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Tracking of Nonlinear Variations of the Parameters of High Mobility Systems

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***Abstract*—Monitoring the parameters of high mobility trains are required for the safe navigation and reliable communication applications of the high speed vehicle. The localization of the train accurately is a challenging task. Hence an effective model and an efficient algorithm must be used. Some of the traditional methods of locating the position of the high speed train include the use of GPS. However, GPS cannot always give precise location as other external factors like climate and mobile environment also plays a major role. So, there is a need for localization of the train by itself. Thus, extra sensors are needed which can increase the precision of localization. Localization of high speed train not only falls into the domain of safety enhancement but also falls into the domain of designing smart trains which are energy efficient and intelligent. This information is also useful for establishing a real time wireless communication link in high mobility system. In this paper, we proposed an algorithm which involves estimating two angles, pitch angle and roll angle in addition to the velocity of the train from the noisy sensors like gyro-meter and accelerometer. This problem falls under the conventional non-linear estimation problem. Choosing the correct estimation algorithm is as important as correctly modeling the train dynamics. Existing literature made use of the extended Kalman filter (EKF) to estimate the state variables accurately for different physical models. In this paper, we propose to use the unscented Kalman filter (UKF). Theoretically, UKF is a better estimator compared to EKF because it captures higher order statistics of the system. The simulation results show that the proposed UKF based tracking algorithm is a reliable estimator by comparing the estimation error of UKF algorithm and EKF algorithm for estimating the pitch angle, roll angle and velocity of the train.**

Keywords*—Detection; Extended Kalman filter; Unscented Kalman filter; High mobility systems; Train localisation; Velocity estimation; Attitude estimation.

I. Introduction

Train localization becomes an important factor for both safety and reliability of train journey. This information is also important for navigation and communication applications of high mobility vehicle. European train control system (ECTS) tries to improve the localization algorithms so that efficient rail collision avoidance system (RCAS) can be designed [1],[2]. These days, smart trains also involve automatic opening and closing of doors, switching lights on and off [3]. Newer requirements like fault detection in dynamically moving train is investigated worldwide, as the traditional method of scheduled-evaluation of train decreases efficiency [4]. For all these performance demands, correct train location is of utmost importance.

Many approaches of localization are present in literature [5],[6] and they mostly involve the fusion of data from multiple sensors. In other example, electrical approaches like track circuit/ axle counters were used in [7] which involved electrical hardware. However, this method suffered from electromagnetic interference (EMI). Another approach is to use global positioning system (GPS) alone with GPS receiver mounted on the train. This system too suffers from lack of accuracy [8].

So, more robust and minimal hardware methods are needed for localization of train. Electronic sensors like gyroscope and accelerometer are low-cost options which are currently being considered. In this paper, we work on the non-linear system model outlined in [9]. The model comprises of obtaining data from 3-axis gyroscope and 3-axis accelerometer. The data is then fused and accurate results are obtained. The authors in [10] have used extended Kalman filter (EKF) for the system model outlined in [9] for better accuracy. They have also obtained velocity and attitude estimations using spectral analysis and complementary filter [10]. For non-linear systems, unscented Kalman filter proposed by Juiler et al. provides better results compared to EKF with increased accuracy [11].

EKF uses linearization of non-linear system with the help of Taylor series. Higher order terms in the series are truncated and this poses a challenge to precision. Moreover, EKF involves the computation of Jacobian of the function. This considerably increases the numerical instability [11],[12].

Julier proposed an alternate method which did not involve the computation of Jacobian and linearization of non-linear system. Instead of linearizing the system equation, the state vector undergoes unscented transformation (UT). The transformed state vector is then passed through the non-linear function. The motivation behind this approach is that, it is easy to approximate the probability distribution rather than approximate non-linear function itself [11], [13].

Thus, we propose UKF to determine the train's location with higher precision.

II. Velocity and Attitude Estimation

The velocity is directly estimated from the vertical accelerometer values [10],[14],[15],[16]. The method involves computing fast Fourier transform (FFT) on the z-axis values of the accelerometer. Then, the sum of Euclidean norms of the FFT components is found out. This is denoted by S_m in Eq.(1).

$$S_m = \sum_{k=0}^{\frac{N}{2}} \|H_{km}\|_2 \tag{1}$$

Then the velocity of the train v_{vel} is found by using Eq.(2).

$$v_{vel} = \rho\sqrt{S_m} + b \tag{2}$$

Where b and ρ are the bias and scale factors respectively. The scale factor is determined during initialization phase of the inertial system. The bias factor is obtained using experimental observations.

The attitude is estimated by using complementary filter which combines values from both the gyrometer and accelerometer to obtain a more accurate result [10],[17]. The complementary filter works on the principle that different sensors work efficiently in different frequency levels and that by frequency-domain filtering of different sensor measurements, we can produce precise results. The ranges of roll and pitch angles are in the range of ±6° for railway applications. They are too high for not considering the gyrometer cross-coupling. The complementary filter can thus be expressed with Eq. (3) and Eq. (4) where $\emptyset_{inc}$ and θ_{inc} are the roll and pitch angles estimated by this filter.

$$\phi_{inc} = H_1(s)\phi_{meas} + H_2(s)\frac{\dot{\phi}_{meas}}{s} \tag{3}$$

$$\theta_{inc} = H_1(s)\theta_{meas} + H_2(s)\frac{\dot{\theta}_{meas}}{s} \tag{4}$$

Here H_1 and H_2 are low-pass and high-pass filters respectively. $\emptyset_{meas}$ and θ_{meas} are the roll and pitch angles obtained from the accelerometer measurements *fx* and *fy* described in Eq. (5) and Eq. (6).

$$\phi_{meas} = \text{atan}\left(-\frac{f_y}{9.81}\right) \tag{5}$$

$$\theta_{meas} = \text{atan}\left(-\frac{f_x}{9.81}\right) \tag{6}$$

$\dot{\phi}_{meas}$ and $\dot{\theta}_{meas}$ are roll and pitch angle rates acquired from gyrometer measurements $(\omega_x, \omega_y, \omega_z)^T$ after using cross coupling compensation.

III. System Model

The system equations of any non-linear system can be written as in Eq. (7) and Eq. (8).

$$st_{k+1} = f(st_k) + w_k \tag{7}$$

$$op_k = h(st_k) + v_k \tag{8}$$

Here, in Eq. (7) and Eq. (8), st_k is the state vector, op_k is the output vector, f is the non-linear function relating present state and next state, h is observation function. w_k and v_k represent process noise and measurement vectors respectively, which are additive-white Gaussian in nature [18]. The variables used in describing dynamics of the train are given in Table I.

The state vector st_k (in Eq. 9) and output vector op_k are given by (in Eq. 10).

$$st_k = \left(\dot{\phi}_k \quad \dot{\theta}_k \quad \dot{\varphi}_k \phi_k \quad \theta_k \ddot{x}^{b}_{n\,k} \quad \ddot{y}^{b}_{n\,k} \quad \ddot{z}^{b}_{n\,k} \dot{x}^{b}_{n\,k} b_{gx\,k} \quad b_{gy_k}\right)^T \tag{9}$$

$$op_k = (\omega_{x_K} \quad \omega_{y\,k} \quad \omega_{z_k} f_{x_k} \quad f_{y_k} f_{z_k} \quad \phi_{inc_k} \theta_{inc_k} \quad v_{vel_k})^T \tag{10}$$

The non-linear state function $f\,(st_k)$ can be defined based on individual components by Eq. (11) to Eq. (21)

TABLE I
VARIABLES USED WITH THEIR PHYSICAL INTERPRETATION

Symbol	*Physical parameter(with unit).*
ϕ_k, θ_k, ψ_k	Roll, pitch and yaw angles (radians).
$\dot{\phi}_k, \dot{\theta}_k, \dot{\psi}_k$	Rate of change of roll, pitch and yaw angles (radians/s).

$\dot{x}^b_{n_k}$	Velocity of train (m/s) defined in the body frame.
$\ddot{x}^b_{n_k}, \ddot{y}^b_{n_k}, \ddot{z}^b_{n_k}$	Linear acceleration (m/s^2)of the train defined in the body frame.
b_{gx_k}, b_{gy_k}	X and Y axis biases of the gyrometer.
w_k	Process noise defined by co-variance matrix Q.
Δt	Sampling time of the Kalman Filter (s).
g_D	Acceleration due to gravity (9.8 m/s^2)
$\omega_{x_k}\omega_{y_k}, \omega_{z_k}$	Gyrometer measurements along X,Y and Z axes respectively (radian/s).
f_x, f_y, f_z	Accelerometer measurements along X,Y and Z axes respectively (N).
$\phi_{inc_k}, \theta_{inc_k}$	Estimates of roll and pitch(radians) obtained using complementary filter.
v_{vel_k}	Velocity of train (m/s^2) obtained after linear fitting of spectral analysis of vertical accelerometer data.
v_k	Measurement noise defined by the co-variance matrix R

$$\dot{\phi}_{k+1} = \dot{\phi}_k + w_{\dot{\phi}_k} \quad (11)$$

$$\dot{\theta}_{k+1} = \dot{\theta}_k + w_{\dot{\theta}_k} \quad (12)$$

$$\dot{\psi}_{k+1} = \dot{\psi}_k + w_{\dot{\psi}_k} \quad (13)$$

$$\phi_{k+1} = \Delta t * \dot{\phi}_k + \phi_k + w_{\phi_k} \quad (14)$$

$$\theta_{k+1} = \Delta t * \dot{\theta}_k + \theta_k + w_{\theta_k} \quad (15)$$

$$\ddot{x}^b_{n_{k+1}} = \ddot{x}^b_{n_k} + w_{\ddot{x}_k} \quad (16)$$

$$\ddot{y}^b_{n_{k+1}} = \dot{x}^b_{n_k}\dot{\psi}_k + w_{\ddot{y}_k} \quad (17)$$

$$\ddot{z}^b_{n_{k+1}} = \dot{x}^b_{n_k}\dot{\theta}_k + w_{\ddot{z}_k} \quad (18)$$

$$\dot{x}^b_{n_{k+1}} = \Delta t * \ddot{x}^b_{n_k} + \dot{x}^b_{n_k} + w_{\dot{x}_k} \quad (19)$$

$$b_{gx_{k+1}} = b_{gx_k} + w_{b_{gx_k}} \quad (20)$$

$$b_{gy_{k+1}} = b_{gy_k} + w_{b_{gy_k}} \quad (21)$$

The components of observation function h(st$_k$) are given by Eq. (22) to Eq. (30).

$$\omega_{x_k} = \dot{\phi}_k - sin\phi_k\dot{\psi}_k + b_{gx_k} + v_{\omega_{x_k}} \quad (22)$$

$$\omega_{y_k} = cos\phi_k\dot{\theta}_k + sin\phi_k cos\theta_k\dot{\psi}_k + b_{gy_k} + v_{\omega_{y_k}} \quad (23)$$

$$\omega_{z_k} = -sin\phi_k\dot{\theta}_k + cos\phi_k cos\theta_k\dot{\psi}_k + v_{\omega_{z_k}} \quad (24)$$

$$f_{x_k} = \ddot{x}^b_{n_k} - g_D sin\theta_k + v_{f_{x_k}} \quad (25)$$

$$f_{y_k} = \ddot{y}^b_{n_k} + g_D sin\phi_k cos\theta_k + v_{f_{y_k}} \quad (26)$$

$$f_{z_k} = \ddot{z}^b_{n_k} + g_D sin\phi_k cos\theta_k + v_{f_{z_k}} \quad (27)$$

$$\phi_{inc_k} = \phi_k + v_{\phi_k} \quad (28)$$

$$\theta_{inc_k} = \theta_k + v_{\theta_k} \quad (29)$$

$$v_{vel_k} = \dot{x}^b_{n_k} + v_{v_{vel_k}} \quad (30)$$

IV. UKF BASED PARAMETER ESTIMATION

The unscented transform is a non-linear transform which converts the state vector st$_k$ to a set of points called sigma points, γ_k based on apriori conditions [18]. The random variable st$_k$ which is assumed to be Gaussian distributed of length L x 1 with a mean $\tilde{s}$ t$_k$ and covariance matrix P_{st} . The length of γ_k is 2L x 1. The scaling parameters like λ (composite scaling parameter), α (primary scaling parameter), β (secondary scaling parameter) and κ (correction factor) which affect the spread of the sigma points, also determine the weight vectors which help in rebuilding the aposteriori statistics. Q and R are the process and measurement noise covariance respectively. $W^{(c)}$ and $W^{(c)}$ are the weights. The algorithm is given by

Step1 Determining weights and scaling parameters as given in Eq. (31), Eq. (32) and Eq. (33) and initializing Q and R.

$$W_0^{(m)} = \frac{\lambda}{L+\lambda} \quad (31)$$

$$W_0^{(c)} = \frac{\lambda}{L+\lambda} + 1 + \beta - \alpha^2 \quad (32)$$

$$W_i^{(m)} = W_i^{(c)} = \frac{\lambda}{2(L+\lambda)}, i = 1, \ldots, 2L \quad (33)$$

Step2. Calculate square-root of P_{k-1} using Cholesky de-composition as in Eq. (34). Then calculate sigma points as in Eq. (35).

$$\sqrt{P_{k-1}} = chol(P_{k-1}) \quad (34)$$

$$\gamma_{k-1} = \begin{bmatrix} \tilde{s}t_{k-1} \\ \tilde{s}t_{k-1} + \sqrt{(\lambda+1)\times P_{k-1}} \\ \tilde{s}t_{k-1} - \sqrt{(\lambda+1)\times P_{k-1}} \end{bmatrix}^T \quad (35)$$

Step3. Prediction transformation

I. Propagate sigma-points through non-linear function using Eq.(36).

$$\gamma_{k|k-1} = f(\gamma_{k-1}) \quad (36)$$

II. Calculate mean of predicted state usingEq.(37).

$$\widetilde{st}_{k|k-1} = \sum_{j=0}^{2L} W_j^{(m)} \times \gamma_{j,k|k-1} \quad (37)$$

III. Calculate co-variance of the predicted state using Eq.(38).

$$P_{st_k} = Q + \sum_{j=0}^{2L} W_j^{(c)} \times [\gamma_{j,k|k-1} - \widetilde{st}_{k|k-1}] \times [\gamma_{j,k|k-1} - \widetilde{st}_{k|k-1}]^{\mathrm{T}} \quad (38)$$

Step4. Transforming the observations

I. Propagate every sigma-point through the observation function using Eq.(39).

$$\Psi_{k|k-1} = h(\gamma_{k|k-1}) \quad (39)$$

II. Predicted output calculation using Eq.(40).

$$\widetilde{op}_{k|k-1} = \sum_{j=0}^{2L} W_j^{(m)} \times \Psi_{j,k|k-1} \quad (40)$$

III. Covariance of predicted output calculation using Eq.(41).

$$P_{op_k} = R + \sum_{j=0}^{2L} W_j^{(c)} \times [\Psi_{j,k|k-1} - \widetilde{op}_{k|k-1}] \times [\Psi_{j,k|k-1} - \widetilde{op}_{k|k-1}]^{\mathrm{T}} \quad (41)$$

IV. Cross-covariance of state and output calculation using Eq.(42).

$$P_{stop} = \sum_{j=0}^{2L} W_j^{(c)} \times [\gamma_{j,k|k-1} - \widetilde{st}_{k|k-1}] \times [\Psi_{j,k|k-1} - \widetilde{op}_{k|k-1}]^{\mathrm{T}} \quad (42)$$

Step5. Updating measurement

I. Calculating Kalman gain K_k using Eq.(43).

$$K_k = P_{stop_k} \times P_{op_k}{}^{-1} \quad (43)$$

II. Updating state estimate using Eq.(44).

$$\widetilde{st}_k = \widetilde{st}_{k|k-1} + K_k \times (op_k - \widetilde{op}_{k|k-1}) \quad (44)$$

III. Updating error co-variance using Eq.(45)

$$P_{st_k} = P_{st_k} - K_k \times P_{op_k}^{-1} \times K_k^T \quad (45)$$

V. RESULT AND DISCUSSIONS

The simulations are performed and the tracking results are shown in Fig.1, Fig.2 and Fig.3. We assume α, β and κ as 1, 2 and 0 respectively. The number of realizations taken is M=100 samples. The sampling time is taken as Δt = 1second. The range of roll and pitch angles are ±6 degrees. The velocity of the train is in the range of 0 to 80 km/hour. We can observe that for the given modeling, the UKF algorithm is efficient in tracking roll, pitch and velocity of the train with high precision as can be seen from Fig.1 to Fig. 3. The tracking of the speed of the train is given in Fig.1. This plot shows that the proposed algorithm is able to track the velocity with minimum error. It also shows that the algorithm start tracking the velocity within few seconds. The tracking of pitch angle of the train is given in Fig. 2. This plot shows that the estimated values are exactly following the original values with an average error of 0.0432 degree. The tracking of the roll angle of the train is given in Fig. 3. From this figure, we can see that the proposed algorithm is able to track the roll angle of the train with very less error of 0.0082 degree. The Fig. 2 and 3 also shows the fast tracking capability of the algorithm, within a few samples itself, the estimated values are tracking the actual values.

Some of the sample values are given in Table II to Table IV. These samples are taken at an interval of 20 samples each. Here the actual values and the estimated values are given and the corresponding errors are also calculated. We can see the estimation error is very small for velocity, pitch and roll angles of the train. We can also note that the maximum error in estimation of roll and pitch angles using proposed UKF based algorithm is better compared to the estimation error obtained using EKF based algorithm [10].

So the simulation results show that the performance of the proposed UKF based algorithm outperforms the existing algorithms in the case of high mobility environments.

TABLE III
TRACKING OF TRAIN VELOCITY v_{vel}

Time sample(k)	**Actual velocity(km /hour)**	**Estimated velocity (km/hour)**	**Error**
20	52.5593	52.3664	0.1929
40	21.1403	21.8257	-0.6854
60	23.4623	23.2596	0.1727
80	19.9801	19.9772	0..0029

100	57.0961	57.8546	-0.7585

TABLE IIIII
TABLE SHOWING DIFFERENCE BETWEEN ACTUAL AND ESTIMATED PITCH ANGLE

Time sample(k)	Actual pitch angle (degree)	Estimated pitch angle(degree)	Error
20	-2.5421	-2.5045	-0.0376
40	2.8395	2.7639	0.0756
60	2.4332	2.3519	0.0813
80	0.4018	0.3479	0.0599
100	-4.7	-4.7512	0.0512

TABLE IVV
TABLE SHOWING DIFFERENCE BETWEEN ACTUAL AND ESTIMATED ROLL ANGLE

Time sample(k)	Actual roll angle (degree)	Estimated roll angle (degree)	Error
20	-2.1666	-2.1619	-0.0047
40	-1.6277	-1.6145	-0.0132
60	5.2179	5.2010	0.0169
80	- 0.2656	-0.2087	-0.0569
100	-1.7144	-1.7081	-0.0063

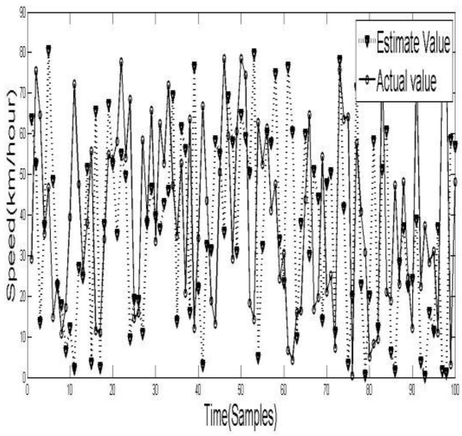

Fig. 1 Tracking of velocity of the train using UKF algorithm.

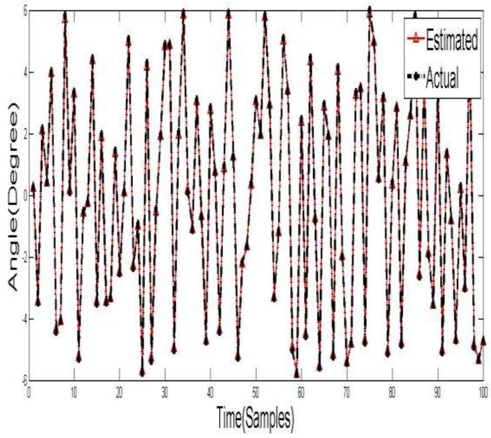

Fig. 2 Tracking of pitch angle using UKF algorithm.

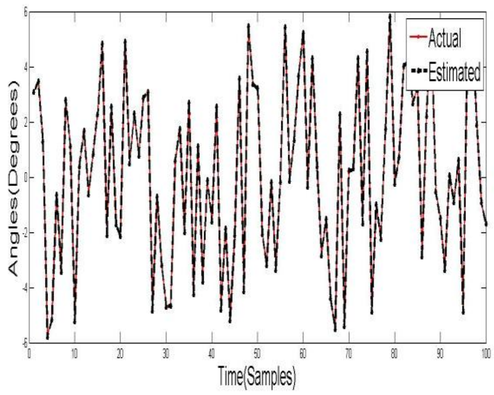

Fig. 3 Tracking of roll angle using UKF algorithm.

VI. CONCLUSIONS

Thus, we could see from the simulations that unscented Kalman filter has very high accuracy in estimating the position for this physical modeling of the train. UKF based algorithm shows better performance accuracy compared to EKF based tracking algorithm for both pitch and roll angles. In the case of velocity estimation, the performance of the proposed algorithm is comparable with existing algorithms. This difference in estimation error between the proposed and existing algorithms may be small in absolute terms but this small error propagates through the algorithm making EKF based algorithm less desirable in the case of mobility environment. Thus in high mobility condition, UKF based algorithm on the other hand has both reduced error and increased accuracy with small amount of increase in computational overhead. We would like to point out that various modifications of UKF have been proposed and we strongly believe that future research will be focusing on highly adaptive and computationally light UKF based methods for high mobility train localization.

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