

Generalized triple entire difference of
rough intuitionistic statistical
convergence in probability of fractional
order defined by
Musielak-Orlicz function

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Abstract

We generalized the concepts in probability of rough intuitionistic convergence and rough intuitionistic statistically convergence by introducing the generalized difference operator Δ_γ^α of fractional order, where α is a proper fraction and $\gamma = (\gamma_{mnk})$ is any fixed sequence of nonzero real or complex numbers. We study some properties of this operator and investigate the topological structures of related of triple difference sequence spaces of $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk})$.

The main focus of the present paper is to generalized rough intuitionistic convergence and rough intuitionistic statistical difference of triple difference sequence spaces of Γ^3 and investigate their topological structures as well as some interesting results concerning the operator Δ_γ^α .

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1 Introduction

A triple sequence (real or complex) can be defined as a function $x : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively, the different types of notion of triple sequence was introduced and investigated by Sahiner et al. [10,11], Esi et al. [1-3], Datta et al. [4], Subramanian et al. [12], Debnath et al. [5] and many others. A triple sequence $x = (x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3 . A triple sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty.$$

The space of all triple entire sequences are usually denoted by Γ^3 . The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [6] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c, c_0$ and ℓ_∞ , where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$.

The difference triple sequence space was introduced by Debnath et al. (see [5]) and is defined as

$$\Delta x_{mnk} = x_{mnk} - x_{m,n+1,k} - x_{m,n,k+1} + x_{m,n+1,k+1} - x_{m+1,n,k} + x_{m+1,n+1,k} + x_{m+1,n,k+1} - x_{m+1,n+1,k+1} \text{ and } \Delta^0 x_{mnk} = x_{mnk}.$$

2 Some new difference triple sequence spaces with fractional order

Let $\Gamma(\alpha)$ denote the Euler gamma function of a real number α . Using the definition $\Gamma(\alpha)$ with $\alpha \notin \{0, -1, -2, -3, \dots\}$, $\Gamma(\alpha)$ can be

expressed as an improper integral as follows: $\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx$, where α is a positive proper fraction. We have defined the generalized fractional triple sequence spaces of difference operator

$$\Delta_\gamma^\alpha(x_{mnk}) = \sum_{u=0}^\infty \sum_{v=0}^\infty \sum_{w=0}^\infty \frac{(-1)^{u+v+w} \Gamma(\alpha+1)}{(u+v+w)! \Gamma(\alpha-(u+v+w)+1)} \times x_{m+u, n+v, k+w}. \quad (1)$$

In particular, we have

- (i) $\Delta_\gamma^{\frac{1}{2}}(x_{mnk}) = x_{mnk} - \frac{1}{16}x_{m+1, n+1, k+1} - \dots$
- (ii) $\Delta_\gamma^{-\frac{1}{2}}(x_{mnk}) = x_{mnk} + \frac{5}{16}x_{m+1, n+1, k+1} + \dots$
- (iii) $\Delta_\gamma^{\frac{2}{3}}(x_{mnk}) = x_{mnk} - \frac{4}{81}x_{m+1, n+1, k+1} - \dots$

Now we determine the new classes of triple difference sequence spaces $\Delta_\gamma^\alpha(x)$ as follows:

$$\Delta_\gamma^\alpha(x) = \{x : (x_{mnk}) \in w^3 : (\Delta_\gamma^\alpha x) \in X\}, \quad (2)$$

where $\Delta_\gamma^\alpha(x_{mnk}) = \sum_{u=0}^\infty \sum_{v=0}^\infty \sum_{w=0}^\infty \frac{(-1)^{u+v+w} \Gamma(\alpha+1)}{(u+v+w)! \Gamma(\alpha-(u+v+w)+1)} \times x_{m+u, n+v, k+w}$ and $(\Delta_\gamma^\alpha X, *, \eta) \in \Gamma_f^{3\Delta}(x, *, \eta) = \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) = \mu_{mnk}(\Delta_\gamma^\alpha x) = \left[f_{mnk} \left(|\Delta_\gamma^\alpha x|^{\frac{1}{m+n+k}}, \bar{0} \right) \right]$.

3 Definitions and Preliminaries

Throughout the article $w^3, \Gamma^3(\Delta), \Lambda^3(\Delta)$ denote the spaces of all, triple entire difference sequence spaces and triple analytic difference sequence spaces respectively.

Subramanian et al. (see [12]) introduced triple entire sequence spaces, triple analytic sequences spaces and triple gai sequence spaces. The triple sequence spaces of $\Gamma^3(\Delta), \Lambda^3(\Delta)$ are defined as follows:

$$\Gamma^3(\Delta) = \left\{ x \in w^3 : |\Delta x_{mnk}|^{1/m+n+k} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty \right\},$$

$$\Lambda^3(\Delta) = \left\{ x \in w^3 : \sup_{m, n, k} |\Delta x_{mnk}|^{1/m+n+k} < \infty \right\}.$$

Definition 1. An Orlicz function ([see [7]]) is a function $M : [0, \infty) \rightarrow [0, \infty)$ which is continuous, non-decreasing and convex

with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$, then this function is called modulus function.

Lindenstrauss and Tzafriri [8] used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = \sup \{ |v| u - (f_{mnk})(u) : u \geq 0 \}, m, n, k = 1, 2, \dots$$

is called the complementary function of a Musielak-Orlicz function f . For a given Musielak-Orlicz function f , [see [9]] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f = \left\{ x \in w^3 : I_f(|x_{mnk}|)^{1/m+n+k} \rightarrow 0 \text{ as } m, n, k \rightarrow \infty \right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk}(|x_{mnk}|)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} f_{mnk} \left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk} \right)$$

is an extended real number.

Definition 2. A ternary operation $*$: $[0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is said to be continuous with metric if it satisfies the following conditions:

(a) $*$ is associative and commutative, (b) $*$ continuous, (c) $a * 1 = a$ for all $a \in [0, 1]$, (d) $a * c \leq b * d$ whenever $a \leq b$ and $c \leq d$ for each $a, b, c, d \in [0, 1]$. For example $d(a, b) = a * b = a \cdot b = d(b, a)$.

Definition 3. A ternary operation η : $[0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ is said to be continuous with co-metric if it satisfies the following conditions:

(a) η is associative and commutative, (b) η continuous, (c) $a\eta 0 = a$ for all $a \in [0, 1]$, (d) $a\eta c \leq b\eta d$ whenever $a \leq b$ and $c \leq d$ for each $a, b, c, d \in [0, 1]$.

Note 4. The three tuple $(X, *, \eta)$ is said to be an intuitionistic metric space (for short, IFMS) if X is a vector space, $*$ is a continuous metric and η is a continuous co-metric on $X \times X \times X \times (0, \infty) \times (0, \infty) \times (0, \infty)$.

Definition 5. Let α be a proper fraction and β be a nonnegative real number. A triple difference sequence spaces of $(\Delta_\gamma^\alpha X, *, \eta) = (\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ is said to be Δ_γ^α rough intuitionistic convergent to $\bar{0}$ with respect to the roughness of degree β if for every $\epsilon > 0$, there exists a natural number such that $|\Delta_\gamma^\alpha x_{mnk}, \bar{0}| < \beta + \epsilon$ for all $m \geq m_0, n \geq n_0, k \geq k_0$ and it is denoted by $(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_\beta^\Delta \bar{0}$. If we take $\beta = 0$, then we obtain the ordinary intuitionistic convergence.

Definition 6. Let α be a proper fraction and β be a nonnegative real number. A triple difference sequence spaces of $(\Delta_\gamma^\alpha X, *, \eta) = (\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ is said to be Δ_γ^α rough intuitionistic statistically convergent to $\bar{0}$ with respect to the roughness of degree β if for every $\epsilon > 0$, the set $\{(m, n, k) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\Delta_\gamma^\alpha x_{mnk}, \bar{0}| \geq \beta + \epsilon\}$ has asymptotic density zero, if the conditions $S\text{-}\lim_{mnk \rightarrow \infty} \sup |\Delta_\gamma^\alpha x_{mnk}, \bar{0}| \leq \beta$ and it is denoted by $(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_\beta^{S\Delta} \bar{0}$. If we take $\beta = 0$, then we obtain the ordinary intuitionistic statistical convergence.

Definition 7. Let α be a proper fraction and β be a nonnegative real number. A triple difference sequence spaces of $(\Delta_\gamma^\alpha X, *, \eta) = (\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ is said to be Δ_γ^α rough intuitionistic statistically convergent in probability to a random variable $(\Delta_\gamma^\alpha X, *, \eta) : W \times W \times W \rightarrow \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ with respect to the roughness of degree β if for each $\epsilon, \delta > 0$ such that $\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{m \leq u, n \leq v, k \leq w : P(|\Delta_\gamma^\alpha x_{mnk}, \bar{0}| \geq \beta + \epsilon) \geq \delta\}| = 0$, or equivalently $\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{m \leq u, n \leq v, k \leq w : 1 - P(|\Delta_\gamma^\alpha x_{mnk}, \bar{0}| < \beta + \epsilon) \geq \delta\}| = 0$ and we write $(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_\beta^{SP\Delta}$. The class of all $\beta\Delta_\gamma^\alpha$ -rough intuitionistic statistically convergent of triple difference sequence spaces of random variables in probability and it is denoted by $\beta SP\Delta$.

Definition 8. Let α be a proper fraction. A triple difference sequence spaces of $(\Delta_\gamma^\alpha X, *, \eta) = (\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ is said to be Δ_γ^α intuitionistic strong Cesàro summable to $\bar{0}$ if $\lim_{uvw \rightarrow \infty} \frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w |\Delta_\gamma^\alpha x_{mnk}, \bar{0}| = 0$. In this we write $(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{[C, 1, 1, 1]}^\Delta \Delta_\gamma^\alpha x_{mnk}$. The set of all Δ_γ^α intuitionistic strong Cesàro summable triple difference sequence spaces is denoted by $[C, 1, 1, 1]$.

Definition 9. Let α be a proper fraction and β be a nonnegative real number. A triple difference sequence spaces of $(\Delta_\gamma^\alpha X, *, \eta) = (\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ is said to be Δ_γ^α rough intuitionistic strong Cesàro summable in probability to a random variable $(\Delta_\gamma^\alpha X, *, \eta) : W \times W \times W \rightarrow \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ with respect to the roughness of degree β if for each $\epsilon > 0$, $\lim_{uvw \rightarrow \infty} \frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w P(|\Delta_\gamma^\alpha x_{mnk}, \bar{0}| \geq \beta + \epsilon) = 0$. In this case we write $(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{[C, 1, 1, 1]^{P\Delta}} \Delta_\gamma^\alpha x_{mnk}$. The class of all $\beta \Delta_\gamma^\alpha$ -rough intuitionistic strong Cesàro summable triple difference sequence spaces of random variables in probability and it will be denoted by $\beta [C, 1, 1, 1]^{P\Delta}$.

4 Main Results

Theorem 10. Let α be a proper fraction, β be a nonnegative real number and f be an Musielak-Orlicz function. If $(\Delta_\gamma^\alpha X, *, \eta)$ rough intuitionistic then the followings are equivalent:

- (i) $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$
- (ii) $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\alpha}^{[C, 1, 1, 1]^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\beta x_{mnk}, *, \eta)$

Proof. (i) $\implies$ (ii) First suppose that $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$. Then we can write $\frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) = \frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1, P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \geq \frac{\delta}{2}}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) + \frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1, P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) < \frac{\delta}{2}}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \leq \frac{1}{uvw} |\{\sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \geq \frac{\delta}{2}\}| + \frac{\delta}{2}$.

(ii) $\implies$ (i): Next suppose that condition (ii) holds. Then $\frac{\sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon)}{\sum_{m=1}^u \sum_{n=1}^v \sum_{k=1, P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \geq \delta}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon)} \delta |\{m \leq u, n \leq v, k \leq w : P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \geq \delta\}|$. Therefore $\frac{1}{uvw} \sum_{m=1}^u \sum_{n=1}^v \sum_{k=1}^w P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon)$

$$\geq \frac{1}{uvw} |\{m \leq u, n \leq v, k \leq w : P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}))] \geq \beta + \epsilon) \geq \delta\}|.$$

Therefore $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$. $\square$

Theorem 11. Let α be a proper fraction, β, η be two nonnegative real numbers and f be an Musielak-Orlicz function. If rough intuitionistic $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta)$ and $\Gamma_f^3(\Delta_\gamma^\alpha y_{mnk}, *, \eta) \rightarrow_{\eta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha y, *, \eta)$ then $P\{|\Delta_\gamma^\alpha x - \Delta_\gamma^\alpha y| \geq \beta + \eta\} = 0$.

Proof. Let ϵ, δ be any two positive real numbers and let

$$|\{(u, v, w) \in \mathbb{N} : P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu(\Delta_\gamma^\alpha x)))] \geq \beta + \frac{\epsilon}{2}) < \frac{\delta}{2}\}|$$

$$\cap |\{(u, v, w) \in \mathbb{N} : P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu(\Delta_\gamma^\alpha y)))] \geq \eta + \frac{\epsilon}{2}) < \frac{\delta}{2}\}|.$$

Then we have $P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x) - \mu_{mnk}(\Delta_\gamma^\alpha y)))] \geq \beta + \eta + \epsilon) \leq$

$$P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))] \geq \beta + \frac{\epsilon}{2}) +$$

$$P([f_{mnk}(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y)))] \geq \eta + \frac{\epsilon}{2} < \delta. \text{ Hence}$$

$$P\{[f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x) - \mu_{mnk}(\Delta_\gamma^\alpha y)))] \geq \beta + \eta\} = 0. \quad \square$$

Theorem 12. Let α be a proper fraction, β be a nonnegative real numbers and f be an Musielak-Orlicz function. If a triple rough intuitionistic difference sequence spaces $(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ of constants $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S\Delta} \Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta)$ then $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta)$.

Proof. A triple rough intuitionistic difference sequence spaces of $(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ can be regarded as a random variable with three element $(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ in the corresponding spectrum. Let ϵ be a positive real number. Since $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta) \rightarrow_{\beta}^{S\Delta} \Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta)$ then

$$\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{m \leq u, n \leq v, k \leq w : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x)))] \geq \beta + \epsilon\}| = 0.$$

$\implies$

$$\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{m \leq u, n \leq v, k \leq w : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x)))] < \beta + \epsilon\}| = 1.$$

Now the event

$|\{w : w \in W \times W \times W : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x)))] < \beta + \epsilon\}|$ is the same as the event $|\{[f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x)))] < \beta + \epsilon\}|$. So

$$P(|\{w : w \in W \times W \times W : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x)))] < \beta + \epsilon\}|) = P(W) = 1$$

$$= \{(m, n, k) \in \mathbb{N} : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))] < \beta + \epsilon\}.$$

Thus for any $\delta > 0$,

$$\{(u, v, w) \in \mathbb{N} : 1 - P([f_{mnk}((\mu_{uvw}(\Delta_\gamma^\alpha x) - \mu(\Delta_\gamma^\alpha x)))] < \beta + \epsilon) \geq \delta\} =$$

$$\{(m \leq u, n \leq v, k \leq w) \in \mathbb{N} : [f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x) - \mu(\Delta_\gamma^\alpha x)))] \geq \beta + \epsilon\}.$$

In general converse is not true. Consider the following example: $\square$

Example 13. Let a triple rough intuitionistic difference sequence spaces of fractional order of random variables $\Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ be defined by, $\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = \{-10^{m+n+k}, 10^{m+n+k}\}$, with probability,

$$P(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = -10^{m+n+k}) = P(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = 10^{m+n+k});$$

if $m = u^2, n = v^2, k = w^2$ for some $u, v, w \in \mathbb{N}$ and

$$\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = \{0, 1^{m+n+k}\}, \text{ with probability,}$$

$$P(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = 0) = P(\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) = 1^{m+n+k}); \text{ if } m \neq u^2, n \neq v^2, k \neq w^2 \text{ for any } u, v, w \in \mathbb{N}. \text{ Let } 0 < \epsilon < 1 \text{ be given.}$$

Then,

$$P(|\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - 1| > 2 + \epsilon) = 1^{m+n+k}, \text{ if } m = u^2, n = v^2, k = w^2 \text{ for some } u, v, w \in \mathbb{N}; \text{ if } m \neq u^2, n \neq v^2, k \neq w^2 \text{ for any } u, v, w \in \mathbb{N}.$$

This implies $(\Delta_\gamma^\alpha x_{mnk}) \rightarrow_2^{S^P\Delta} 1$. But it is not ordinary rough intuitionistic statistical convergence of a triple difference sequence spaces of numbers of fractional order to 1.

Theorem 14. Let α be a proper fraction, β be a nonnegative real numbers and f be an Musielak-Orlicz function. If $(\Delta_\gamma^\alpha X, *, \eta)$ be a triple rough intuitionistic difference sequence spaces of random variable then, $\Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta) \rightarrow_\beta^{S^P\Delta} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ if and only if there exists a triple difference sequence spaces of random variables $(\Delta_\gamma^\alpha y_{mnk}, *, \eta)$ such that $\Gamma_f^3(\Delta_\gamma^\alpha y_{mnk}, *, \eta) \rightarrow^{S^P\Delta} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ and $S\text{-}\lim_{mnk \rightarrow \infty} P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk})))] > \beta) = 0$.

Proof. Let $\Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta) \rightarrow_\beta^{S^P\Delta} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$.

$$\text{Let } A = \{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk})))] \geq \beta + \epsilon) \geq \delta\}.$$

Then $d(A) = 0$.

$$\text{Now we define } \Delta_\gamma^\alpha y_{mnk} = \begin{cases} x^{m+n+k}, & \text{if } m, n, k \in \mathbb{N} \setminus A \\ x^{m+n+k} + Z; & \text{if otherwise} \end{cases}.$$

Then it is very obvious that

$$d(\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x_{mnk})))] \geq \epsilon) \geq \delta\}) =$$

0

and

$$d(\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk})))]) \geq \beta + \epsilon \geq \delta\}) \leq$$

$$d(\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \beta + \frac{\epsilon}{2} \geq \frac{\delta}{2}\}) +$$

$$d(\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \beta + \epsilon \geq \delta\}) = 0.$$

Conversely, let $\Gamma_f^3(\Delta_\gamma^\alpha y_{mnk}, *, \eta) \xrightarrow{S^{P\Delta}} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta)$ and

$$S - \lim_{mnk \rightarrow \infty} P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x) - \mu_{mnk}(\Delta_\gamma^\alpha y)))]) > \beta) = 0.$$

Then for each $\epsilon, \delta > 0$,

$$\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \frac{\epsilon}{2} \geq \frac{\delta}{2}\}| = 0$$

$$\lim_{uvw \rightarrow \infty} \frac{1}{uvw} |\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}))]) \geq \beta + \frac{\epsilon}{2} \geq \frac{\delta}{2}\}| = 0.$$

We know the inequality

$$P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}))]) \geq \beta + \epsilon) \leq$$

$$P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \frac{\epsilon}{2} +$$

$$P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}))]) \geq \beta + \frac{\epsilon}{2}).$$

This implies

$$\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \beta + \epsilon \geq \delta\} \subseteq$$

$$\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha x)))]) \geq \frac{\epsilon}{2} \geq \frac{\delta}{2}\} \cup$$

$$\{P([f_{mnk}((\mu_{mnk}(\Delta_\gamma^\alpha x_{mnk}) - \mu_{mnk}(\Delta_\gamma^\alpha y_{mnk}))]) \geq \beta + \frac{\epsilon}{2} \geq \frac{\delta}{2}\}.$$

$$\text{Hence } \Gamma_f^3(\Delta_\gamma^\alpha x, *, \eta) \xrightarrow{\beta} \Gamma_f^3(\Delta_\gamma^\alpha x_{mnk}, *, \eta).$$

□

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