

Thermal and Solutal Dispersion Effects on Heat and Mass Transfer in a Viscous MHD Flow over a Vertical Plate

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Abstract

A Magneto hydrodynamic incompressible viscous fluid flow over a vertical porous plate is considered. The influence of double dispersion and viscous dissipation on heat and mass transfer are studied. The flow constitutive equations are converted to a set of non-linear ordinary differential equations using similarity transformation and then are solved numerically by MATLAB fourth order Boundary Value Problem Solver. Flow characteristics are analyzed graphically for various parameters.

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1 Introduction

Convective flow over a flat surface having pores is of very much interested for as its wide applications in environmental, geophysical

and energy related engineering problems eg., migration of moisture in fibrous insulation, drying of porous solid, welding in manufacturing processes. [1] examined the combined heat and mass transfer effects in a free convective boundary layer flow. Later [2], studied heat and mass transfer in free convection in a porous enclosure. Recently, much amount of work has been credited towards the mixed convective boundary layer problems as it plays a key role in astrophysical and geophysical problems. More recently the problem of fusion power, involves containment of hot plasmas by electromagnetic forces. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields. [3] noticed the behaviour of Viscous dissipation and Joule heating effects of the fluid over heat and mass transfer. [4] analyzed double dispersion effects on heat and mass transfer in a non-Darcy free convection flow. [5] explained the effect of viscous dissipation on the Darcian free convection over a non-isothermal body. Many problems of Darcian and non-Darcian mixed convection about a vertical plate had been reported, as in [6]. Double Dispersion on Mixed Convection Heat and Mass Transfer in Non-Darcy Porous Medium has been studied by [7]. The present paper deals heat and mass transfer pattern when a viscous fluid flow over a vertical plate is subjected to both magnetic and dispersion effects. In this context we solved the governing equations of motion numerically through 'BVP4C' routine programme. Heat and mass transfer across the boundary layer are analyzed graphically against viscous dissipation and Lewis number.

2 Mathematical Formulation

Consider a two dimensional incompressible viscous laminar flow over a vertical flat porous plate. A uniform magnetic field of strength H_0 is applied normal to the direction of flow, ignoring induced magnetic field. Also assume the fluid is experienced by both thermal and solute dispersions. It is also assumed that both temperature (T_w) and concentration (C_w) of the plate are constant. Further the temperature (T_∞) and concentration (C_∞) of the ambient fluid are assumed to be constant. (x, y) are coordinates measured along and normal to the leading edge of the plate. Under these conditions the

governing equations of motion for the boundary layer flow in the region $x \geq 0$ and $y \geq 0$ are derived as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\left[1 + \left(\frac{K \sigma u_e^2 H_0^2}{\mu} \right) \right] \frac{\partial u}{\partial y} + \frac{a \sqrt{K}}{\nu} \frac{\partial u^2}{\partial y} = \pm \frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y} \right) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\alpha_e \frac{\partial T}{\partial y} \right) + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left(D_e \frac{\partial C}{\partial y} \right) - K_1 (C - C_\infty) \quad (4)$$

where u, v -velocity components along x and y directions, K -porous permeability, σ -electrical conductivity, μ_e -magnetic permeability, μ -dynamic viscosity, a -Forchheimer empirical constant, ν -kinematic viscosity, g -acceleration due to gravity, β_T, β_C -coefficient of thermal and solute expansions respectively, K_1 -coefficient of first order chemical reaction, T, C -temperature and concentration of the fluid within boundary layer, c_p -specific heat at constant pressure. Further thermal diffusivity $\alpha_e = \alpha_m + \gamma u d$ where α_m -dispersion thermal diffusivity, γ -coefficient of dispersion, d -mean diameter of particle. Mass diffusivity $D_e = D_m + \zeta u d$ where D_m -dispersion solutal diffusivity, β -coefficient of solutal dispersion. The boundary conditions are

$$v = 0, T = T_w, C = C_w \text{ at } y = 0; u \rightarrow u_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (5)$$

Introducing a stream function $\psi = f(\eta)(\alpha_m u_\infty x)^{0.5}$ which satisfy the velocity components $u = \frac{\partial \psi}{\partial x}$ $v = -\frac{\partial \psi}{\partial y}$ where $f(\eta)$ is known as dimensionless stream function and $\eta(x, y) = \frac{y}{x} \left(\frac{u_\infty x}{\alpha_m} \right)^{\frac{1}{2}}$, called similarity variable. Further define non-dimension quantities temperature and concentration in terms of similarity variable as $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ and $\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$. These dimensionless quantities transform the governing equations of fluid flow given in (2) – (4) into the following set of second order coupled non-linear ordinary differential equations.

$$(1 + M + F f') f'' = \frac{Ra}{Pe} (\theta' + N \phi') \quad (6)$$

$$2\theta'' + f\theta' + 2PrEc_c f''^2 + 2D(f''\theta' + f'\theta'') = 0 \quad (7)$$

$$2\phi'' + 2LeB(f'\phi'' + f''\phi') + Lef\phi' - 2\gamma Le\phi = 0 \quad (8)$$

Also the boundary conditions(5)are converted to the following dimensionless form

$$f = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \text{ and } f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (9)$$

where $F = \frac{2a\sqrt{K}u_\infty}{\nu}$ -Forchhemier parameter,

$Ra_x = \frac{Kg\beta_T(T_w - T_\infty)x}{\nu\alpha_m}$ -local Rayleigh number,

$M = \frac{K\sigma\mu_e^2 H_0^2}{\nu\alpha_m}$ - Hartmann number,

$N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ - buoyancy ratio parameter,

$Ec = \frac{u_\infty^2}{C_p((T_w - T_\infty))}$ - viscous dissipation parameter,

$Le = \frac{\alpha_m}{D_m}$ -Lewis number,

$Pr = \frac{\nu}{\alpha_m}$ -Prandtl number, $\frac{Ra}{Pe}$ -mixed convection parameter,

$B = \frac{\zeta du_\infty}{\alpha_m}$ -solute dispersion,

$D = \frac{\gamma du_\infty}{\alpha_m}$ -thermal dispersion and $Pe_x = \frac{u_\infty x}{\alpha_m}$ -local Peclet number.

Heat and mass transfer are measured in terms of local Nusselt and Sherwood numbers respectively. These quantities in dimensionless form are derived as $\frac{Nu_x}{Ra_x^{0.5}} = -\theta'(0)$ and $\frac{Sh_x}{Ra_x^{0.5}} = -\phi'(0)$

3 Results and discussion

The set of coupled non-linear ordinary differential equations are solved numerically by MATLAB 'bvp4c', 4th order boundary value problem solver. This is a residual control based adaptive mesh solver [8]. Instead of uniform mesh, this provides a uniform grid of data points x over the interval $\frac{1}{2x}$ and solve accordingly. The relative tolerance is taken upto 10^{-10} . The maximum value for similarity variable is $\eta_{max} = 15$ which represents the conditions far away from the fluid. The computations are carried out for $Pr = 0.73, 0 \leq Ec \leq 4, 0 \leq Le \leq 4$

Fig.1 witnesses the effect of Hartmann number M on momentum boundary layer. A significant velocity profile descent is noticed in assisting flow with an increase in Hartmann number. In point of physics this retardation occurs due to the impact of dragging force developed by the transverse magnetic field. But for opposing flow, a

slight increase in velocity profile is observed by increasing magnetic parameter. This is because of opposing buoyancy on the fluid. Further from the same figure we observe that velocity is enhanced by increasing thermal dispersion for the case of assisting flow and the reversal is observed for opposing fluid flow.

From fig.2 and fig.3, it is noticed that increasing Hartmann number results an increment in both temperature and concentration profiles. This change is due to, Lorentz force, an opposing force that resists the fluid motion within the boundary layer. further a substantial hike in temperature and concentration profiles is observed with respect to thermal and solutal dispersion. Fig.4 explains the influence of magnetic parameter on heat transfer. At low magnetic number ($0 < M < 1.5$), heat transfer rate is more in forced convection rather than mixed convection. The same is reversed for $M > 1.5$. This may be explained as follows. Increasing magnetic parameter leads to more resistance in fluid motion as a result more temperature gradient occurs. Further, dispersion enhances the heat transfer and this is more significant in opposing flows.

Fig.5 depicts the influence of Lewis number on mass transfer. A remarkable increase in mass transfer is observed in aiding flows with increasing of Lewis number. For aiding flows, Sherwood number decreases with increase in solutal dispersion. It is interesting to note that in case of opposing flows, for $0 < Le < 2$, mass transfer increases by increasing solute dispersion but for $Le > 2$ simply this profile is reversed and behaves like mass transfer profile in aiding flows. From fig.6 we can see that heat transfer decreases with increasing thermal dispersion coefficient. A linear descent profile is observed in heat transfer profile with increase in Eckert number. Viscous dissipation acts like heat generating source so the raise in temperature will decrease heat transfer rate. A transition takes place at $Ec = 3$ for opposing flows. The effect of Lewis number on heat transfer can be viewed in fig.7. There is a markable rate of heat transfer at low Lewis numbers and decreases as Lewis number increases. It is quite remarkable to note that the heat transfer for opposing flow is much lower and insignificant than that of aiding flow. Lewis number, ratio of thermal diffusivity to mass diffusivity, is more implies more thermal diffusivity takes place and hence rate of heat transfer is less. Mass transfer rate across magnetic parameter

is picturised in fig.8. Mass transfer is high when there is no magnetic effect. But as soon as magnetic field is applied, there is fall in Sherwood number indicating that rate of mass transfer is low at high Hartmann number. This pattern is reversed in the case of opposing flows.

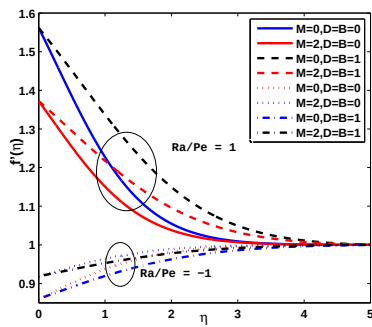


Figure 1:

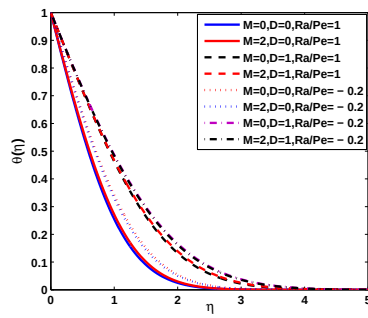


Figure 2:

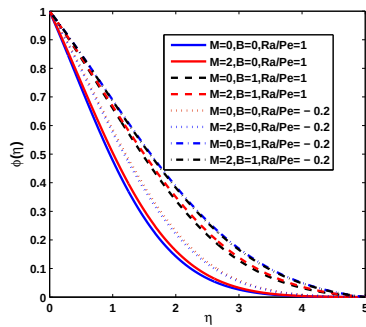


Figure 3:

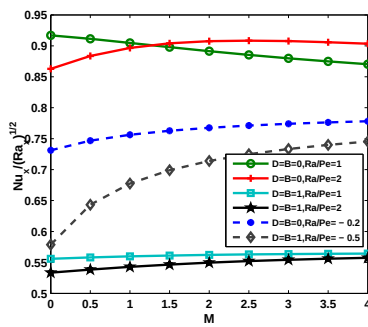


Figure 4:

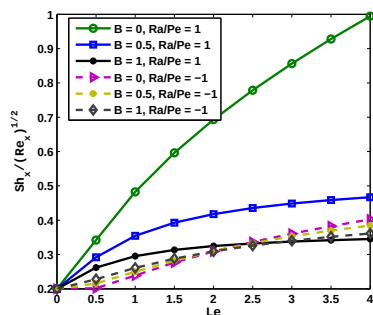


Figure 5:

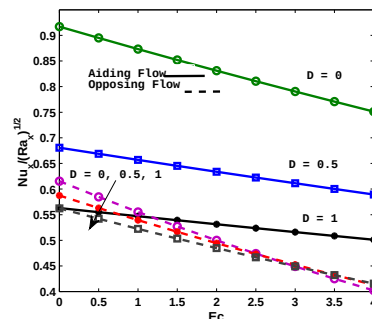


Figure 6:

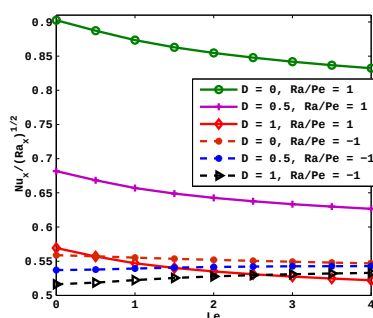


Figure 7:

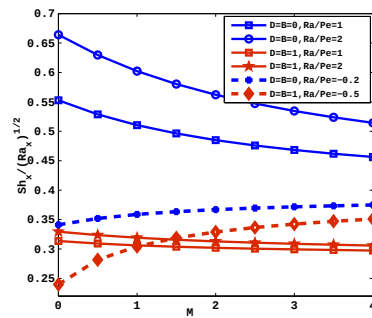


Figure 8:

References

- [1] Lai and Kulacki, Coupled Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media, *Int. J. Heat Mass Transfer*, **34** (1991) 1189-1194.
- [2] Angirasa et al, Combined Heat and Mass Transfer by Natural Convection with opposing Buoyancy Effects in a Fluid Saturated Porous Medium, *Int. J. Heat Mass Transfer*, **40**(2) (1997) 2755-2773.
- [3] Alim MA, Alam Md. M, Mamun AA and Hossain Md. B, Combined Effect of Viscous Dissipation and Joule Heating on the Coupling of Conduction and Free Convection along a Vertical flat Plate, *Int. J. Heat Mass Transfer*, **35**(3) (2008) 338-346.

- [4] El-Amin MF, Aissa WA and Amgad Salama, Effects of Chemical Reaction and Double Dispersion on Non-Darcy Free Convection Heat and Mass Transfer, *Transp Porous Med.*, **75** (2008) 93-109.
- [5] Nakayama A and Pop I, Free convection over a non-isothermal body in a porous medium with viscous dissipation, *Int. Commun. in Heat and Mass Transfer*, **16** (1989) 173-180.
- [6] Hsu CT and Cheng P, The Brinkmann model for natural convection about a semi-infinite vertical plate in a porous medium, *Int. J. Heat Mass Transf.*, **28** (1985) 683-697.
- [7] Murthy PVS, Effect of Double Dispersion on Mixed Convection Heat and Mass Transfer in Non-Darcy Porous Medium, *ASME. J Heat Transf.*
- [8] Kameswaran PK, Hemalatha K, Madhavi MVDNS, Melting effect on convective heat transfer from a vertical plate embedded in a non-Darcy porous medium with variable permeability, *Advanced Powder technology*, **27** (2016) 417-425.

