

gs^{**} -closed set

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Abstract

A new class of set called gs^{**} -closed set has been introduced in this paper and its various characteristics are discussed. Further study on gs^{**} -closed set with other existing closed sets are made.

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1 Introduction

The notion of closed set is fundamental in the study of topological spaces. The generalized closed set has been first studied and initiated by N. Levine in the year 1970[1]. This generalized closed set has lead to significant contribution to generalization of continuity. Bhattacharya and Lahiri [3] introduced a closed set named sg-closed set. Further Arya and Nour [2] defined generalized semi open sets i.e. gs-open using semi-open sets. Furthermore Elvina Mary. L and R. Myvizhi [4] has worked on gs^* -closed set. As an extension of this closed set the concept of gs^{**} -closed set is to be introduced.

2 Preliminaries

Throughout this paper, topological space is represented by (X, τ) or X on which no separation axioms are assumed. The closure and interior for a subset A of a space X is denoted by $cl(A)$ and $int(A)$ respectively.

Definition 2.1 A subset A of a topological space (X, τ) is called a

- (1) pre-open [7] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
- (2) semi-open set [8] if $A \subseteq cl(int(A))$ and semi-closed set if $cl(int(A)) \subseteq A$.
- (3) semi-preopen set [9] if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set if $int(cl(int(A))) \subseteq A$.
- (4) α -open set [10] if $A \subseteq int(cl(int(A)))$ and α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition 2.2 A subset A of a topological space (X, τ) is called a

- (1) g -closed [1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (2) gs -closed [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
- (3) αg -closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (4) gp -closed set [11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (5) gsp -closed set [12] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (6) wg -closed set [13] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- (7) $s\alpha g^*$ -closed set [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in (X, τ) .
- (8) αg^{**} -closed set [15] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^{**} -open in (X, τ) .
- (9) sg^* -closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in (X, τ) .
- (10) sg^{**} -closed set [16] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^{**} -open in (X, τ) .

(11) g^*p -closed set [?] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .

3 gs^{**} -closed set and gs^{**} -open set

Definition 3.1 A subset A of a topological space (X, τ) is said to be **gs^{**} -closed** if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs^* -open in (X, τ) .

Example 3.1 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. The gs^{**} -closed sets are $\{\phi, X, \{a\}, \{a, b\}, \{c, a\}\}$.

Theorem 3.1 Every closed set is gs^{**} -closed set but not conversely.

proof: Let A be a closed set in X . Then $A = cl(A)$. Let $A \subseteq U$ and U is gs^* -open. Then $cl(A) = A \subseteq U$ whenever $A \subseteq U$ and U is gs^* -open. Hence $cl(A) \subseteq U$.
 $\therefore A$ is gs^{**} -closed.

Example 3.2 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{a, c\}$ is gs^{**} -closed but not closed.

Theorem 3.2 Every gs^{**} -closed set is gs -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is gs^* -open and A is gs^{**} -closed we have, $scl(A) \subseteq cl(A) \subseteq U$. Hence A is gs -closed set in X .

Example 3.3 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is gs -closed set but not gs^{**} -closed set.

Theorem 3.3 Every gs^{**} -closed set is αg -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is gs^* -open and A is gs^{**} -closed we have, $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is αg -closed.

Example 3.4 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is αg -closed set but not gs^{**} -closed set.

Theorem 3.4 Every gs^{**} -closed set is gp-closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is gs^* -open and A is gs^{**} -closed we have, $pcl(A) \subseteq cl(A) \subseteq U$. Hence A is gp-closed.

Example 3.5 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is gp-closed set but not gs^{**} -closed set.

Theorem 3.5 Every gs^{**} -closed set is gsp-closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is gs^* -open and A is gs^{**} -closed we have, $spcl(A) \subseteq cl(A) \subseteq U$. Hence A is gsp-closed.

Example 3.6 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is gsp-closed set but not gs^{**} -closed set.

Theorem 3.6 Every gs^{**} -closed set is wg-closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is gs^* -open and A is gs^{**} -closed we have, $cl(int(A)) \subseteq cl(A) \subseteq U$. Hence A is wg-closed.

Example 3.7 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is wg-closed set but not gs^{**} -closed set.

Theorem 3.7 Every gs^{**} -closed set is sag^* -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let $A \subseteq U$ and U be g^* -open. Then $A \subseteq U$ and U is gs^* -open and $cl(A) \subseteq U$, since A is gs^{**} -closed. Then $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is sag^* -closed.

Example 3.8 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is sag^* -closed set but not gs^{**} -closed set.

Theorem 3.8 Every gs^{**} -closed set is ag^{**} -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let $A \subseteq U$ and U be g^{**} -open. Then $A \subseteq U$ and U is gs^* -open and $cl(A) \subseteq U$, since A is gs^{**} -closed. Then $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is αg^{**} -closed.

Example 3.9 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is αg^{**} -closed set but not gs^{**} -closed set.

Theorem 3.9 Every gs^{**} -closed set is sg^* -closed but not conversely..

proof: Let A be gs^{**} -closed set in X . Let $A \subseteq U$ and U be g^* -open. Then $A \subseteq U$ and U is gs^* -open and $cl(A) \subseteq U$, since A is gs^{**} -closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is sg^* -closed.

Example 3.10 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is sg^* -closed set but not gs^{**} -closed set.

Theorem 3.10 Every gs^{**} -closed set is sg^{**} -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let $A \subseteq U$ and U be g^{**} -open. Then $A \subseteq U$ and U is gs^* -open and $cl(A) \subseteq U$, since A is gs^{**} -closed. Then $scl(A) \subseteq cl(A) \subseteq U$. Hence A is sg^{**} -closed.

Example 3.11 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is sg^{**} -closed set but not gs^{**} -closed set.

Theorem 3.11 Every gs^{**} -closed set is g^*p -closed but not conversely.

proof: Let A be gs^{**} -closed set in X . Let U be open set such that $A \subseteq U$. Since every open set is g -open and A is gs^{**} -closed we have, $pcl(A) \subseteq cl(A) \subseteq U$. Hence A is g^*p -closed set in X .

Example 3.12 Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. Then $A = \{b\}$ is g^*p -closed set but not gs^{**} -closed set.

The above results can be illustrated by the figure given at the end of this section.

Definition 3.2 A subset A of a topological space (X, τ) is called gs^{**} -open set if and only if A^c is gs^{**} -closed in X .

Theorem 3.12 (i) Every open set is gs^{**} -open set.
(ii) Every gs^{**} -open set is gs -open, αg -open, gp -open, sg^* -open, sg^{**} -open.
(iii) Every gs^{**} -open set is gsp -open, wg -open, αg^{**} -open, $s\alpha g^*$ -open.

4 Characteristics of gs^{**} -closed and gs^{**} -open sets

Theorem 4.1 The finite union of gs^{**} -closed set is gs^{**} -closed.

proof: Let A and B be gs^{**} -closed sets in X . Let U be gs^* -open in X such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are gs^{**} -closed sets, we have $cl(A) \subseteq U$ and $cl(B) \subseteq U$. $cl(A) \cup cl(B) \subseteq U$. i.e. $cl(A \cup B) \subseteq U$. Therefore, $A \cup B$ is gs^{**} -closed.

Theorem 4.2 Intersection of any two gs^{**} -closed sets is closed.

proof: Let A and B be gs^{**} -closed sets in X . Let U be a gs^* -open set in X such that $A \cap B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. since A and B are gs^{**} -closed sets, we have $cl(A) \subseteq U$ and $cl(B) \subseteq U$. $cl(A) \cap cl(B) \subseteq U$. i.e. $cl(A \cap B) \subseteq U$. Hence $A \cap B$ is closed.

Remark 4.1 The finite intersection of gs^{**} -closed set need not to be gs^{**} -closed set. It can be seen from the following example.

Example 4.1 Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, X, \{c\}\}$
The gs^{**} -closed sets are $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{c, a\}\}$.
Let A and B be two gs^{**} -closed sets.
Let $A = \{b, c\}$ and $B = \{a, c\}$
Then $A \cap B = \{c\}$
which is not gs^{**} -closed set.

Theorem 4.3 Let A and B be subsets of X such that $A \subseteq B \subseteq cl(A)$. If A is gs^{**} -closed then B is gs^{**} -closed.

proof: Let A and B be subsets such that $A \subseteq B \subseteq cl(A)$. suppose that A is gs^{**} -closed. Then $A \subseteq U$ and U is gs^* -open in X . Since A is gs^{**} -closed, we have $cl(A) \subseteq U$. Since $B \subseteq cl(A)$, we have $cl(B) \subseteq cl(cl(A)) = cl(A) \subseteq U$. $cl(B) \subseteq U$. Hence B is gs^{**} -closed set.

Theorem 4.4 If A and B are gs^{**} -open sets in X then $A \cap B$ is also gs^{**} -open set in X

proof: Suppose A and B are gs^{**} -open sets in X . Then A^c and B^c are gs^{**} -closed sets in X . $A^c \cup B^c$ is gs^{**} closed set in X . we know that $A^c \cup B^c = (A \cap B)^c$. $(A \cap B)^c$ is gs^{**} -closed set. $A \cap B$ is gs^{**} -open set.

Theorem 4.5 A set A is gs^{**} -open if and only if $F \subseteq int(A)$, where F is gs^* -closed and $F \subseteq A$.

proof: Necessity: Suppose $F \subseteq int(A)$ where F is gs^* -closed and $F \subseteq A$. Let $A^c \subseteq G$, where $G = F^c$ i.e. G is gs^* -open. Then $G^c \subseteq A$, where G^c is gs^* -closed. By assumption, $G^c \subseteq int(A)$.

$cl(A^c) \subseteq G$. Therefore A^c is gs^{**} -closed which implies A is gs^{**} -open.

Sufficiency: Suppose A is gs^{**} -open with $F \subseteq A$ where F is gs^* -closed. Then F^c is gs^* -open and $A^c \subseteq F^c$. Therefore $cl(A^c) \subseteq F^c$ implies $F \subseteq int(A)$.

Theorem 4.6 If a set A is gs^{**} -closed the $cl(A)$ - A contains no non-empty closed set.

proof: Let F a closed subset of $cl(A)$ - A . Then $A \subseteq F^c$. Since A is gs^{**} -closed, $cl(A) \subseteq F^c$. Hence $F \subseteq (cl(A))^c$. We have $F \subseteq cl(A) \cap (cl(A))^c = \phi$. And hence F is empty.

Theorem 4.7 If A is gs^* -open and gs^{**} -closed in X then A is closed in X .

proof: Since A is gs^* -open and gs^{**} -closed in X , then $cl(A) \subseteq A$ and hence A is closed in X .

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