

Soft $g^{**}r$ -closed sets

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Abstract

In various fields, like engineering, social science, medical science etc, soft sets have many applications with real time problems. Recently, the concept of soft topological space has been developed with the help of soft sets. Here, we introduce the concepts of soft generalized star star regular closed sets and soft generalized star star regular open sets in soft topological spaces and their basic properties have been investigated further.

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1 Introduction

Molodtsov [1] introduced the concept of soft sets from which the difficulties of fuzzy sets [2], intuitionistic fuzzy sets [3], vague sets, interval mathematics [4, 5] and rough sets [6] have been rectified. A soft set over the universe U is a parametrized family of subsets of the universe U . Application of soft sets in decision making problems has been found by Maji et al. [7, 8] whereas Chen [9] gave a parametrization reduction of soft sets and a comparison of it with attribute reduction in rough set theory. Further, soft sets are a class of special information [10].

Shabir and Naz [11] introduced soft topological spaces in 2011 and studied some basic properties of them. Meanwhile, generalized closed sets [12] in topological spaces were introduced by Levine in 1970 and recent survey of them is in [13] which is extended to soft topological spaces in the year 2012 [14]. Further, Kannan and Rajalakshmi have introduced soft g -locally closed sets [15] and soft semi star generalized closed sets [16]. Soft strongly g -closed sets have been studied by Kannan, Rajalakshmi and Srikanth [17]. Chandrasekhara Rao and Palaniappan [18] introduced generalized star star regular closed sets in topological spaces and it is extended to the bitopological context by Chandrasekhara Rao and Kannan [19]. Now we shall give some known concepts as prerequisites.

Let X be an initial universe set, E be the set of parameters. Let τ be the collection of soft sets over X , then is said to be a soft topology on X if (a) $\tilde{\phi}, \tilde{X}$ belong to τ , (b) the union of any number of soft sets in τ belongs to τ , (c) the intersection of any two soft sets in τ belongs to τ . The triplet (X, τ, E) is called a soft topological space over X . Let (X, τ, E) be a soft space over X , then the members of τ are said to be soft open sets in X . A soft set (F, E) over X is said to be a soft closed set in X , if its relative complement $(F, E)'$ belongs to τ . The closure and interior of a soft set in soft topological spaces is similar to that of in the classical topological spaces. A soft subset (A, E) of a soft topological space (X, τ, E) is called soft g -closed if $(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open in X . A soft set (A, E) is soft g -open if $(A, E)'$ is soft g -closed in X .

2 Soft generalized star star regular closed sets

Definition 1. A soft subset $(A, E) \subseteq (X, \tau, E)$ is called soft generalized star star regular closed (soft g^{**r} -closed) in X if and only if $\overline{[(A, E)^o]} \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open in X .

Example 2. Let $X = \{a, b, c\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\phi}, \tilde{X}, (A, E), (B, E), (C, E)\}$ be a soft topology defined on X , where $(A, E), (B, E), (C, E)$ are soft sets over X , defined as follows:

$A(e_1) = \{b\}$, $A(e_2) = \{a\}$, $B(e_1) = \{b, c\}$, $B(e_2) = \{a, b\}$, $C(e_1) = \{a, b\}$, $C(e_2) = \{a, c\}$. Then (D, E) is soft $g^{**}r$ -closed set in (X, τ, E) , where $D(e_1) = \{a\}$, $D(e_2) = \{b, c\}$.

Theorem 3. If (A, E) is soft $g^{**}r$ -closed then $\overline{[(A, E)^o]} - (A, E)$ contains no nonempty soft closed set.

Proof. Let $(F, E) \subseteq \overline{[(A, E)^o]} - (A, E)$ and (F, E) is soft closed. Then, $(F, E) \subseteq \overline{[(A, E)^o]} \cap (A, E)'$. Then, $(A, E) \subseteq (F, E)'$ and $(F, E) \subseteq \overline{[(A, E)^o]}$. Since $(F, E)'$ is soft open and (A, E) is soft $g^{**}r$ -closed, $\overline{[(A, E)^o]} \subseteq (F, E)'$. Therefore, $(F, E) \subseteq \overline{[(A, E)^o]}$. Hence $(F, E) \subseteq \phi$. But $\phi \subseteq (F, E)$. Therefore, $(F, E) = \phi$. $\square$

Theorem 4. If (A, E) is soft $g^{**}r$ -closed and (B, E) is soft g -closed, then $(A, E) \cup (B, E)$ is soft $g^{**}r$ -closed.

Proof. Let $(A, E) \cup (B, E) \subseteq (U, E)$ and (U, E) is soft open in X . Since $(A, E) \subseteq (U, E)$ and (A, E) is $g^{**}r$ -closed, $\overline{[(A, E)^o]} \subseteq (U, E)$. Similarly, $\overline{[(B, E)^o]} \subseteq (U, E)$. Now, $\overline{[(A, E) \cup (B, E)]^o} \subseteq \overline{[(A, E)^o] \cup [(B, E)^o]} \subseteq \overline{[(A, E)^o]} \cup \overline{[(B, E)^o]} \subseteq (U, E)$. Therefore, $(A, E) \cup (B, E)$ is soft $g^{**}r$ -closed. $\square$

Remark 5. The intersection {resp. union} of two soft $g^{**}r$ -closed sets is not $g^{**}r$ -closed in general. In Example 2, the soft sets $(P, E), (Q, E)$ are soft $g^{**}r$ -closed sets in (X, τ, E) where $P(e_1) = \{b, c\}$, $P(e_2) = \{a, c\}$, $Q(e_1) = \{a, b\}$, $Q(e_2) = \{a, b\}$, but $(P, E) \cap (Q, E) = (R, E)$ is not soft $g^{**}r$ -closed where (R, E) is defined as follows: $R(e_1) = \{b\}$, $R(e_2) = \{a\}$. Moreover, the soft sets $(S, E), (T, E)$ are soft $g^{**}r$ -closed sets in (X, τ, E) where $S(e_1) = \{b\}$, $S(e_2) = \phi$, $T(e_1) = \{c\}$, $T(e_2) = \{a\}$, but $(S, E) \cap (T, E) = (Z, E)$ is not soft $g^{**}r$ -closed where (Z, E) is defined as follows: $Z(e_1) = \{b, c\}$, $Z(e_2) = \{a\}$.

Definition 6. Let $(B, E) \subseteq (Y, E) \subseteq (X, \tau, E)$. A soft subset (B, E) of (Y, E) is said to be soft $g^{**}r$ -closed relative to (Y, τ_Y, E) if (B, E) is soft $g^{**}r$ -closed in the soft subspace Y .

Theorem 7. Let a soft subset (B, E) of soft subspace (Y, τ_Y, E) of (X, τ, E) be soft $g^{**}r$ -closed relative to (Y, τ_Y, E) and (Y, E) be soft open and soft g -closed in (X, τ, E) . Then (B, E) is soft

$g^{**}r$ -closed in (X, τ, E) .

Proof. Let $(B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X . Since (Y, E) is soft open in X , $(U, E) \tilde{\cap} (Y, E)$ is soft open in Y and $(B, E) \tilde{\subseteq} (U, E) \tilde{\cap} (Y, E)$. Since (B, E) is soft $g^{**}r$ -closed relative to (Y, τ_Y, E) , $cl_Y \{int_Y[(B, E)]\} \tilde{\subseteq} (U, E) \tilde{\cap} (Y, E)$. Hence $(B, E)^o \tilde{\cap} (Y, E) \tilde{\subseteq} (U, E) \tilde{\cap} (Y, E)$. Let $(G, E) = (U, E) \tilde{\cup} [(B, E)^o]'$. Then (G, E) is soft open and $(Y, E) \tilde{\subseteq} (G, E)$. Since (Y, E) is soft g -closed, $(Y, E) \tilde{\subseteq} (G, E)$. Now, $(B, E)^o = (Y, E) \tilde{\cap} [(B, E)^o] \tilde{\subseteq} (G, E) \tilde{\cap} [(B, E)^o] \tilde{\subseteq} (U, E)$. Therefore, (B, E) is soft $g^{**}r$ -closed in (X, τ, E) . $\square$

Theorem 8. Suppose that a soft subset (B, E) of (Y, τ_Y, E) is soft $g^{**}r$ -closed in (X, τ, E) and (Y, E) is soft open in (X, τ, E) then (B, E) is soft $g^{**}r$ -closed relative to (Y, τ_Y, E) .

Proof. Let $(B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in (Y, τ_Y, E) . Since (Y, E) is soft open, we have $(U, E) = (Y, E) \tilde{\cap} (W, E)$ for some soft open set (W, E) in (X, τ, E) . Since $(B, E) \tilde{\subseteq} (Y, E) \tilde{\cap} (W, E) \tilde{\subseteq} (W, E)$ and (B, E) is soft $g^{**}r$ -closed in (X, τ, E) , we have $[(B, E)^o] \tilde{\subseteq} (W, E)$. Therefore, $cl_Y[int_Y(B, E)] = [(B, E)^o] \tilde{\cap} (Y, E) \tilde{\subseteq} (W, E) \tilde{\cap} (Y, E) = (U, E)$. Hence (B, E) is soft $g^{**}r$ -closed relative to (Y, τ_Y, E) . $\square$

Theorem 9. Let (A, E) be soft $g^{**}r$ -closed and $(A, E) \tilde{\subseteq} (B, E) \tilde{\subseteq} [(A, E)^o]$, then (B, E) is soft $g^{**}r$ -closed.

Proof. Let $(B, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X . Since $(A, E) \tilde{\subseteq} (B, E)$, $(A, E) \tilde{\subseteq} (U, E)$. Since (A, E) is soft $g^{**}r$ -closed, $[(A, E)^o] \tilde{\subseteq} (U, E)$. Since $(B, E) \tilde{\subseteq} [(A, E)^o]$, $[(B, E)^o] \tilde{\subseteq} [(A, E)^o] \tilde{\subseteq} (U, E)$. Therefore, (B, E) is soft $g^{**}r$ -closed. $\square$

Theorem 10. Suppose that $\tau \tilde{\subseteq} C(X, \tau, E)$. Then every soft subset of X is soft $g^{**}r$ -closed.

Proof. Let $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft open in X . Since $\tau \tilde{\subseteq} C(X, \tau, E)$, (U, E) is soft closed in X . Since $(A, E) \tilde{\subseteq} (U, E)$, $(A, E) \tilde{\subseteq} (U, E) = (U, E)$. Therefore, $[(A, E)^o] \tilde{\subseteq} (A, E) \tilde{\subseteq} (U, E)$. Hence (A, E) is soft $g^{**}r$ -closed. $\square$

3 Soft generalized star star regular open sets

Definition 11. A soft subset $(A, E) \widetilde{\subseteq} (X, \tau, E)$ is called soft generalized star star regular open (soft $g^{**}r$ -open) in X if and only if its complement is soft $g^{**}r$ -closed in X .

Example 12. In Example 2, the soft set (M, E) is a soft $g^{**}r$ -open set in (X, τ, E) where $M(e_1) = \{b, c\}$, $M(e_2) = \{a\}$.

Theorem 13. A soft subset $(A, E) \widetilde{\subseteq} (X, \tau, E)$ is soft $g^{**}r$ -open if and only if $(F, E) \subseteq \left[\overline{(A, E)} \right]^o$ whenever $(F, E) \widetilde{\subseteq} (A, E)$ and (F, E) is soft closed in X .

Proof. Necessity: Let $(F, E) \widetilde{\subseteq} (A, E)$ and (F, E) is soft closed in X . Then $(A, E)' \widetilde{\subseteq} (F, E)'$ and $(F, E)'$ is soft open in X . Since (A, E) is soft $g^{**}r$ -open, $(A, E)'$ is soft $g^{**}r$ -closed. Hence, $\{[(A, E)']^o\} \widetilde{\subseteq} (F, E)'$. Consequently, $(F, E) \subseteq \left[\overline{(A, E)} \right]^o$.

Sufficiency: Let $(A, E)' \widetilde{\subseteq} (U, E)$ and (U, E) is soft open in X . Then $(U, E)' \widetilde{\subseteq} (A, E)$ and $(U, E)'$ is soft closed in X . By our assumption, $(U, E)' \subseteq \left[\overline{(A, E)} \right]^o$. Hence $\{[(A, E)']^o\} \widetilde{\subseteq} (U, E)$. Consequently, $(A, E)'$ is soft $g^{**}r$ -closed. $\square$

Theorem 14. If (A, E) is soft $g^{**}r$ -open and (B, E) is soft g -open, then $(A, E) \widetilde{\cap} (B, E)$ is soft $g^{**}r$ -open.

Proof. Let $(F, E) \widetilde{\subseteq} (A, E) \widetilde{\cap} (B, E)$ and (F, E) is soft closed in X . Since $(F, E) \widetilde{\subseteq} (A, E)$ and (A, E) is soft $g^{**}r$ -open, $(F, E) \subseteq \left[\overline{(A, E)} \right]^o$. Similarly, $(F, E) \subseteq (B, E)^o$. Now, $(F, E) \subseteq \left[\overline{(A, E)} \right]^o \widetilde{\cap} (B, E)^o \subseteq \left[\overline{(A, E) \widetilde{\cap} (B, E)} \right]^o \subseteq \left[\overline{(A, E) \widetilde{\cap} (B, E)} \right]^o \subseteq \left[\overline{(A, E) \widetilde{\cap} (B, E)} \right]^o$. Therefore, $(A, E) \widetilde{\cap} (B, E)$ is soft $g^{**}r$ -open. $\square$

The intersection {resp. union} of two soft $g^{**}r$ -open sets is not $g^{**}r$ -open in general. For Example, in Example 2, the soft sets (F, E) , (G, E) are soft $g^{**}r$ -open sets in (X, τ, E) where $F(e_1) = \{a, c\}$, $F(e_2) = \{a, b, c\}$, $G(e_1) = \{a, b\}$, $G(e_2) = \{b, c\}$, but $(F, E) \widetilde{\cap} (G, E) = (H, E)$ is not soft $g^{**}r$ -open where (H, E) is

defined as follows: $H(e_1) = \{a\}$, $H(e_2) = \{b, c\}$. Moreover, the soft sets $(I, E), (J, E)$ are soft $g^{**}r$ -open sets in (X, τ, E) where $I(e_1) = \{a\}$, $I(e_2) = \{b\}$, $J(e_1) = \{c\}$, $J(e_2) = \{c\}$, but $(I, E) \tilde{\cup} (J, E) = (K, E)$ is not soft $g^{**}r$ -open where (K, E) is defined as follows: $K(e_1) = \{a, c\}$, $K(e_2) = \{b, c\}$.

Theorem 15. Let (A, E) be soft $g^{**}r$ -open and $\left[\overline{(A, E)}\right]^o \tilde{\subseteq} (B, E) \tilde{\subseteq} (A, E)$, then (B, E) is soft $g^{**}r$ -open.

Proof. Let $(F, E) \tilde{\subseteq} (B, E)$ and (F, E) is soft closed in X . Since $(B, E) \tilde{\subseteq} (A, E)$, we have $(F, E) \tilde{\subseteq} (A, E)$. Since (A, E) is soft $g^{**}r$ -open, $(F, E) \tilde{\subseteq} \left[\overline{(A, E)}\right]^o$. Since $\left[\overline{(A, E)}\right]^o \tilde{\subseteq} (B, E)$, $(F, E) \tilde{\subseteq} \left[\overline{(A, E)}\right]^o \tilde{\subseteq} \left[\overline{(B, E)}\right]^o$. This completes the proof. $\square$

Theorem 16. If (A, E) is soft $g^{**}r$ -closed, then $\left[\overline{(A, E)^o}\right] - (A, E)$ is soft $g^{**}r$ -open.

Proof. Let $(F, E) \tilde{\subseteq} \left[\overline{(A, E)^o}\right] - (A, E)$ and (F, E) is soft closed. Since (A, E) is soft $g^{**}r$ -closed, $\left[\overline{(A, E)^o}\right] - (A, E)$ does not contain nonempty soft closed set $\{ \text{By Theorem 3} \}$. Then, $(F, E) = \tilde{\phi} \tilde{\subseteq} \left[\overline{(A, E)^o}\right] - (A, E)$. Hence $\left[\overline{(A, E)^o}\right] - (A, E)$ is soft $g^{**}r$ -open. $\square$

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References

- [1] D. Molodtsov, Soft set theory first results. *Computers & Mathematics with Applications* **37**, (1999), 19–31.
- [2] L.A. Zadeh, Fuzzy sets, *Information and Control*, **8**, (1965), 338–353.

- [3] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **20** (1986), 87–96.
- [4] K. Atanassov, Operators over interval valued intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **64** (1994), 159–174.
- [5] M.B. Gorzalzany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, *Fuzzy Sets and Systems*, **21**, (1987), 1–17.
- [6] Z. Pawlak, Rough sets, *International Journal of Computer & Information Sciences*, **11**, (1982), 341–356.
- [7] P.K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem, *Computers & Mathematics with Applications*, **44**, (2002), 1077–1083.
- [8] P. K. Maji, R. Biswas, R. Roy, Soft set theory, *Computers & Mathematics with Applications*, **45**, (2003), 555–562.
- [9] D. Chen, The parametrization reduction of soft sets and its applications, *Computers & Mathematics with Applications*, **49**, (2005), 757–763.
- [10] D. Pie, D. Miao, From soft sets to information systems, *Granular computing, 2005 IEEE International Conference*, **2**, 617–621.
- [11] Muhammad Shabir, Munazza Naz, On soft topological spaces. *Computers & Mathematics with Applications* **61**, (2011), 1786–1799.
- [12] N. Levine, Generalized closed sets in topology, *Rendiconti del Circolo Matematico di Palermo* **19**, No.2 (1970), 89–96.
- [13] K. Kannan, On Levine’s generalized closed sets: a survey, *Research Journal of Applied Sciences, Engineering and Technology*, **4**, N0. 11 (2012), 1612–1615.
- [14] K. Kannan, Soft Generalized closed sets in soft topological spaces, *Journal of Theoretical and Applied Information Technology*, **37**, No. 1 (2012), 17–21.

- [15] K. Kannan and D. Rajalakshmi, Soft g -locally closed sets, *National Academy of Science Letters*, **Vol.38**, No. 3 (2015), 239–241.
- [16] K. Kannan K and D. Rajalakshmi, Soft semi star generalized closed sets, *Malaysian Journal of Mathematical Sciences*, **9** No. 1 (2015), 77–88
- [17] K. Kannan, D. Rajalakshmi and R. Srikanth, Soft strongly g -closed sets, *Indian Journal of Science and Technology*, **8**, No. 18 (2015), 6 Pages.
- [18] N. Palaniappan, Ph. D. Thesis, *Alagappa University*, Karaikudi, (2000).
- [19] K. Chandrasekhara Rao, K. Kannan K, D. Narasimhan and R. Ravikumar, Generalized star star regular closed sets, *National Academy of Science Letters* **31** No. 1 & 2 (2008), 31–33.

