

3- SOFT PRIME IDEALS OVER A NEAR - RINGS

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Abstract

In this paper 3-soft prime ideals and 3- soft prime ideal near ring are defined and their basic algebraic characterization are studied. Also we identified some related properties of this soft substructures.

Keywords: 3-prime ideal, 3-soft prime ideal, 3- soft prime ideal near ring

1. Introduction

The concept of soft set was submitted via Molodtsov to deal with the uncertainties. Since then many studies are exploring the generalizations of the notion of soft set. The algebraic structure of set theories dealing with uncertainties has also been studied by some researchers. They act the notion of the soft and derived its some basic properties. This search is organized as follows. In the preliminaries, we give the concept of 3-soft prime ideals and 3- soft prime ideal near ring. In section three, we introduce the definition of 3-soft prime ideals and give some fundamental properties of them.

2. Preliminaries

In this part we view some basic definitions and preliminaries lemmas of near ring 3-prime ideal and soft near rings:

Definition (2.1)[1]

A right near-ring, we shall mean an algebraic system $(N, +, \cdot)$, where

1) $(N, +)$ forms a group (not necessarily abelian)

2) $(N, \cdot)$ forms a semi-group and

3) $(a + b) \cdot c = a \cdot c + b \cdot c$ for all $a, b, c \in N$

Definition(2.2) [1]

Let N be a near ring, A normal subgroup I of $(N, +)$ is called a right ideal of N if

1) $N \subseteq I$

2) $n(s + i) = ns + is$ for all $i \in I$

Definition(2.3)[2]

An ideal I of a near ring N is called a 3-prime ideal if $a.N.b \subseteq I$ implies

$a \in I$ or $b \in I$ for all $a, b \in N$.

Definition (2.4) [3]

Let $(N, +, \cdot)$ and $(M, +', \cdot')$ be two near rings. The mapping $F : N \rightarrow M$ is called a near ring homomorphism if $F(n_1 + n_2) = F(n_1) +' F(n_2)$ and $F(n_1 \cdot n_2) = F(n_1) \cdot' F(n_2)$ for all $n_1, n_2 \in N$.

Theorem (2.5) [3]

Let $F : (N, +, \cdot) \rightarrow (M, +', \cdot')$ is homomorphism

- 1) If I is ideal of a near ring N , then $f(I)$ is ideal of a near ring M .
- 2) If J is ideal of a near ring M , then $f^{-1}(J)$ is ideal of a near ring N .

theorem (2.6) [2]

Let $\{I_j\}_{j \in J}$ be a family of ideals of a near ring N , then

- 1) $\bigcap_{j \in J} I_j$ is an ideal of N
- (2) if $\{I_j\}_{j \in J}$ is a chain, then $\bigcup_{j \in J} I_j$ is an ideal of N .

Definition (2.7) [4]

Let $\{N_i\}_{i \in I}$ be a family of near rings, i is an index set and $\prod_{i \in I} N_i = \{(x_i) : x_i \in N_i\}$ for all $i \in I$ be

the directed product of N_i with the component wise defined operations '+' and '.', is called the direct product near ring of the near rings N_i .

Definition (2. 8) [5]

If I_1 and I_2 are ideals of a near ring N , then $I_1 \cdot I_2 = \{i_1 \cdot i_2 : i_1 \in I_1, i_2 \in I_2\}$

Definition (2.9) [5]

If I_1 and I_2 are ideals of a near ring N , then $I_1 \vee I_2 = \{i_1 \vee i_2 : i_1 \in I_1, i_2 \in I_2\}$.

Definition (2. 10) [4]

The factor near ring N/I is defined as in case of a ring.

Definition(2.11)[6]

Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$, A pair (F, A) is called a soft set over U , where F is a

mapping $F : A \rightarrow P(U)$.

Definition(2.12) [7] For a soft set (F, A) the set $\text{Supp}(F, A) = \{x \in A : F(x) \neq \emptyset\}$ is called the support of the soft set (F, A) . If $\text{Supp}(F, A) \neq \emptyset$ then the soft set (F, A) is called non-null.

Definition(2.13) [9]

The $\vee$ -union of two soft sets (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \vee (G, B)$ over U , where $C = A \times B$ and $H(a, b) = F(a) \vee G(b)$ for all $(a, b) \in A \times B$.

Definition (1.14)[8]

The *intersection* of two soft sets (F, A) and (G, B) over a common universe set U is the soft set (H, C) , where $C = A \cap B$, and for all $x \in C$, $H(x) = F(x)$ or $G(x)$, (as both are the same set). We write $(F, A) \cap (G, B) = (H, C)$.

Definition (2.15)[9]

Let (F, A) and (G, B) be two soft sets over a common universe U . The *extended intersection* of (F, A) and (G, B) is defined to be the soft set (H, C) , where $C = A \cup B$ and for all $x \in C$

$$H(x) = \begin{cases} F(x) & \text{if } x \in A / B \\ G(x) & \text{if } x \in B / A \\ F(x) \cap G(x) & \text{if } x \in A \cap B \end{cases}$$

This relation is denoted by $(F, A) \cap_e (G, B) = (H, C)$

Definition (2.16) [9]

The union or extended union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) , where $C = A \cup B$ and for all $x \in C$

$$H(x) = \begin{cases} F(x) & \text{if } x \in A / B \\ G(x) & \text{if } x \in B / A \\ F(x) \vee G(x) & \text{if } x \in A \cap B \end{cases}$$

We write $(F, A) \cup_e (G, B) = (H, C)$

3. 3-soft prime ideal of soft near ring

In this section, we will introduce a new notion called 3-soft prime ideal of soft near ring (F, A) , 3- soft prime near ring and study some properties of this.

Definition (3.1)

Let (F, A) be a soft near-ring over N . A non-null soft set (G, I) over N is called a 3-soft prime ideal of soft near ring (F, A) if it satisfies:

- 1) $I \subseteq A$
- 2) $F(x.n.y) \subseteq G$ implies $F(x) \subseteq G$ or $F(y) \subseteq G$ for all $n \in N$ and $x, y \in \text{Supp}(F, A)$.

Definition (3.2)

Let (F, A) be a non-null soft set over a near ring N . Then (F, A) is called a 3- soft prime near ring over R if $F(x)$ is 3- soft prime ideal of N for all $x \in \text{Supp}(F, A)$.

Example(3.3)

Let us Consider the set $Z_5 = \{0, 1, 2, 3, 4\}$ be a near ring with addition and multiplication defined by the following tables

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	2	0	1	2
4	4	0	1	2	3

.	0	1	2	3	4
0	0	0	0	0	0
1	1	2	1	2	1
2	2	0	2	0	2
3	3	2	1	0	4
4	4	0	3	2	1

Let (F, A) be a soft set over Z_5 , where $A = Z_5$ and $F: A \rightarrow P(Z_5)$ is defined by

$F(x) = \{y \in Z_5 : x \cdot y \in \{0, 2\}\}$ for all $x \in A$, then $F(0) = F(2) = Z_5$ and $F(1) = F(3) = F(4) = \{0, 1, 3\}$ are sub near ring over Z_5 .

Hence (F, A) be a soft near ring over Z_5 .

Let (G, I) be a soft set over Z_5 where $G: I \rightarrow P(Z_3)$ is defined by $G(x) = \{y \in Z_3 : x \cdot y \in \{0, 2\}\}$ for all $x \in A$

Theorem (3.4)

Let (G_1, I_1) and (G_2, I_2) be 3- soft prime ideal of a soft near-ring (F, A) over a near-ring N . Then the soft set $(G_1, I_1) \cap_e (G_2, I_2)$ is a 3- soft prime ideal of (F, A) .

Proof:

Assume that $(G_1, I_1) \cap_e (G_2, I_2) = (G, I)$ where $I_1 \cup I_2 = I$

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in I_1 / I_2 \\ G_2(x) & \text{if } x \in I_2 / I_1 \\ G_1(x) \cap G_2(x) & \text{if } x \in I_1 \cap I_2 \end{cases}$$

Since $I_1 \subseteq A$ and $I_2 \subseteq A$, it is obvious that $I \subseteq A$.

Suppose that for all $n \in N$ and $x, y \in \text{Supp}(F, A)$

Let $F(x \cdot n \cdot y) \subseteq G$, If $x \cdot n \cdot y \subseteq I_1$ then $F(x \cdot n \cdot y) \subseteq G_1$

, If $x \cdot n \cdot y \subseteq I_2$ then $F(x \cdot n \cdot y) \subseteq G_2$, And If $x \cdot n \cdot y \subseteq I_1 \cap I_2$ then $F(x \cdot n \cdot y) \subseteq G_1 \cap G_2$

assumption, we deduce that the nonempty sets $G_1(x)$ and $G_2(x)$ are both 3- soft prime ideal of $F(x)$.

It follows that $G_1(x) \cap G_2(x)$ for all $x \in \text{Supp}(F, A)$.

Therefore extended intersection of a 3- soft prime ideal is a 3- soft prime ideal of (F, A) .

Theorem (3.5)

Let (G_1, I_1) and (G_2, I_2) be 3- soft prime ideal of a soft near-ring (F, A) over a near-ring N . Then the soft set $(G_1, I_1) \cup_e (G_2, I_2)$ is a 3- soft prime ideal of (F, A) .

Proof:

Assume that $(G_1, I_1) \cup_e (G_2, I_2) = (G, I)$ where $I_1 \cup I_2 = I$

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in I_1 / I_2 \\ G_2(x) & \text{if } x \in I_2 / I_1 \\ G_1(x) \vee G_2(x) & \text{if } x \in I_1 \cap I_2 \end{cases}$$

Since $I_1 \subseteq A$ and $I_2 \subseteq A$, it is obvious that $I \subseteq A$.

Suppose that for all $n \in N$ and $x, y \in \text{Supp}(F, A)$

Let $F(x.n.y) \subseteq G$, If $x.n.y \subseteq I_1$ then $F(x.n.y) \subseteq G_1$

, If $x.n.y \subseteq I_2$ then $F(x.n.y) \subseteq G_2$, And If $x.n.y \subseteq I_1 \cap I_2$ then $F(x.n.y) \subseteq G_1 \vee G_2$

assumption, we deduce that the nonempty sets $G_1(x)$ and $G_2(x)$ are both 3- soft prime ideal of $F(x)$.

It follows that $G_1(x) \vee G_2(x)$ for all $x \in \text{Supp}(F, A)$.

Therefore extended union of a 3- soft prime ideal is a 3- soft prime ideal of (F, A) .

Theorem (3.6)

Let $(G_i, I_i)_{i \in I}$ be a family 3-soft prime ideal of soft near rings $(F_i, A_i)_{i \in I}$ over $\{N_i\}_{i \in I}$ for all $i \in I$. Then $\prod_{i \in I} (G_i, I_i)$ is 3-soft prime ideal of the direct product soft near rings $\prod_{i \in I} (F_i, A_i)_{i \in I}$ over $\prod_{i \in I} N_i$.

Proof:

Let $(a_j), (b_j), (x_j) \in \prod_{i \in I} N_i$, and $\prod_{i \in I} I_i$ is a 3- prime ideal of soft near rings $\prod_{i \in I} A_i$ over $\prod_{i \in I} N_i$

Suppose $(G_i, I_i)_{i \in I}$ be 3-soft prime ideal of soft near rings $(F_i, A_i)_{i \in I}$ for all $i \in I$

$\rightarrow F_i(a_i \times_i b_i) \subseteq G_i$ for all $i \in I$ implies $F_i(a_i) \subseteq G_i$ or $F_i(b_i) \subseteq G_i$ for all $i \in I$

$\rightarrow \prod_{i \in I} F_i(a_i \times_i b_i) \subseteq \prod_{i \in I} G_i$ implies $\prod_{i \in I} F_i(a_i) \subseteq \prod_{i \in I} G_i$ or $\prod_{i \in I} F_i(b_i) \subseteq \prod_{i \in I} G_i$ for all $i \in I$

$\rightarrow \prod_{i \in I} (G_i, I_i)$ is 3-soft prime ideal of the direct product soft near rings $\prod_{i \in I} (F_i, A_i)_{i \in I}$ over $\prod_{i \in I} N_i$

Theorem (3.7)

Let $(N, +, \cdot)$ and $(M, +, \cdot)$ be two near rings, $f: N \rightarrow M$ is epimorphism

1) If (G, I) is 3-soft prime ideal of a soft near ring (A, F) over N , then $(f(G), f(I))$ is 3-soft prime ideal of a soft near ring (B, g) over M .

2) If (G, I) is 3-soft prime ideal of a soft near ring (B, g) over M , then $(f^{-1}(G), f^{-1}(I))$ is 3-soft prime ideal of a soft near ring (A, F) over N .

Proof:

1) Let (G, I) is 3-soft prime ideal of a soft near ring (A, F) over N by using theorem (1.5)

Let $g(x.m.y) \subseteq f(G)$ for all $x, m, y \in M$, $\exists a, b, n \in N$ such that

$f(a) = x, f(n) = m, f(b) = y: x, m, y \in M$ Since f be an epimorphism

$$\rightarrow g(f(a).f(n).f(b)) \subseteq f(G) \longrightarrow g(f(a.n.b)) \subseteq f(G)$$

$$\rightarrow F((a.n.b)) \subseteq G$$

$$\text{implies } F(a) \subseteq G \quad \text{Or } F(b) \subseteq G$$

$$\rightarrow f(F(a)) \subseteq f(G) \text{ or } f(F(b)) \subseteq f(G)$$

$$\rightarrow g(a) \subseteq f(G) \text{ or } g(b) \subseteq f(G)$$

then $(f(G), f(I))$ is 3-soft prime ideal of a soft near ring (B, g) over M

2) If (G, I) is 3-soft prime ideal of a soft near ring (B, g) over M and let $F(x.n.y) \subseteq f^{-1}(G)$

for all $x, n, y \in N$ $\exists a = f^{-1}(x), n = f^{-1}(m), b = f^{-1}(y) \in N$

$$\rightarrow F(f^{-1}(a).f^{-1}(m).f^{-1}(b)) \subseteq f^{-1}(G)$$

$$\rightarrow F(f^{-1}(a.m.b)) \subseteq f^{-1}(G)$$

$$\rightarrow g(a.m.b) \subseteq G$$

$$\rightarrow F(a) \subseteq f^{-1}(G) \text{ or } F(b) \subseteq f^{-1}(G)$$

$$\text{implies } g(a) \subseteq G \text{ or } g(b) \subseteq G$$

$$\rightarrow f^{-1}(g(a)) \subseteq f^{-1}(G) \text{ or } f^{-1}(g(b)) \subseteq f^{-1}(G)$$

then $(f^{-1}(G), f^{-1}(I))$ is 3-soft prime ideal of a soft near ring (A, F) over N .

Proposition (3.8)

Let (F, A) be a 3-soft prime ideal near ring over N then I is 3-soft prime ideal of (F, A) .

Theorem (3.9)

Let (G, I) be a 3-soft prime ideal of a soft near ring (F, A) over N , Then the factor soft near ring

F/I is 3-soft near ring over N/I .

Proof:

The natural homomorphism $\text{nat}_G : F \rightarrow F/G$ which is defined by $\text{nat}_G(F(x)) = F(x) \cap G$ for all $x \in N$, is an epimorphism.

Let (K, J) be a soft ideal of the factor soft near ring $(F/G, A/I)$.

Then by theorem (3.4) we have $\text{nat}_G^{-1}((K, J))$ is a soft ideal of the soft near ring (F, A) over $N \Rightarrow \text{nat}_G^{-1}((K, J))$ is a 3-soft prime ideal of the soft near ring (F, A) over N [since (F, A) is a 3-soft prime ideal near ring over N].

By theorem (3.4) we have $\text{nat}_G(\text{nat}_G^{-1}((K, J))) = (K, J)$ is a 3-soft prime ideal of soft near ring $(F/G, A/I)$ over $N/I \Rightarrow (K, J)$ is a 3-soft prime ideal of soft near ring $(F/G, A/I)$ over N/I .

Then F/G is a 3-soft prime near ring over N/I .

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