_{mnk} \left(q \left(\frac{1}{m+n+k} \right) \right) \right]^{p_{mnk}} \rightarrow 0$ as $m, n, k \rightarrow \infty$. Which implies

$(x_{mnk}) = 0$. Also $u_{mnk} \left[f_{mnk} \left(q \left(\frac{m-n-k}{m+n+k} \right) \right) \right]^{p_{mnk}} \rightarrow 0$ as $m, n, k \rightarrow \infty$. But $(y_{mnk}) \not\rightarrow 0$. Hence the space $\Gamma_f^3(p, q, u)$ are not convergence free in general. This completes the proof.

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