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Intuitionistic Fuzzy Graph Metric Space

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Abstract

Intuitionistic Fuzzy graph theory has variety of applications in modern science and technology, especially in the fields of neural networks artificial intelligence and decision making. An intuitionistic fuzzy set is a generalization of the concept of fuzzy set. The intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the classical and fuzzy models. In this paper, the concept of intuitionistic fuzzy graph metric space is introduced. Sequence, convergent sequence, Cauchy sequence, complete intuitionistic fuzzy graph metric, Continuous, Uniformly Continuous and Isometric Functions are studied.

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Key Words and Phrases: Intuitionistic Fuzzy Graph, Intuitionistic Fuzzy Graph Metric, Convergent

1 INTRODUCTION

In 1965, Zadeh [10] introduced the concept of fuzzy set as a method of finding uncertainty. Rosenfeld [6] introduced the concept of fuzzy graphs in 1975. Yeh and Bang [9] also introduced fuzzy graphs independently. Vaishnaw et al.[8] discussed some analogies results on fuzzy graphs. Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graph. Parvathi and Karunambigai [5] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Atanassov and Shannon [7]. Parvathi and Thamizhendhi [4] were introduced cardinality of an intuitionistic fuzzy graph. The important graph theory approach to metric fixed point theory introduced so far is attributed to Jachymski [2]. In 1996, Kada et al. [3] defined the notion of w-distance in metric spaces. In this paper, the concept of intuitionistic fuzzy graph metric space is introduced. Sequence, convergent sequence, Cauchy sequence, complete intuitionistic fuzzy graph metric, Continuous, Uniformly Continuous and Isometric Functions are studied.

2 PRELIMINARIES

Definition 1. An intuitionistic fuzzy graph is of the form $G = (V, E)$, where

- $V = \{v_1, v_2, ..., v_n\}$ such that $\sigma_1 : V \to [0, 1]$ and $\sigma_2 : V \to [0, 1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively and $0 \leq \sigma_1(v_i) + \sigma_2(v_i) \leq 1$ for every $v_i \in V(i = 1, 2, ..., n)$.
- $E \subseteq V \times V$ where $\mu_1 : V \times V \to [0, 1]$ and $\mu_2 : V \times V \to [0, 1]$ are defined by $\mu_1(v_i, v_j) \leq \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) \geq \sigma_2(v_i) \vee \sigma_2(v_j)$ such that $0 \leq \mu_1(v_i, v_j) + \mu_2(v_i, v_j) \leq 1$,for every $(v_i, v_j) \in E$.

Definition 2. In an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, the weight of a vertex $u \in V$ is defined by

$$w(u) = \frac{1 + \sigma_1(u) - \sigma_2(u))}{2}$$

and also the weight of an edge $e = (u, v) \in E$ is defined by

$$w(e) = w(u, v) = \frac{1 + \mu_1(u, v) - \mu_2(u, v)}{2}$$

Definition 3. Let $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is an intuitionistic fuzzy graph, then the distance $d(u, v)$ between two of its vertices u and v is the length of shortest path between them.

$$d(u, v) = \bigwedge(\sum_{i,j} w(u_i, v_j))$$

Definition 4. Let V be an arbitrary non-empty set and IF(V) be the intuitionistic fuzzy subsets of V.

$$IF(V) = \{v = (\sigma_1(v), \sigma_2(v)) \in I^X \times I^X : 0 \leq \sigma_1(v) + \sigma_2(v) \leq 1, x \in X\}.$$

Definition 5. Let $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph of the graph $G : (V, E)$, and then a function $d : IF(V) \times IF(V) \to [0, \infty)$ is said to be metric in intuitionistic fuzzy graph if it satisfied the following conditions: For all $u, v, w \in IF(V)$,

1. $d(u, v) > 0$ if $\sigma_1(u) \neq \sigma_1(v); \sigma_2(u) \neq \sigma_2(v)$
2. $d(u, v) = 0 \iff \sigma_1(u) = \sigma_1(v) = 0; \sigma_2(u) = \sigma_2(v) = 1$
3. $d(u, v) = d(v, u)$
4. $d(u, w) \leq d(u, v) + d(v, w)$

Then d is called an intuitionistic fuzzy graph metric on V. The function $d(u, v)$ is the length of the shortest path between u and v. The pair $(IF(X), d)$ is called an intuitionistic fuzzy graph metric space.

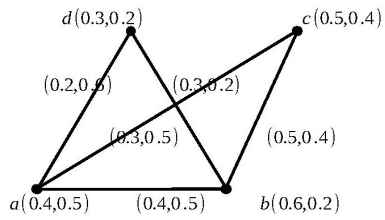

Figure 1: Example of Intuitionistic Fuzzy Graph Metric Space

Example 1. *Consider the intuitionistic fuzzy graph as given in Fig.1,* $IF(V) = \{a(0.4, 0.5), b(0.6, 0.2), c(0.5, 0.4), d(0.3, 0.2)\}$ $d(a, b) = 0.45 > 0$ *(since* $a \to b$ *is the shortest path)* $d(a, b) = d(b, a)$ *since shortest path between* a *and* b = *shortest path between* b *and* a.
$d(a, b) \leq d(a, c) + d(c, b) \implies 0.45 \leq 0.4 + 0.55,$
$d(a, b) \leq d(a, d) + d(d, b) \implies 0.45 \leq 0.3 + 0.4$
Hence, $(IF(V), d)$ *is an intuitionistic fuzzy graph metric space.*

Example 2. *Consider a connected intuitionistic fuzzy graph of the graph* $G : (V, E)$. *The function* d *defined by*

$$d(u, v) = \begin{cases} 1 & if \;\; \sigma_1(u) \neq \sigma_1(v) \;\; and \;\; \sigma_2(u) \neq \sigma_2(v) \\ 0 & if \;\; \sigma_1(u) = \sigma_1(v) \;\; and \;\; \sigma_2(u) = \sigma_2(v)), \forall u, v \in IF(V) \end{cases}$$

is a metric in intuitionistic fuzzy graph and it is called discrete metric.

Example 3. *Let* $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ *be a connected intuitionistic fuzzy graph of the graph* $G : (V, E)$ *and* d *be a metric on intuitionistic fuzzy graph, and then* $\overline{d}(u, v) = \min\{d(u, v), 1\}$, *for all* $u, v \in IF(V)$ *is also a metric on intuitionistic fuzzy graph.*

Theorem 6. *Let* $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ *be a connected intuitionistic fuzzy graph then distance between vertices is metric in intuitionistic fuzzy graph.*

Proof. Since $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is a connected intuitionistic fuzzy graph with $G = (V, E)$, then we have to show that distance between vertices of intuitionistic fuzzy graph is metric, for this let $d : IF(V) \times IF(V) \to [0, \infty)$. Then by the definition of distance on intuitionistic fuzzy graph the condition 1 and 2 is trivial. The shortest distance between u and v is same as shortest distance between v and u. Condition 3 holds. Now, since distance $d(u, v)$ is the length of the shortest path between u and v and this path cannot be greater than any path between u and v, which goes through another vertex w. $d(u, v) \leq d(u, w) + d(w, v)$. This completes the proof. □

Definition 7. Let $G : ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be a connected intuitionistic fuzzy graph with $G : (V, E)$ and $d : IF(V) \times IF(V) \to [0, \infty)$ be a metric in intuitionistic fuzzy graph. The sequence $\{v_n\}$ of vertices of $IF(V)$ is said to converge to a vertex

$v \in IF(V)$, if for each $\epsilon > 0$, there exists a positive integer m such that $d(v_n, v) < \epsilon$ for all $n \geq m$.

$$\lim_{n\to\infty} d(v_n, v) \to 0$$

Example 4. *Let us consider the sequence $v_n = (\sigma_1(v_n), \sigma_2(v_n))$ of vertices in intuitionistic fuzzy graph where $\sigma_1(v_n) = \frac{1}{n}$ and $\sigma_2(v_n) = \frac{n-1}{n}$. Then the sequence converges to $v = (\sigma_1(v), \sigma_2(v))$ where $\sigma_1(v) = 0$ and $\sigma_2(v) = 1$ as $n \to \infty$.*

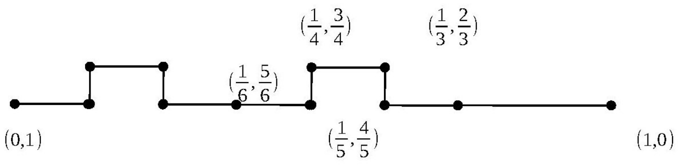

Figure 2: Illustration of Convergence Sequence

Example 5. *The sequences $\{v_n\} = (\sigma_1(v_n), \sigma_2(v_n)) = \{(1/(n+1), n/(n+1))\}, \{(1 - 1/n, 1/n)\}$ are some examples of convergent sequence in intuitionistic fuzzy graph metric space $(IF(V), d)$.*

Definition 8. A sequence $\{v_n\}$ of vertices of $IF(V)$ is said to be a Cauchy sequence if for each $\epsilon > 0$, there exists a positive integers N such that $d(v_n, v_m) < \epsilon,\ \forall\ n, m \geq N,\ d(v_n, v_m) \to 0,\ \ as\ \ n \to \infty$.

Example 6. *Consider the sequence $\{v_n\} = \{\sigma_1(v_n), \sigma_2(v_n)\} = \{(1/n, (n-1)/n)\}$ of vertices in $IF(V)$ is an example for Cauchy sequence.*

3 COMPLETE INTUITIONISTIC FIZZY GRAPH METRIC SPACE

Definition 9. An intuitionistic fuzzy graph metric space$(IF(V), d)$ is said to be complete intuitionistic fuzzy graph metric space if every Cauchy sequence is converges to a vertex of $IF(V)$.

Lemma 1. *In intuitionistic fuzzy graph metric space (IF(V),d), any sequence is convergent sequence if and only if it is Cauchy sequence.*

Lemma 2. *Every Intuitionistic fuzzy graph metric space is always complete.*

Proof. For any Intuitionistic fuzzy graph metric space (IF(V),d), a sequence in (IF(V),d) is convergent sequence if and only if it is Cauchy sequence. Therefore, every intuitionistic fuzzy graph metric space is always complete. □

Definition 10. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f : IF(V) \to IF(V)$ is said to be continuous at a vertex $u \in IF(V)$ if for given $\epsilon > 0$, there exists a $\delta > 0$ such that $d_2(f(u), f(v)) < \epsilon$ whenever $d_1(u, v) < \delta$, for all $u, v \in IF(V)$.

Example 7. *Let G be an intuitionistic fuzzy graph on the set* $V = \{v_1/ < \sigma_1(v_1) = 1, \sigma_2(v_1) = 0 >, v_2/ < \sigma_1(v_2) = 0.75, \sigma_2(v_2) = 0.25 >, v_3/ < \sigma_1(v_3) = 1, \sigma_2(v_3) = 0 >, v_4/ < \sigma_1(v_4) = 0.25, \sigma_2(v_4) = 0.75 >\}$ *such that* $f(\sigma_1(v_1)) = 0.75, f(\sigma_2(v_2)) = 0.25, f(\sigma_1(v_2)) = 0.75, f(\sigma_2(v_2)) = 0.25, f(\sigma_1(v_3)) = 0.75, f(\sigma_2(v_3)) = 0.25, f(\sigma_1(v_4)) = 0.75, f(\sigma_2(v_4)) = 0.25$ *Clearly, f is continuous functions.*

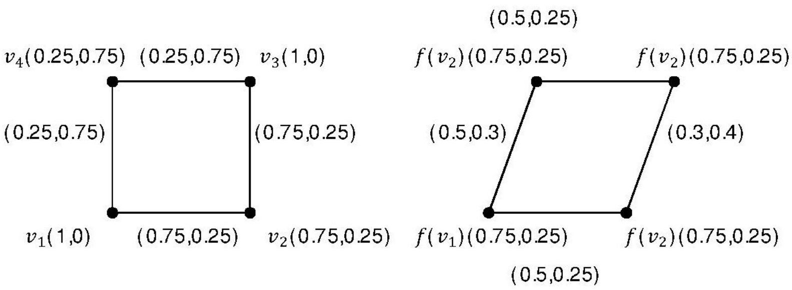

Figure 3: Illustration of Continuous function

Definition 11. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f : (IF(V), d_1) \to (IF(V), d_2)$ is said to be uniformly continuous if for given $\epsilon > 0$, there exists a $\delta > 0$ such that $d_2(f(u), f(v)) < \epsilon$ whenever $d_1(u, v) < \delta$ for all $u, v \in IF(X)$.

Lemma 3. *Any function defined on $(IF(V), d)$ is continuous if and only if it is uniformly continuous.*

Example 8. *Consider the following two intuitionistic fuzzy graph metrics,* $f : (IV(F), d_1) \to (IV(F), d_2)$ $G_1 : IF(V) = \{[a < 0.5, 0.3 >, b < 0.4, 0.4 >, c < 0.6, 0.2 >, d < 0.2, 0.8 >], [ab < 0.4, 0.4 >, bc < 0.4, 0.4 >, ca < 0.5, 0.3 >, ad < 0.2, 0.8 >, dc < 0.2, 0.8 >, db < 0.2, 0.8 >]\}$ $G_2 : IF(V) = \{[f(a) < 0.5, 0.3 >, f(b) < 0.4, 0.4 >, f(c) < 0.6, 0.2 >, f(d) < 0.2, 0.8 >], [f(a)f(b) < 0.3, 0.4 >, f(b)f(c) < 0.3, 0.4 >, f(c)f(a) < 0.4, 0.3 >, f(a)f(d) < 0.1, 0.8 >, f(d)f(c) < 0.2, 0.8 >, f(d)f(b) < 0.1, 0.7 >]\}$

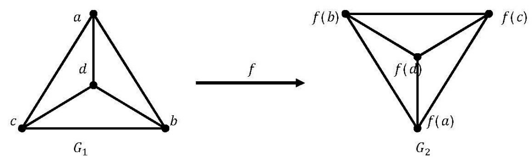

Figure 4: Example of ‘”continuous ⇔ uniformly continuous”
f is continuous $\iff$ f is uniformly continuous

Definition 12. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f : IF(V) \to IF(V)$ is said to be an isometric if $d_1(u, v) = d_2(f(u), f(v))$ for all $u, v \in IF(V)$.

Example 9. *Consider the two intuitionistic fuzzy graph metric spaces* $(IF(V), d_1)$ *and* $(IF(V), d_2)$ *as given below. Here,* $f : IF(V) \to IF(V)$ *is an isometric function.*

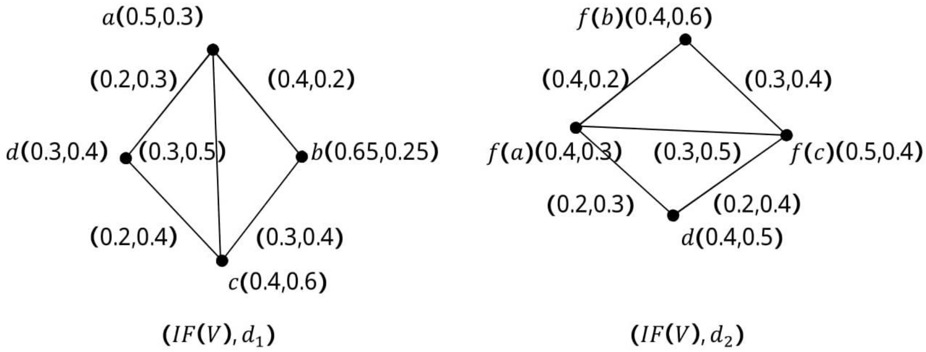

Figure 5: Illustration of Isometric Function

4 CONCLUSION

In this paper, we introduced Intuitionistic fuzzy graph metric space and complete Intuitionistic fuzzy graph metric space which will useful for further research. Sequence, convergent sequence, Cauchy sequence, Continuous, Uniformly Continuous and Isometric functions are studied in the Intuitionistic fuzzy graph metric spaces. Related theorems and examples are discussed in detail. In future, we elaborate the new concepts and derive more results from this.

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