

An application of Intuitionistic Fuzzy Soft Matrix theory based on the Reference Function in Decision Making Problem

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November 9, 2017

Abstract

In this paper we study intuitionistic fuzzy soft matrix based on reference function. We define some new operations such as intuitionistic fuzzy soft complement matrix and intuitionistic product of fuzzy soft matrix based on the reference function. Then, we introduced some related properties, and some examples are given. A new approach in intuitionistic fuzzy soft matrix decision method based on the reference function has been devised.

Keywords : Soft set, fuzzy soft set, intuitionistic fuzzy soft set based on reference function, intuitionistic fuzzy soft matrix based on reference function.

AMS Subject Classification : 03E72, 03F55

1 Introduction

Fuzzy set theory was proposed by Lotfi A. Zadeh [1] in 1965, where each element is associated to a real value in $[0, 1]$ which is a degree

of membership defined on the universe of discourse X , the theory finds extensive application in various fields to handle uncertainty. Therefore, several researches were conducted on the generalization on the notions of fuzzy sets such as intuitionistic fuzzy set proposed by Atanassov[2,3], interval valued fuzzy set[4]. In the literature we found many wellknown theories to describe uncertainty rough set theory[5] , but all of these theories have their inherit difficulties as pointed by Molodtsov in his pioneer work[6]. Fuzzy membership value is the difference between fuzzy membership function and reference function. Fuzzy membership function and fuzzy membership value are two different things. In [13,14] M.Dhar applied this concept to fuzzy square matrix and developed some interesting properties such as determinant, trace and so on. Thereafter, in [15] , T.J.Neog, D. K.Sutwere extended this new concept to soft set theory, introducing a new concept called fuzzy soft set based on fuzzy reference function. Recently, Neog. T.J, Sut D. Kumar, M.Bora[16] combined fuzzy soft set based on reference function with soft matrices. The next section briefly introduces some definitions related to soft set, fuzzy soft set, and fuzzy soft set based on reference function. Section 4 presents fuzzy soft complement matrix based on reference function. Section 5 presents the Operations on Intuitionistic Fuzzy Soft Matrix Theory. Section 6 presents new fuzzy soft matrix theory in decision making. Conclusions appear in the last section.

II. PRELIMINARIES In this section first we review some concepts and definitions of soft set, fuzzy soft set, and fuzzy soft set based on reference function from [8,11,12,16] which will be needed in the sequel. Remark: For the sake of simplicity we adopt the following notation of fuzzy soft set based on reference function defined in our way as: Fuzzy soft set based on reference $= (F, A)_{rf}$, to make the difference between the notation (F, A) defined for classical soft set or its variants as fuzzy soft set.

2 Definitions and Preliminaries:

The basic definitions of Intuitionistic fuzzy soft set based on reference function that are useful for subsequent discussions are given.

2.1 Soft set [1]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U . Let $A \subseteq E$. A pair (F_A, E) is called a soft set over U , where F_A is a mapping given by $E \rightarrow P(U)$ Such that $F_A(e) = \varphi$ if $e \notin A$. Here F_A is called approximate function of the soft set (F_A, E) . The set $F_A(e)$ is called e-approximate value set which consist of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Example 2.1: Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of shirts and $E = \{e_1, e_2, e_3, e_4\} = \{black(e_1), red(e_2), blue(e_3), green(e_4)\}$ be a set of parameters. If $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Let $F_A(e_1) = \{u_1, u_2, u_3, u_4\}$ and $F_A(e_2) = \{u_1, u_4\}$, $F_A(e_3) = \{u_1, u_3, u_4\}$, $F_A(e_4) = \{u_4\}$ then we write the soft set as $(F_A, E) = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_4\}), (e_3, \{u_1, u_3, u_4\}), (e_4, \{u_4\})\}$ over U which describes the "color of the shirts",? Mr. A going to buy. We may represent the fuzzy soft set in the following form:

U	e_1	e_2	e_3	e_4
u_1	1	1	1	0
u_2	1	0	0	0
u_3	1	0	1	0
u_4	1	1	1	1

2.2 Fuzzy Soft Set [6]

Let U be an initial universe, E be the set of all parameters and $A \subseteq E$. A pair (F_A, E) is called a fuzzy set over U where F_A is a mapping given by, $F_A : E \rightarrow P(U)$ such that $F_A(e) = \varphi$ if $e \notin A$, Where φ is a null fuzzy set and $\tilde{P}(U)$ denotes the collection of all subsets of U .

Example 2.2 Consider the example 2.1, here we cannot express the members of the sets with only two real numbers 0 and 1, we can characterize it by a membership function instead of crisp number 0 and 1, which associates with each element, a real number in the

interval $[0, 1]$.

Then $(F_A, E) = \{F_A(e_1)\} = \{(u_1, 0.8), (u_2, 0.7), (u_3, 0.6), (u_4, 0.5)\}$,
 $F_A(e_2) = \{(u_1, 0.5), (u_4, 0.4)\}$, $F_A(e_3) = \{(u_1, 0.6), (u_3, 0.7), (u_4, 0.8)\}$,
 $F_A(e_4) = \{(u_4, 0.2)\}$ is the fuzzy soft set representing the "color of the pens" which Mr. A is going to buy. We may represent the fuzzy soft set in the following form:

U	e_1	e_2	e_3	e_4
u_1	0.8	0.5	0.6	0
u_2	0.7	0	0	0
u_3	0.6	0	0.7	0
u_4	0.5	0.4	0.8	0.2

2.3 Fuzzy Soft Matrices (FSM) [5]

Let (F_A, E) be fuzzy soft set over U . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$, which is called relation form of (F_A, E) . The characteristic function of R_A is written by $\mu_{RA} : U \times E \rightarrow [0, 1]$, where $\mu_{RA}(u, e) \in [0, 1]$ is the membership value of $u \in U$ for each $e \in U$. If $\mu_{ij} = \mu_{RA}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \dots & \dots & \dots & \dots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

which is called a $m \times n$ soft matrix of the soft set (F_A, E) over U . Therefore we can say that a fuzzy soft set (F_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

Example 2.3 Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters.

If $A \subseteq E = \{e_1, e_2, e_3, e_4\}$ and

$$F_A(e_1) = \{(u_1, 0.7), (u_2, 0.6), (u_3, 0.8), (u_4, 0.2), (u_5, 0.7), (u_6, 0.8)\}$$

$$F_A(e_2) = \{(u_1, 0.5), (u_3, 0.8), (u_4, 0.1), (u_5, 0.2), (u_6, 0.9)\}$$

$$F_A(e_3) = \{(u_1, 0.5), (u_2, 0.7), (u_4, 0.5), (u_5, 0.6), (u_6, 0.7)\}$$

$$F_A(e_4) = \{(u_1, 0.9), (u_6, 0.1)\}.$$
 Then the fuzzy soft set (F_A, E) is a

parameterized family $\{F_A(e_1), F_A(e_2), F_A(e_3), F_A(e_4)\}$ of all fuzzy sets over U . Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} 0.7 & 0.5 & 0.5 & 0.9 \\ 0.6 & 0.0 & 0.7 & 0.0 \\ 0.8 & 0.8 & 0.0 & 0.0 \\ 0.2 & 0.1 & 0.5 & 0.0 \\ 0.7 & 0.2 & 0.6 & 0.0 \\ 0.8 & 0.9 & 0.7 & 0.1 \end{bmatrix}$$

2.4 Row- Fuzzy Soft Matrix

A fuzzy soft matrix of order $1? \times n$ i.e., with a single row is called a row-fuzzy soft Matrix.

2.5 Column -Fuzzy Soft Matrix

A fuzzy soft matrix of order $m? \times 1$ i.e., with a single column is called a column?-fuzzy soft matrix.

2.6 Fuzzy soft complement set [11]

The complement of fuzzy soft sets (F_A, E) denoted by $(F_A, E)?^o$ is defined by $(F_A, E)?^o = (F_A^o, E)$, where $F_A^o : E? \rightarrow I^U$ is a mapping given by $F_A?^o(e) = [F_A(e)]?^o, \forall e? \in E$.

2.7 Definition [29]

We define the membership value matrix corresponding to the matrix A as $MV(A) = [\delta_{ji}(c_i)]$ Where $\delta(A)_{ij} = \mu_{j1}(c_i) - \mu_{j2}(c_i)$, $i = 1, 2, 3, ? \dots, m$ and $j = 1, 2, 3, \dots?, n$, where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$.

3 Intuitionistic Fuzzy Soft Matrix Theory

3.1 Intuitionistic Fuzzy Soft Set (IFSS)

Let U be an initial universe, E be the set of parameters and $A? \subseteq E$. A pair (F_A, E) is called an intuitionistic fuzzy soft set (IFSS) over U , where F_A is a mapping given by $F_A : E? \rightarrow I^U$, I^U denotes the collection of all intuitionistic fuzzy subsets of U .

Example 3.1: Suppose that $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{white(e_1), blue(e_2), green(e_3)\}$ be a set of parameters. If $A = \{e_1, e_2\} \in E$. Let

$$F_A(e_1) = \{(u_1, 0.3, 0.7), (u_2, 0.8, 0.1), (u_3, 0.4, 0.2), (u_4, 0.6, 0.2)\}$$

$$F_A(e_2) = \{(u_1, 0.8, 0.1), (u_2, 0.9, 0.1), (u_3, 0.4, 0.5), (u_4, 0.2, 0.3)\}$$

then we write intuitionistic fuzzy soft set as

$$(F_A, E) = \{F_A(e_1) = \{(u_1, 0.3, 0.7), (u_2, 0.8, 0.1), (u_3, 0.4, 0.2), (u_4, 0.6, 0.2)\}$$

$$F_A(e_2) = \{(u_1, 0.8, 0.1), (u_2, 0.9, 0.1), (u_3, 0.4, 0.5), (u_4, 0.2, 0.3)\}\}.$$

We would represent this intuitionistic fuzzy soft set in matrix form as

$$\begin{bmatrix} (0.3, 0.7) & (0.8, 0.1) & (0.0, 0.0) \\ (0.8, 0.1) & (0.9, 0.1) & (0.0, 0.0) \\ (0.4, 0.2) & (0.4, 0.5) & (0.0, 0.0) \\ (0.6, 0.2) & (0.2, 0.3) & (0.0, 0.0) \end{bmatrix}$$

3.2 Intuitionistic Fuzzy Soft Matrix (IFSM) [5]

Let U be an initial universe, E be the set of parameters and $A? \subseteq E$. Let (F_A, E) be an intuitionistic fuzzy soft set (IFSS) over U . Then a subset of $U? \times E$ is uniquely defined by $R_A = \{(u, e) : e? \in A, u? \in F_A(e)\}$ which is called the relation form of (F_A, E) . The membership and nonmembership functions are written by $\mu_{RA} : U? \times E? \rightarrow [0, 1]$ and $\gamma_{RA} : U? \times E? \rightarrow [0, 1]$ where $\mu_{RA} : (u, e) \in [0, 1]$ and $\gamma_{RA} : (u, e) \in [0, 1]$ are the membership value and nonmembership value of $u? \in U$ for each $e? \in E$. If $(\mu_{ij}, v_{ij}) = (\mu_{RA}(u_i, e_j), \gamma_{RA}(u_i, e_j))$ we can define a matrix

$$[(\mu_{ij}, v_{ij})]_{\mu \times n} = \begin{bmatrix} (\mu_{11}, v_{11}) & (\mu_{12}, v_{12}) & \dots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (\mu_{22}, v_{22}) & \dots & (\mu_{2n}, v_{2n}) \\ \dots & \dots & \dots & \dots \\ (\mu_{m1}, v_{m1}) & (\mu_{m2}, v_{m2}) & \dots & (\mu_{mn}, v_{mn}) \end{bmatrix}$$

which is called a $m? \times n$ IFSM of the IFSS (F_A, E) over U . Therefore, we can say that IFSS (F_A, E) is uniquely characterized by the matrix $[(\mu_{ij}, \gamma_{ij})]_{m? \times n}$ and both concepts are interchangeable. The set of all $m? \times n$ IFS matrices will be denoted by IFSM $m? \times n$.

Example 3.2:

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$, is a set of parameters. If $A = \{e_1, e_3, e_4, e_5\} \subseteq E$ and

$$F_A(e_1) = \{(u_1, .8, .4), (u_2, .8, .1), (u_3, .5, .5), (u_4, .5, .4), (u_5, .2, .1)\},$$

$$F_A(e_3) = \{(u_1, .4, .6), (u_3, .2, .2), (u_4, 1, 0), (u_5, .6, .2)\},$$

$$F_A(e_4) = \{(u_1, .6, .2), (u_2, 1, 0), (u_3, .8, .2), (u_4, .6, .3), (u_5, .7, .3)\},$$

$$F_A(e_5) = \{(u_1, .7, .8), (u_2, 1, 0), (u_3, .6, .5), (u_4, .5, .3), (u_5, .9, .2)\}$$

Then the IFS set (F_A, E) is a parameterized family $\{F_A(e_1), F_A(e_2), F_A(e_3), F_A(e_4)\}$ of all IFS sets over U . Hence IFSM $[(\mu_{ij}, \gamma_{ij})]$ can be written as

$$[(\mu_{ij}, \gamma_{ij})] = \begin{bmatrix} (0.8, 0.4) & (0, 0) & (0.4, 0.6) & (0.6, 0.2) & (0.7, 0.8) \\ (0.8, 0.1) & (0, 0) & (0, 0) & (1, 0) & (1, 0) \\ (0.5, 0.5) & (0, 0) & (0.2, 0.2) & (0.8, 0.2) & (0.6, 0.5) \\ (0.5, 0.4) & (0, 0) & (1, 0) & (0.6, 0.3) & (0.5, 0.3) \\ (0.2, 0.1) & (0, 0) & (0.6, 0.2) & (0.7, 0.3) & (0.9, 0.2) \end{bmatrix}$$

3.3 Intuitionistic Fuzzy Soft Set Complement Matrix:

Let $A = [a_{ij}]$ IFSM $m \times n$, where $a_{ij} = (\mu_j(c_i), \gamma_j(c_i))$ for all i, j . Then A^C IFSM is called a Intuitionistic Fuzzy Soft Complement Matrix if $A^C = [d_{ij}]_{m \times n}$, where $d_{ij} = (v_j(c_i), \mu_j(c_i))$ for all i, j .

3.4 Intuitionistic Fuzzy Soft Sub Matrix:

Let $A = [a_{ij}]$ IFSM $m \times n$, $B = [b_{ij}]$ IFSM $m \times n$, Then A is a intuitionistic fuzzy soft submatrix of B, denoted by $A? \subseteq B$. If $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B? \forall i, j$.

3.5 Intuitionistic Fuzzy Soft Null (Zero) Matrix:

An intuitionistic fuzzy soft matrix of order $m \times n$ is called intuitionistic fuzzy soft null (zero) matrix, if all its elements are $(0, 1)$. It is denoted by ϕ .

3.6 Intuitionistic Fuzzy Soft Universal Matrix:

An intuitionistic fuzzy soft matrix of order $m \times n$ is called intuitionistic fuzzy soft universal matrix if all its elements are $(1, 0)$. It is denoted by U .

3.7 Intuitionistic Fuzzy Soft Equal Matrix:

$A = [a_{ij}]IFSM_{m \times n}, B = [b_{ij}]IFSM_{m \times n}$, Then A is equal to B , denoted by $A = B$. If $\mu_A = \mu_B$ and $\gamma_A = \gamma_B \forall i, j$.

3.8 Intuitionistic Fuzzy Soft Transpose Matrix:

Let $A = [a_{ij}]IFSM_{m \times n}$. Then A^T is an intuitionistic fuzzy soft transpose matrix of A , if $A^T = [a_{ji}]$

3.9 Intuitionistic Fuzzy Soft Rectangular Matrix:

Let $A = [a_{ij}]IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \gamma_j(c_i))$. Then A is called a Intuitionistic Fuzzy Soft rectangular Matrix, if $m \neq n$.

3.10 Intuitionistic Fuzzy Soft Upper Triangular Matrix:

Let $A = [a_{ij}]IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \gamma_j(c_i))$. Then A is called a Intuitionistic Fuzzy Soft upper rectangular Matrix, if $m = n$ and $a_{ij} = (0, 1), i > j$.

3.11 Intuitionistic Fuzzy Soft Lower Triangular Matrix:

Let $A = [a_{ij}]IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \gamma_j(c_i))$. Then A is called an Intuitionistic Fuzzy Soft lower rectangular Matrix, if $m = n$ and $a_{ij} = (0, 1), i < j$.

4 Intuitionistic Fuzzy Soft Complement Matrix Based on Reference Function

In this section, we start by introducing the notion of the intuitionistic fuzzy soft complement matrix based on reference function, and we prove some formal properties.

4.1 Definition

Let $A = [(a_{ij}, 0)_{m \times n}] = [a_{ij}]IFSM_{m \times n}$, where $a_{ij} = (\mu_j(c_i), \gamma_j(c_i))$, for all i, j . Then A^c IFSM is called an Intuitionistic Fuzzy Soft Complement Matrix, if $A^c = [d_{ij}]_{m \times n}$, where $d_{ij} = (\gamma_j(c), \mu_j(c)) \forall i, j \in [0, 1]$

4.2 Proposition

Let A and B be two intuitionistic fuzzy soft matrix based on fuzzy reference function. Then

- (i) $(A^c)^T = (A^T)^c$
- (ii) $(A^c + B^c)^T = (A^T)^c + (B^T)^c$

Proof. To show (i) $(A^c)^T = (A^T)^c$

We have, let $A \in IFSF_{m \times n}$, then

$$A = [(\mu_{j1}(c_i), \mu_{j2}(c_i))]$$

$$A^c = [1, \mu_{j1}(c_i)]$$

$$(A^c)^T = [1, \mu_{j1}(c_i)]$$

$$\text{For } A^T = [(\mu_{j1}(c_i), \mu_{j2}(c_i))],$$

$$\text{we have } (A^T)^c = [1, \mu_{j1}(c_i)]$$

$$\text{Hence } (A^c)^T = (A^T)^c$$

The proof of (ii) follows similar lines as above.

□

4.3 Example

$$\text{Let } A = \begin{bmatrix} (0.3, 0.0) & (0.6, 0.0) \\ (0.4, 0.0) & (0.5, 0.0) \end{bmatrix}, B = \begin{bmatrix} (0.7, 0.0) & (0.4, 0.0) \\ (0.8, 0.0) & (0.2, 0.0) \end{bmatrix}$$

$$A^c = \begin{bmatrix} (1, 0.3) & (1, 0.6) \\ (1, 0.4) & (1, 0.5) \end{bmatrix}, B^c = \begin{bmatrix} (1, 0.7) & (1, 0.4) \\ (1, 0.8) & (1, 0.2) \end{bmatrix}$$

$$(A^c) = \begin{bmatrix} (1, 0.3) & (1, 0.4) \\ (1, 0.6) & (1, 0.5) \end{bmatrix}, (B) = \begin{bmatrix} (1, 0.7) & (1, 0.8) \\ (1, 0.4) & (1, 0.2) \end{bmatrix}$$

Then $(A^c + B^c)^T = (A^T)^c + (B^T)^c$

5 Operations on Intuitionistic Fuzzy Soft Matrix Theory

5.1 Addition and Subtraction of Intuitionistic Fuzzy Soft Matrix:

If $A = [a_{ij}] \in IFSM_{m \times n}, B = [b_{ij}] \in IFSM_{m \times n}$, then we define the addition and Subtraction of Intuitionistic Fuzzy Soft Matrices of A and B as:

$$A + B = \{ \max [(\mu_A(a_{ij}), \mu_B(b_{ij})), \min [v_A(a_{ij}), v_B(b_{ij})]] \} \forall i, j.$$

$$A - B = \{ \min [(\mu_A(a_{ij}), \mu_B(b_{ij})), \max [v_A(a_{ij}), v_B(b_{ij})]] \} \forall i, j.$$

5.2 Product of Intuitionistic Fuzzy soft Matrix:

If $A = [a_{ij}] \in IFSM_{m \times n}, B = [b_{ij}] \in IFSM_{m \times n}$, then we define multiplication of A and B as $A * B = [c_{ij}]_{m \times n}$ $S = \{ \max \min [(\mu_A(a_{ij}), \mu_B(b_{ij})), \min \max [v_A(a_{ij}), v_B(b_{ij})]] \} \forall i, j.$

5.3 Value Matrix:

Let $\tilde{A} = [(\mu_{ij}^A, v_{ij}^A)] \in IFSM_{m \times n}$. Then $\tilde{A}$ is said to be value of Intuitionistic fuzzy soft matrix denoted by $V(\tilde{A})$ and is defined as $V(\tilde{A}) = [(\mu_{ij}^A - v_{ij}^A)]$, if $i = 1, 2, 3, \dots, m, j = 1, 2, 3, \dots, n \forall i$ and j .

5.4 Score Matrix:

If $A = [(\mu_{ij}^A, v_{ij}^A)] \in IFSM_{m \times n}, B = [(\mu_{ij}^B, v_{ij}^B)] \in IFSM_{m \times n}$. Then A and B is said to be Intuitionistic fuzzy soft score matrix denoted by $S(A, B)$ and is defined as $S(A, B) = V(\tilde{A}) - V(B)$.

5.5 Total Score:

If $\tilde{A} = [(\mu_{ij}^A, v_{ij}^A)] \in IFSM_{m \times n}, B = [(\mu_{ij}^B, v_{ij}^B)] \in IFSM_{m \times n}$. Let the corresponding value matrix be $V(\tilde{A}), V(B)$ and their score matrix

is $S_{(A,B)}$. Then the total score of each u_i in U is $S_i = (V(\tilde{A}) - V(B))$

$$\sum_{j=1}^n [\mu_{ij}^A \cdot v_{ij}^A - (\mu_{ij}^B \cdot v_{ij}^B)]$$

6 New approach in a Decision Making Problem

ALGORITHM

Step 1: Input the intuitionistic fuzzy soft set $(F_A, E), (G_B, E)$ and obtain the intuitionistic fuzzy soft matrices A, B corresponding to $(F_A, E), (G_B, E)$ respectively.

Step 2: Write the intuitionistic fuzzy soft complement sets $(F_A, E)^c, (G_B, E)^c$ and obtain the intuitionistic fuzzy soft matrices $\tilde{A}^c, \tilde{B}^c$ corresponding to $(F_A, E)^c$ and $(G_B, E)^c$ respectively.

Step 3: Compute $(A * B), (A^c * B^c), V(A * B), V(A^c * B^c)$ and $S_{(A*B, (A^c*B^c))}$.

Step 4: Compute the total score S_i for each u_i in U .

Step 5: Find $S_k = \max(S_i)$, then we conclude that the best candidate u_k has the maximum value, since u_k is suitable for the post.

Step 6: If S_k has more than one value, then go to step (1) so as to repeat the process by reassessing the parameter.

Case Study:

Let (F_A, E) and (G_B, E) be the two intuitionistic fuzzy soft sets based on reference function representing the selection of General Secretary from the universal set $U = \{u_1, u_2, u_3, u_4\}$, the set of four persons related with a political party of a particular district by the experts X and Y . The party high command has to select only one person for nominating a candidate in the upcoming election. Let $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters for selecting a person, who stands for commitment to the society, experience in the field of politics, relation with the people, and clean image. Then the intuitionistic fuzzy soft matrix is constructed on the basis of the parameters as follows:

$$\begin{aligned} (F_A, E) &= \{ \{F_A(e_1) = \{(u_1, 0.1, 0.8), (u_2, 0.8, 0.1), (u_3, 0.6, 0.4), (u_4, 0.2, 0.7)\} \\ F_A(e_2) &= \{(u_1, 0.6, 0.2), (u_2, 0.2, 0.6), (u_3, 0.6, 0.3), (u_4, 0.8, 0.2)\} \\ F_A(e_3) &= \{(u_1, 0.5, 0.5), (u_2, 0.7, 0.2), (u_3, 0.5, 0.4), (u_4, 0.5, 0.3)\} \\ F_A(e_4) &= \{(u_1, 0.1, 0.7), (u_2, 0.3, 0.4), (u_3, 0.7, 0.6), (u_4, 0.6, 0.3)\} \} \end{aligned}$$

$$\begin{aligned}
(G_B, E) &= \{\{G_B(e_1) = \{(u_1, 0.6, 0.2), (u_2, 0.3, 0.3), (u_3, 0.7, 0.2), \\
&\quad (u_4, 0.5, 0.5)\}\} \\
G_B(e_2) &= \{(u_1, 0.5, 0.4), (u_2, 0.1, 0.2), (u_3, 0.7, 0.5), (u_4, 0.4, 0.7)\} \\
G_B(e_3) &= \{(u_1, 0.4, 0.4), (u_2, 0.3, 0.7), (u_3, 0.1, 0.5), (u_4, 0.8, 0.5)\} \\
G_B(e_4) &= \{(u_1, 0.6, 0.1), (u_2, 0.4, 0.3), (u_3, 0.2, 0.2), (u_4, 0.6, 0.2)\}
\end{aligned}$$

These two intuitionistic fuzzy soft sets are represented by the following intuitionistic fuzzy soft matrices respectively.

$$\begin{aligned}
A &= \begin{bmatrix} (0.1, 0.8) & (0.8, 0.1) & (0.6, 0.4) & (0.2, 0.7) \\ (0.6, 0.2) & (0.2, 0.6) & (0.6, 0.3) & (0.8, 0.2) \\ (0.5, 0.5) & (0.7, 0.2) & (0.5, 0.4) & (0.5, 0.3) \\ (0.1, 0.7) & (0.3, 0.4) & (0.7, 0.6) & (0.6, 0.3) \end{bmatrix} \\
B &= \begin{bmatrix} (0.6, 0.2) & (0.3, 0.3) & (0.7, 0.2) & (0.5, 0.5) \\ (0.5, 0.4) & (0.1, 0.2) & (0.7, 0.5) & (0.4, 0.7) \\ (0.4, 0.4) & (0.3, 0.7) & (0.1, 0.5) & (0.8, 0.5) \\ (0.6, 0.1) & (0.4, 0.3) & (0.2, 0.2) & (0.6, 0.2) \end{bmatrix}
\end{aligned}$$

Then the intuitionistic fuzzy soft complement matrices are

$$\begin{aligned}
A^C &= \begin{bmatrix} (0.8, 0.1) & (0.1, 0.8) & (0.4, 0.6) & (0.7, 0.2) \\ (0.2, 0.6) & (0.6, 0.2) & (0.3, 0.6) & (0.2, 0.8) \\ (0.5, 0.5) & (0.2, 0.7) & (0.4, 0.5) & (0.3, 0.5) \\ (0.5, 0.1) & (0.4, 0.3) & (0.6, 0.7) & (0.3, 0.6) \end{bmatrix} \\
B^C &= \begin{bmatrix} (0.2, 0.6) & (0.3, 0.3) & (0.2, 0.7) & (0.5, 0.5) \\ (0.4, 0.5) & (0.2, 0.1) & (0.5, 0.7) & (0.7, 0.4) \\ (0.4, 0.4) & (0.7, 0.3) & (0.5, 0.1) & (0.5, 0.8) \\ (0.1, 0.6) & (0.3, 0.4) & (0.2, 0.2) & (0.2, 0.6) \end{bmatrix}
\end{aligned}$$

Then the subtraction matrices are

$$\begin{aligned}
A - B &= \begin{bmatrix} (0.1, 0.8) & (0.3, 0.3) & (0.6, 0.4) & (0.2, 0.7) \\ (0.5, 0.4) & (0.1, 0.6) & (0.6, 0.5) & (0.4, 0.7) \\ (0.4, 0.5) & (0.3, 0.7) & (0.1, 0.5) & (0.5, 0.5) \\ (0.1, 0.7) & (0.3, 0.4) & (0.2, 0.6) & (0.6, 0.3) \end{bmatrix} \\
A^C - B^C &= \begin{bmatrix} (0.2, 0.6) & (0.1, 0.8) & (0.2, 0.7) & (0.5, 0.5) \\ (0.2, 0.6) & (0.2, 0.2) & (0.3, 0.7) & (0.2, 0.8) \\ (0.4, 0.5) & (0.2, 0.7) & (0.4, 0.5) & (0.3, 0.8) \\ (0.1, 0.6) & (0.3, 0.4) & (0.2, 0.7) & (0.2, 0.6) \end{bmatrix}
\end{aligned}$$

$$V(A - B) = \begin{bmatrix} -0.7 & 0.0 & 0.2 & -0.5 \\ 0.1 & -0.5 & 0.1 & -0.3 \\ -0.1 & -0.4 & -0.4 & 0.0 \\ -0.6 & -0.1 & -0.4 & 0.3 \end{bmatrix}$$

$$V(A^o - B^o) = \begin{bmatrix} -0.4 & -0.7 & -0.5 & 0.0 \\ -0.4 & 0.0 & -0.4 & -0.6 \\ -0.1 & -0.5 & -0.1 & -0.5 \\ -0.5 & -0.1 & -0.5 & -0.4 \end{bmatrix}$$

Calculate the score matrix $S_{(A-B, (A^C-B^C))}$ and total score for the best course for his son is given by

$$S_{(A-B, (A^C-B^C))} = \begin{bmatrix} -0.3 & 0.7 & 0.7 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.3 \\ 0.0 & 0.1 & -0.3 & 0.5 \\ -0.1 & 0.0 & 0.1 & 0.7 \end{bmatrix}$$

$$Totalscore = \begin{bmatrix} 0.6 \\ 0.8 \\ 0.4 \\ 0.7 \end{bmatrix}$$

We know that S_2 , the second candidate has the maximum value and thus we conclude that from both the experts opinion, candidate u_2 is selected for the post.

7 Conclusion

In our work, we have put forward some new concepts such as complement based on reference function. Some related properties have been established with example. An application of intuitionistic fuzzy soft matrix based on reference function in decision making problem is given.

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