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An Upper Bound For New Fuzzy Graph Operations

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Abstract

In this paper ,an upper bound for new fuzzy graph operations α-product,β-product and γ-product are discussed by using fuzzy vertex order colouring.

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Key Words and Phrases: Alpha,Beta and Gamma Product , Alpha strong,Beta strong and Gamma strong vertex, and Underlying crisp graph.

1 Introduction

In 1965,Lotfi.A.Zadeh[10] introduced the notion of fuzzy set.Fuzzy Graph theory was introduced by Rosenfeld[8] .In this paper we discuss an upper bound of the chromatic number for the new fuzzy graph operations Alpha, Beta and Gamma-product of fuzzy graphs, by using the fuzzy vertex order colouring.Regular properties of Alpha ,Beta and Gamma product of fuzzy graphs are used here to illustrate an upper bound of the chromatic number.

2 Results From Fuzzy Graph Operations and Fuzzy Vertex Order Colouring

Theorem 1. *[5] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs .If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$,then $G_1 \times_\alpha G_2$ is regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Theorem 2. *[5] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs and its underlying crisp graphs G_1* and G_2* are complete graphs.If $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$ then alpha product of two fuzzy graphs are regular if and only if G_1 and G_2 are regular fuzzy graphs*

Theorem 3. *[5] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs and its underlying crisp graphs G_1* and G_2* are complete graphs,then alpha product becomes cartesian product.*

Theorem 4. *[6] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs such that underlying crisp graphs G_1* and G_2* are complete graphs,then $G_1 \times_\beta G_2$ is regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Theorem 5. *[6] Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs and its underlying crisp graphs G_1* and G_2* are complete graphs.If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ then $G_1 \times_\gamma G_2$ is a regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Theorem 6. *[7] Let G be a fuzzy graph.If G is regular fuzzy graph then all vertices of G is α-strong vertices.*

3 Upper Bound for Alpha,Beta and Gamma Product of Fuzzy Graphs

Theorem 7. *Let G_1 and G_2 be two regular fuzzy graphs with the condition that $\sigma_1 \geq \mu_2$,$\sigma_2 \geq \mu_1$ and whose underlying crisp graphs are complete.Then the chromatic number of alpha product of fuzzy graph is $\chi(G_1 \times_\alpha G_2) \leq [\frac{p+q}{2}] + 1$,where p and q are the degree of the vertices of regular graphs G_1^* and G_2^**

Proof. Let G_1 and G_2 be two regular fuzzy graphs with the condition $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ having m and n vertices. Let G_1* and G_2* be underlying crisp graphs of G_1 and G_2 such that both G_1* and G_2* are complete graphs. Let p and q be the degree of vertices of G_1* and G_2*.
(ie)

$$d_{G1} * (u_1) = p, \forall u_1 \in V_1$$
$$d_{G2} * (u_2) = q, \forall u_2 \in V_2$$

By theorem 4.2[5],$G_1 \times_\alpha G_2$ is regular fuzzy graph. Also by theorem 4.6[7], all vertices of $G_1 \times_\alpha G_2$ is alpha strong vertices. If a fuzzy graph having more alpha strong vertices , then by the vertex order colouring, its chromatic number is maximum.

$$d_{(G1 \times_\alpha G2)} * (u_1, u_2) = \sum_{((u_1,u_2),(v_1,v_2)) \in E} (E_1 \times_\alpha E_2)((u_1, u_2), (v_1, v_2)) \quad (1)$$

$$= \sum_{\substack{u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \notin E_2 \text{ (or)} \\ u_1v_1 \notin E_1, u_2v_2 \in E_2}} (E_1 \times_\alpha E_2)((u_1, u_2), (v_1, v_2)) \quad (2)$$

By theorem 4.3[5],if G_1* and G_2* are complete then $G_1 \times_\alpha G_2$ is Cartesian product. Thus

$$d_{(G1 \times_\alpha G2)} * (u_1, u_2) = \sum_{\substack{u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1}} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \quad (3)$$

$$d_{(G1 \times_\alpha G2)} * (u_1, u_2) = d_{G1} * (u_1) + d_{G2} * (u_2), \forall u_1 \in V_1 and \forall u_2 \in V_2 \quad (4)$$

$$= \text{p+q}$$

This is true for all vertices of $V_1 \times_\alpha V_2$ .Thus each vertex in $G_1 \times_\alpha G_2$ is adjacent with p+q vertices. This shows that $\chi(G_1 \times_\alpha G_2) \leq p+q$. We know that $|V_1 \times_\alpha V_2| = mn$. That is the number of non-adjacent vertices with each vertex = mn-(p+q)-1

= (p+1)(q+1)-(p+q)-1
= pq+p+q+1-p-q-1
= pq

Hence each vertex in alpha product of fuzzy graph having non-adjacency with pq vertices. From this pq vertices ,only $[\frac{pq}{2}]$ vertices are adjacent. Since alpha product doesn't satisfy the condition $u_1v_1 \in E_1, u_2v_2 \in E_2$. Thus $[\frac{pq}{2}] + 1$ vertices are attain the same colour.
That is

$$\chi(G_1 \times_\alpha G_2) = [\tfrac{pq}{2}] + 1$$
$$\leq [\tfrac{p+q}{2}] + 1$$

Hence the chromatic number of regular alpha product of fuzzy graph is at most $[\frac{p+q}{2}] + 1$

□

Theorem 8. *Let G_1 and G_2 be two regular fuzzy graphs with the condition that $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$*
(1) *If G_1* is complete and G_2* is regular then $\chi(G_1 \times_\alpha G_2) \leq [\frac{p+q}{2}] + p|E_2| + 1$*
(2) *If G_2* is complete and G_1* is regular then $\chi(G_1 \times_\alpha G_2) \leq [\frac{p+q}{2}] + q|E_1| + 1$*

Proof. **(1)**Given G_1* is complete and G_2* is regular .Also G_1 and G_2 be two regular fuzzy graphs with the condition that $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$. Hence by theorem 4.1[5] , alpha product of two fuzzy graphs is regular fuzzy graph. Let p and q be the degree of the vertex of regular graphs G_1* and G_2*.

$$\text{(i.e) } d_{G1} * (u_1) = p, \forall u_1 \in V_1$$
$$d_{G2} * (u_2) = q, \forall u_2 \in V_2$$

.

If G_1* is complete and G_2* is regular then

$$(E_1 \times_\alpha E_2)((u_1,u_2),(v_1,v_2)) = \left\{ ((u_1,u_2),(v_1,v_2))/ \quad \begin{array}{r} u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \notin E_2 \end{array} \right\} \quad (5)$$

$$d_{(G1\times_\alpha G2)} * (u_1,u_2) = \sum_{((u_1,u_2),(v_1,v_2)) \in E} (E_1 \times E_2)((u_1,u_2),(v_1,v_2)) \quad (6)$$

$$= \sum_{\substack{u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \notin E_2}} (E_1 \times E_2)((u_1,u_2),(v_1,v_2)) \quad (7)$$

$$= d_{G1} * (u_1) + d_{G2} * (u_2) + d_{G1} * (u_1)|E_2|, \forall u_1 \in V_1 and \forall u_2 \in V_2$$

where $|E_2|$ is the degree of a vertex of the complement graph G_2* .
$d_{(G1\times_\alpha G2)} * (u_1,u_2) = q + p + p|E_2|$.
This is true for all vertices of $V_1 \times_\alpha V_2$. Thus each vertex in $G1 \times_\alpha G2$ is adjacent with $q + p + p|E_2|$ vertices . Therefore $\chi(G_1 \times_\alpha G_2) \leq p + q + p|E_2|$. We know that $|V_1 \times_\alpha V_2| = mn$,where m and n is the number of vertices of G_1 and G_2. That is $|v_1| = m$,$|v_2| = n$. Number of non-adjacent vertices for each vertex is equal to $mn - (p + q + p|E_2|) - 1$. Clearly G_1* is complete graph,thus $m = p + 1$ and G_2* is regular ,therefore $n = q + |E_2| + 1$. Hence , the number of non-adjacent vertices for each vertex

$$= (p+1)(q+|E_2|+1) - (q+p+p|E_2|) - 1$$
$$= pq + |E_2|$$

Thus we conclude that , these $pq + |E_2| + 1$ vertices have attained the same colour. Out of these $pq + |E_2|$ vertices ,only $[\frac{pq}{2}]$ vertices are adjacent. Hence , $[\frac{pq}{2}] + |E_2|$ vertices have only attained the same colour.
Thus the chromatic number of alpha product of fuzzy graph is $\frac{|V_1||V_2|}{[\frac{pq}{2}]+|E_2|}$

$$\text{(i.e) } \chi(G_1 \times_\alpha G_2) = \frac{|V_1||V_2|}{[\frac{pq}{2}]+|E_2|}$$
$$= \frac{(p+1)(q+|E_2|+1)}{[\frac{pq}{2}]+|E_2|}$$
$$\leq [\frac{p+q}{2}] + p|E_2| + 1$$

Hence the chromatic number of alpha product of fuzzy graph is at most $[\frac{p+q}{2}] + p|E_2| + 1$
(2) The proof is similar from (1) Hence the chromatic number of alpha product of fuzzy graph is at most $[\frac{p+q}{2}] + q|E_1| + 1$

□

Theorem 9. *Let G_1 and G_2 be two regular fuzzy graphs with $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.If both G_1* and G_2* are regular but not complete then*
$\chi(G_1 \times_\alpha G_2) \leq [\frac{p+q}{2}] + [\frac{p||E_2|+q|E_1|}{|E_1|+|E_2|}] + 1$

Proof. Given G_1 and G_2 be two regular fuzzy graphs with the condition $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$. Then by theorem 4.1 alpha product of fuzzy graph is regular fuzzy graph. Given underlying crisp graphs G_1* and G_2* are regular. Let p and q be the degree of the vertex of regular graphs G_1* and G_2*

$$\begin{aligned}\text{(i.e) } d_{G1}*(u_1) &= p, \forall u_1 \in V_1 \\ d_{G2}*(u_2) &= q, \forall u_2 \in V_2\end{aligned}$$

.

Thus

$$(E_1 \times_\alpha E_2)((u_1,u_2),(v_1,v_2)) = \left\{ ((u_1,u_2),(v_1,v_2))/ \begin{array}{r} u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \notin E_2 \text{ (or)} \\ u_1v_1 \notin E_1, u_2v_2 \in E_2 \end{array} \right\} \quad (8)$$

$$d_{(G1\times_\alpha G2)}*(u_1,u_2) = \sum_{((u_1,u_2),(v_1,v_2)) \in E} (E_1 \times E_2)((u_1,u_2),(v_1,v_2)) \quad (9)$$

$$= \sum_{\substack{u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \notin E_2 \text{ (or)} \\ u_1v_1 \notin E_1, u_2v_2 \in E_2}} (E_1 \times E_2)((u_1,u_2),(v_1,v_2)) \quad (10)$$

$$d_{(G1\times_\alpha G2)}*(u_1,u_2) = d_{G1}*(u_1) + d_{G2}*(u_2) + d_{G2}*(u_2)|E_1| + d_{G1}*(u_1)|E_2| \quad (11)$$

$\forall u_1 \in V_1$ and $\forall u_2 \in V_2$ where $|E_1|$ and $|E_2|$ are the degree of the vertex of the complement graphs G_1* and G_2*
$d_{(G1\times_\alpha G2)}*(u_1,u_2) = q + p + p|E_2| + q|E_1|$
This is true for all vertices of $V_1 \times_\alpha V_2$. Thus each vertex is adjacent with $q + p + p|E_2| + q|E_1|$ vertices .The maximum chromatic number is $q + p + p|E_2| + q|E_1|$. We know that $|V_1 \times_\alpha V_2| = mn$,where m and n is the number of vertices of G_1 and G_2. That is $|v_1| = m$,$|v_2| = n$ Here both G_1* and G_2* are regular graph, thus $m = p + |E_1| + 1$ and $n = q + |E_2| + 1$. Number of non-adjacent vertices for each vertex is equal to $mn - (q + p + p|E_2| + q|E_1|) - 1$

$$\begin{aligned}&= (p + |E_1| + 1)(q + |E_2| + 1) - (q + p + p|E_2| + q|E_1|) - 1 \\ &= pq + |E_1||E_2| + |E_1| + |E_2|\end{aligned}$$

That is , the number of non-adjacent vertices for each vertex is $pq + |E_1||E_2| + |E_1| + |E_2|$. Hence $pq + |E_1||E_2| + |E_1| + |E_2|$ vertices have the same colour. But $pq + |E_1||E_2| + |E_1| + |E_2|$ vertices are non-adjacent which are in pair. Hence $\frac{pq+|E_1||E_2|+|E_1|+|E_2|}{2} + 1$ vertices have attained the same colour. Thus the chromatic number of alpha product of fuzzy graph is $\frac{mn}{pq+|E_1||E_2|+|E_1|+|E_2|}$.

$$\begin{aligned}\chi(G_1 \times_\alpha G_2) &= \frac{|V_1||V_2|}{pq+|E_1||E_2|+|E_1|+|E_2|} \\ &= \frac{(p+|E_1|+1)(q+|E_2|+1)}{pq+|E_1||E_2|+|E_1|+|E_2|} \\ &\le [\frac{p+q}{2}] + [\frac{p|E_2|+q|E_1|}{|E_1|+|E_2|}] + 1\end{aligned}$$

Thus the chromatic number of alpha product of fuzzy graph is at most

$$[\frac{p+q}{2}] + [\frac{p|E_2|+q|E_1|}{|E_1|+|E_2|}] + 1$$

□

Theorem 10. *Let G_1 and G_2 be two regular fuzzy graphs with $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.If both G_1* and G_2* are complete graphs,then the chromatic number of regular beta product of fuzzy graph is $Min\{\chi(G_1), \chi(G_2)\}$*

Proof. Given G_1 and G_2 be two regular fuzzy graphs with $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$ and the underlying crisp graphs G_1* and G_2* are complete graphs.

$$d_{(G1\times_\beta G2)} * (u_1, u_2) = \sum_{((u_1,u_2),(v_1,v_2)) \in E} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \quad (12)$$

$$= \sum_{\substack{u_1 \neq v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 \neq v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \in E_2}} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \quad (13)$$

$$= \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \quad (14)$$

Since G_1* and G_2* are complete graphs. Thus $d_{(G1\times_\beta G2)} * (u_1, u_2) = d_{G1} * (u_1)d_{G2} * (u_2)$ .This is true for all vertices of $V_1 \times_\beta V_2$. That is each vertex has adjacency with $d_{G1} * (u_1)d_{G2} * (u_2)$ vertices. Let p and q be the degree of vertices of regular graphs G_1* and G_2* . We know that $|V_1| = p + 1$,$|V_2| = q + 1$ .By theorem 4.4[6],$G_1 \times_\beta G_2$ is regular fuzzy graph.We know that from theorem 4.6,all vertices of regular beta product of fuzzy graph is alpha strong vertices and if underlying crisp graphs G_1* and G_2* are complete then the chromatic number of G_1* and G_2* is $|V_1|$ and $|V_2|$. That is $\chi(G_1) = \chi(G_1*) = |V_1| = p + 1$.Similarly $\chi(G_2) = \chi(G_2*) = |V_2| = q + 1$.
Case(i) :Let $d_{G1} * (u_1) \leq d_{G2} * (u_2)$
If the degree of a vertex of complete graph G_1* is less than G_2* ,then the number of edges of G_1 must be less than the number of edges of G_2. In beta product of fuzzy graph each vertex is adjacent to all vertices except from this two types of edges.
1) $u_1 = v_1, u_2v_2 \in E_2$ and 2) $u_2 = v_2, u_1v_1 \in E_1$.
For $p \leq q$, $u_1 = v_1, u_2v_2 \in E_2$ gives the result. That is $|V_1|$ is the chromatic number of $G1 \times_\beta G2$. Hence $\chi(G_1 \times_\beta G_2) = |V_1| = Min\{|V_1|, |V_2|\}$ (or)
$\chi(G_1 \times_\beta G_2) = \chi(G_1) = Min\{\chi(G_1), \chi(G_2)\}$
Case (ii) :Let $d_{G1} * (u_1) \geq d_{G2} * (u_2)$
Clearly the number of edges of G_2 must be less than the number of edges of G_1 .That is the number of vertices of G_1 is greater than the number of vertices of G_2 . As per case (i) , $u_2 = v_2, u_1v_1 \in E_1$ is the only choice for finding the chromatic number of beta product of fuzzy graph, since $q \leq p$
Thus $\chi(G_1 \times_\beta G_2) = |V_2|$
$\chi(G_1 \times_\beta G_2) = |V_2| = Min\{|V_1|, |V_2|\}$ (or)
$\chi(G_1 \times_\beta G_2) = \chi(G_2) = Min\{\chi(G_1), \chi(G_2)\}$
Case (iii):Let $d_{G1} * (u_1) = d_{G2} * (u_2)$
From cases (i) and (ii) ,it is clear that $\chi(G_1 \times_\beta G_2) = Min\{\chi(G_1), \chi(G_2)\}$
Thus from the above cases $\chi(G_1 \times_\beta G_2) = \chi(G_1)$ (or)$\chi(G_2) = Min\{\chi(G_1), \chi(G_2)\}$
□

Theorem 11. *Let G_1 and G_2 be two regular fuzzy graphs with $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$.*
If both G_1* and G_2* are complete graphs,then the chromatic number of regular gamma product of fuzzy graph is the product of two chromatic numbers $\chi(G_1)$ and $\chi(G_2)$ (or) $\chi(G_1 \times_\gamma G_2) = |V_1|.|V_2|$

Proof. Given G_1 and G_2 be two regular fuzzy graphs with the conditions $\sigma_1 \geq \mu_2$ and $\sigma_2 \geq \mu_1$. Both underlying crisp graphs G_1* and G_2* are complete graphs, then by theorem 4.5[6],$G_1 \times_\gamma G_2$ is regular fuzzy graph. Therefore all vertices of $G_1 \times_\gamma G_2$ are alpha strong vertices. Let the number of vertices of G_1 and G_2 be m and n. That is $|V_1| = m$ and $|V_2| = n$. Let p and q be the degree of the vertex of regular graphs G_1* and G_2* . If G_1* and G_2* are complete then $|V_1| = m = d_{G1} * (u_1) + 1$,$\forall u_1 \in V_1$
$|V_2| = n = d_{G2} * (u_2) + 1$,$\forall u_2 \in V_2$

$$d_{(G1\times_\gamma G2)} * (u_1, u_2) = \sum_{((u_1,u_2),(v_1,v_2)) \in E} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \tag{15}$$

$$= \sum_{\substack{u_1 = v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 = v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1 \neq v_1, u_2v_2 \in E_2 \text{ (or)} \\ u_2 \neq v_2, u_1v_1 \in E_1 \text{ (or)} \\ u_1v_1 \in E_1, u_2v_2 \in E_2}} (E_1 \times E_2)((u_1, u_2), (v_1, v_2)) \tag{16}$$

$$d_{(G1\times_\gamma G2)}*(u_1, u_2) = d_{G1}*(u_1)+d_{G2}*(u_2)+d_{G2}*(u_2)|E_1|+d_{G1}*(u_1)|E_2|+d_{G2}*(u_2)d_{G1}*(u_1) \tag{17}$$

$\forall u_1 \in V_1 and \forall u_2 \in V_2$
where $|E_1|$ and $|E_2|$ are the degree of the vertex of the complement graphs G_1* and G_2*
$d_{(G1\times_\gamma G2)} * (u_1, u_2) = q + p + p|E_2| + q|E_1| + pq.$
This is true for all vertices of $V_1 \times_\gamma V_2$. Here both G_1* and G_2* are complete graphs.
Thus $\chi(G_1) = m = d_{G1} * (u_1) + 1$,$\forall u_1 \in V_1$
and $\chi(G_2) = n = d_{G2} * (u_2) + 1$,$\forall u_2 \in V_2$. Hence the number of vertices of $G1 \times_\gamma G2$ is $mn = |V_1||V_2|$. Each vertex in $G1 \times_\gamma G2$ is adjacent with mn-1 vertices.
Thus the chromatic number of the gamma product of fuzzy graph is (mn-1)+1 which is equal to mn .
(i.e) $\chi(G_1 \times_\gamma G_2) = mn = |V_1|.|V_2| = \chi(G_1).\chi(G_2)$

□

4 Conclusion

In this paper we proposed an upper bound of a chromatic number of Alpha,Beta and Gamma product of fuzzy graphs by using fuzzy vertex order colouring .Our bounds favourably with Brooks bound $\chi(G) \leq [\Delta(G)] + 1$. Our future work is to find the application for vertex order colouring and fuzzy graph operations in the field of graph folding and networking.

References

[1] Assia Gueham,Anass Nagih, Two Bounds of Chromatic Number in Graph Coloring Problem, *IEEE*,(2014),292-296.

[2] David R Wood,Attila Por, Colourings of the Cartesian Product of Graphs and Multiplicative Sidon Sets,*Combinatorica* **29**(4),(2009),449-466.

[3] Changiz Eslahchi and B N Onagh , Vertex Strength Of Fuzzy Graphs,*International Journal of Mathematics and Mathematical Sciences*,(2006), 19

[4] Mordeson J N,Peng C S, Fuzzy Graphs and Fuzzy Hyper Graphs,*Information Sciences***79**,(1994),159-170.

[5] Nagoor Gani A,Fathima Kani B, Alpha Product of Fuzzy Graphs, *Advances in Fuzzy Sets And Systems* **17**(1),(2014),27-48 .

[6] Nagoor Gani A,Fathima Kani B, Beta and Gamma Product of Fuzzy Graphs,*International Journal of Fuzzy Mathematical Archive* **4**(1),(2014), 20-36.

[7] Nagoor Gani A,Fathima Kani B, Fuzzy Vertex Order Colouring, *International Journal of Pure and Applied Mathematics* **107** (3),(2016),601-614.

[8] Rosenfeld A, *Fuzzy Graphs* , In:L A Zadeh ,K S Fu, M Shimura, Eds, Fuzzy Sets and Their Applications, Academic Press,(1975),77-95.

[9] Yeh R T and Banh S Y, *Fuzzy relations, fuzzy graphs and their applications to clustering analysis*,In Fuzzy sets and their Applications to Cognitive and Decision Process. L A Zadeh, K S Fu, M Shimura Eds: Academic Press, New York,(1975),125-149.

[10] Zadeh L A, *Fuzzy Sets*,Information and Control **8**, (1965),338-353.