

Algorithm for the bitopological spaces to corresponding to directed graphs

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Abstract

The main aim of the present communication is to develop the algorithm to find the bitopological spaces corresponding to the directed graphs due to Ahlborn and Bhargava. The algorithm, flow chart and suitable examples are also explored.

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Key Words and Phrases: Bitopological Spaces,
directed graphs, generalized closed sets

1 Introduction

The directed graph consists a set of points (vertices) and the set of edges joining particular ordered pair of points, whereas a topology is a family of sets in which the intersection of any two sets is a member of the family and the union of the members of each subfamily is a member of the family. The bitopological space is the set together with two topologies which was initiated in 1963 by Kelly [1]. Ahlborn and Bhargava [2] define bitopological space corresponding to directed graph. A directed graph is an ordered pair (P, Λ) where P is a set and Λ is a binary relation on P . Now, we are going to determine a topology on a set P from (P, Λ) . A set M of the directed graph (P, Λ) is open if there is no edge from M^C to M . A set M is closed if M^C is open. Thus, a unique topological space (X, τ_Λ^+) can be determined where $\tau_\Lambda^+ = \{M : M \subseteq P \text{ of } (P, \Lambda) \text{ is open}\}$.

For instant, consider the the directed graph (M, Λ) where $P = \{1, 2, 3, 4, 5\}$.

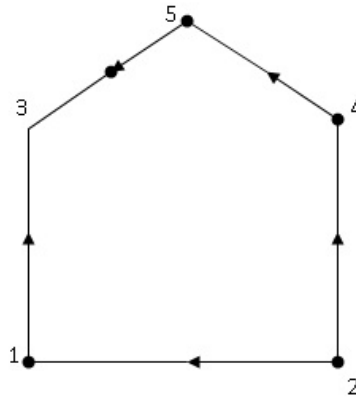


Figure 1: Digraph

Then the topology corresponding to the above directed graph is $\tau_\Lambda^+ = \{\phi, P, \{2\}, \{1, 2\}, \{2, 4\}, \{1, 2, 4\}, \{2, 4, 5\}, \{1, 2, 4, 5\}\}$. Consequently, we can form a unique topology such that $\tau_\Lambda^- = \{M : M \subseteq P \text{ and there is no edge from } M \text{ to } M^C \text{ in } (P, \Lambda)\}$. Thus the topology associated to the directed graph mentioned in

Figure 1 is $\tau_{\Lambda}^{-} = \{\phi, P, \{3\}, \{1, 3\}, \{3, 5\}, \{1, 3, 5\}, \{3, 4, 5\}, \{1, 3, 4, 5\}\}$. Thus, we have a bitopological space $(P, \tau_{\Lambda}^{+}, \tau_{\Lambda}^{-})$ associated to the directed graph (P, Λ) .

2 Proposed Algorithm

The proposed algorithm for finding bitopological space takes a directed graph as input for which the total number of vertices (n) of the directed graph and its adjacency matrix (A) are given as input. The vertices from 1 to n are stored in a vertex array. The adjacency matrix is a $n \times n$ matrix where each $A_{ij} = 1$ if there exists a directed path between i and j else $A_{ij} = 0$. Subsequently, the algorithm generates the powerset. To generate the powersets, a loop is iterated from 1 to $2n$. Each value (1 to $2n$) is internally represented as integer. The value is first converted into its binary representation and the corresponding positions that contain "1" are concatenated and stored as powerset. The process is repeated for $2n$ values. Then the null set is appended in the powerset. The powersets are sorted in ascending order.

For first n powersets, the corresponding column in the adjacency matrix is added and if the sum is equal to zero, then there are no vertices incident on that vertex and it is included in the $T1$ -open set. For the rest of the powersets, the complement set ($s1$) for each subset s is found. The set in Powerset is parsed into individual numbers and stored in a temporary array. The vertex array is compared with this temporary array and each unmatched vertex is concatenated and stored as the complement set $s1$. The intersection of rows (s) and columns ($s1$) are checked and if $s \cap s1 = 0$ then the corresponding subset s is included in $T1$ -Open and it is implemented by using a method `adjacent()`.

The data set is $T1$ -open for finding $T1$ -closed sets in the following way. For each set in the $T1$ -Open, the complement set $s1$ is found. The set in $T1$ -open is parsed into individual numbers and stored in a temporary array. The vertex array is compared with this temporary array and each unmatched vertex is concatenated and stored as the complement set $s1$ which in turn is included in the $T1$ -closed. Thus, we develop the algorithm using Java to construct the bitopological spaces corresponding to digraphs. One can easily

obtain the bitopological spaces associated to the digraphs with the following algorithm.

Algorithm Open_Closed Set($A[][]$, N):Set

```

1. for all  $v \in V[ ]$  do
    Compute powerset  $PS[ ]$ 
    [power set]  $\leftarrow 2^n - 2$ 
2. for each  $s \in ps[ ]$  do
    Compute  $s^c$  in  $V[ ]$ 
    If is not adjacent( $s, s^c$ ) then
         $os[ ] \leftarrow s$ 

3. for each set  $t \in os[ ]$  do
    Compute  $set^c$ 
    If union( $set, set^c$ ) then
         $os[ ] \leftarrow set^c$ 
end

```

Algorithm is not adjacent(s, s^c):Boolean

```

1. for all  $col \in s$  do
    for all  $row \in s^c$  do
        if ( $A[col] \cap A[row] = 0$ )
            return true

```

end

The present algorithm is implemented in JAVA programming language. Since the vertices are internally represented as numbers, the algorithm is restricted to directed graph with maximum of 9 vertices.

3 Results and discussion

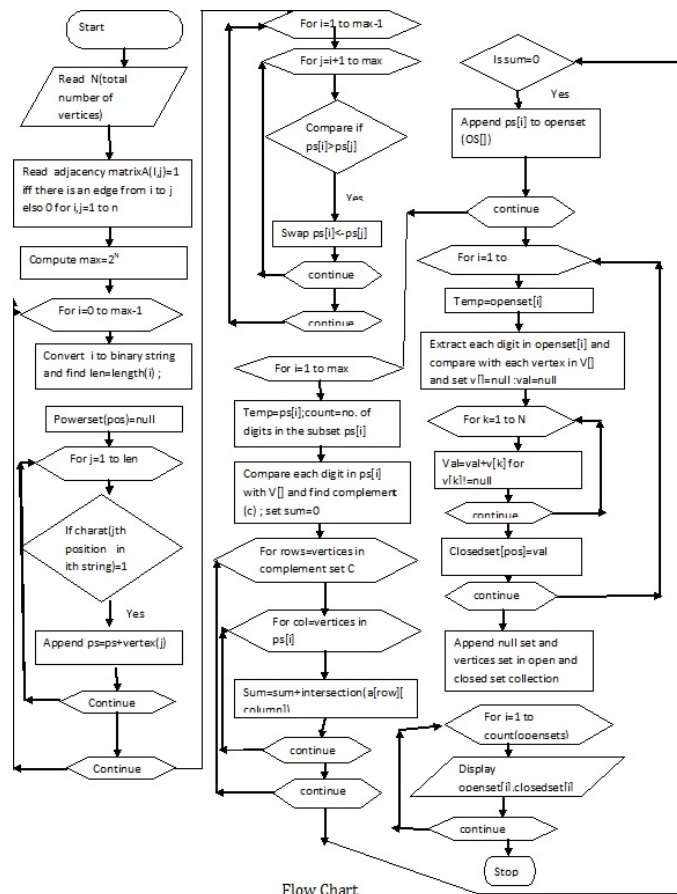
Every digraph determines a unique topological space in which the intersection of any number of open sets is open. Further, the complements of open sets form the another topology. Hence, the bitopological space will be obtained corresponding to each digraph. The following figure shows the output for the digraph given in Figure 1 by the above algorithm. Further, the flow chart is also given for the algorithm.

```

C:\Windows\system32\cmd.exe
Adjacency Matrix
0 0 1 0 0
1 0 0 1 0
0 0 0 0 0
0 0 0 0 1
0 0 1 0 0

The corresponding bitopological space is (X,T1,T2) where
X={1,2,3,4,5}
T1={ {}, {2}, {1,2}, {2,4}, {1,2,4}, {2,4,5}, {1,2,4,5}, {1,2,3,4,5} }
T2={ {}, {1,3,4,5}, {3,4,5}, {1,3,5}, {3,5}, {1,3}, {3}, {1,2,3,4,5} }
0 -> \_

```



4 Conclusion

The closed sets are extended to generalized closed sets [3] in bitopological settings. Kannan [4] studied the properties of g -closed sets in bitopological spaces corresponding to the directed graphs. But finding the examples for such generalized closed sets to support the results are very difficult as well as time taken process. To overcome these practical difficulties, the algorithm is developed for constructing bitopological spaces associated to digraphs as a beginning. Further, this algorithm can be extended to find the generalized closed sets in this direction. Hence the future study may be mainly focussed in algorithmic approach not in the mathematical view. The present algorithm can be enhanced further to examine a directed graph with more than 9 vertices by representing the powersets using the data structure “linked list” which is under process.

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