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Fixed Point Theorems in 2-Fuzzy 2-Anti Normed Linear Space

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Abstract

In this paper fixed point theorems are developed for self mappings and for the pair of weakly commuting self mappings on 2- fuzzy 2- anti normed linear space.

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1 Introduction

Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy set handle such situation by attributing a degree to which certain object belongs to a set. The idea of fuzzy set was initiated by Zadeh[13] in 1965 and . The concept of fuzzy norm was investigated by Katsaras [4]in 1984. Cheng and Mordeson [2] 1994 established an idea of a fuzzy norm on a linear space.A satisfactory theory of 2- norm on a linear space has been brought out and developed by Gahler[3]. R.M Somasundaram and Thangaraj Beaula[12] defined the concept of 2-fuzzy 2- normed linear space in 2009. Later, the idea of fuzzy anti norm was introduced by Bag and Samanta[1] and investigated their important properties.In 2011 B.Surender Reddy [7, 8] introduced the idea of fuzzy anti 2- normed linear space. In 2012 Parijit Sinha et.al[6, 10, 11] developed some results on fuzzy anti 2- normed linear space.In 2010, Kailash Namdeo et.al[5] have proved some theorems related to common fixed points for fuzzy 2- metric space for rational expression. In 2013 Savitha Rathee et.al[9] have established a common fixed point theorem for a pair of operators in fuzzy normed space. In this paper fixed point theorems are developed for self mappings and for the pair of weakly commuting self mappings on 2- fuzzy 2- anti normed linear space. Some important theorem in fixed points in 2- fuzzy 2- anti normed linear space is developed.

2 Preliminaries

Definition 1. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is a t-concorm if $\diamond$ satisfies the following conditions

(i) $\diamond$ is commutative and associative.

(ii) $a \diamond 0 = a, \forall$ a $\in$ [0,1].

(iii) $a \diamond b \leq c \diamond d$ whenever a $\leq$ c, b $\leq$ d and a, b, c, d $\in$ [0,1].

Definition 2. Let X be the linear space over the real field K. A fuzzy subset N^* of $F(X) \times F(X) \times R$. (R, the set of all real numbers) is called a 2- fuzzy 2 - anti norm on X if and only iff

(A-N1) For all t $\in$ R with t $\leq$ 0, $N^*(f_1, f_2, t) = 1$.

(A-N2) For all t $\in$ R with t $\geq$ 0, $N^*(f_1, f_2, t) = 0$ if and only if f_1 and f_2 are linearly dependent.

(A-N3) $N^*(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2.

(A-N4) For all t $\in$ R with t $\geq$ 0, $N^*(f_1, cf_2, t) = N^*(f_1, f_2, \frac{t}{?c?})$ if c $\neq$ 0, c $\in$ K (field).

(A-N5) For all s, t $\in$ R, $N^*(f_1, f_2 + f_{3,} s + t) \leq \max \{N^*(f_1, f_2, s), N^*(f_1, f_3, t)\}$.

(A-N6) $N^*(f_1, f_2, \cdot) : (0, \infty) \to [0,1]$ is continuous.

(A-N7) $\lim_{t \to \infty} N^*(f_1, f_2, t) = 1$.

Then $(F(X), N^*)$ is a fuzzy 2-anti normed linear space, or (X, N^*) is a 2-fuzzy 2-anti normed linear space.

Definition 3. A sequence $\{f_n\}$ in a 2- fuzzy 2- anti normed linear space, (X, N^*) is said to be a Cauchy sequence if for given $t > 0$,$0 < r < 1$,there exist a $n_0 \in N$ such that $N^*(f_n - f_m, g, t) < r$ for all $n \geq n_0$.

3 Fixed points Theorems in 2- fuzzy 2- anti normed linear space

Definition 4. Let A , B be self mappings in a 2- fuzzy 2-anti normed linear space $(X, N^*, \diamond)$. The pair A,B are weakly commuting if $N^*(ABf - BAf, g, t) \geq N^*(Af - Bf, g, t)$ for every $f, g \in F(X)$ and $t > 0$

Theorem 5. *Let (X, N^*) be a 2-fuzzy 2- anti normed linear space. Let T : $F(X) \to F(X)$ be a map satisfying the following condition,*
(i.e) there exist a $\lambda \in (0,1)$ such that for all $f \in F(X)$ and for all $t > 0$,

$$N^*(f, g, t) < t \Rightarrow N^*(T(f), g, \lambda t) < \lambda t \tag{1}$$

(i) For any real number $\varepsilon > (0,1)$ there exist $k_0(\varepsilon) \in N$ such that $f^k(f) \to \theta$

(ii) T has atmost fixed point that is the null vector of F(X). Moreover if T is a linear mapping

then T has exactly one fixed point.

Proof. Assume that T satisfies the condition(i) then for every $\varepsilon \in (0,1)$ there exists a $k_0 = k_0(\varepsilon)$ such that for all $k \geq k_0$ and for every $f \in F(X)$, $N^*(T^k(f), g, \varepsilon) < \varepsilon$ holds. That is $N^*(f, g, 1+\varepsilon) < 1+\varepsilon$ By condition (i)$N^*(f, g, 1+\varepsilon) < 1+\varepsilon \implies N^*(Tf, g, \lambda(1+\varepsilon)) < \lambda(1+\varepsilon)$ Consider,

$$\begin{aligned} N^*(T^2f, g, \lambda^2(1+\varepsilon)) &= N^*(T(Tf), g, \lambda(\lambda(1+\varepsilon))) \\ &< \lambda(\lambda(1+\varepsilon)) \\ &= \lambda^2(1+\varepsilon) \end{aligned}$$

Continuing this way, $N^*(T^kf, g, \lambda^k(1+\varepsilon)) < \lambda^k(1+\varepsilon)$
Indeed for each $\varepsilon > 0$ there exist a $k = k_0$ which implies that $\lambda^k(1+\varepsilon) \leq \varepsilon$ because of (A-N7)

$$\begin{aligned} &\lim_{t\to\infty} N^*(f, g, t) = 0 \ \ \textit{there exist a } k_0 \in N \textit{ such that for } k \geq k_0 \\ &N^*(T^k(f), g, \varepsilon) \leq N^*(T^k(f), g, \lambda^k(1+\varepsilon)). \\ &\qquad < \lambda^k(1+\varepsilon). \\ &\qquad \leq \varepsilon. \end{aligned}$$

Since ε is arbitrary, we have $T^k(f) \to \theta$ as required.
Assume that $T(f) = f$, from (i) for all $\varepsilon \in (0,1)$,
$N^*(f, g, \varepsilon) < \varepsilon \Longrightarrow N^*(T^k(f), g, \varepsilon)$ for every $f \in F(X)$
this implies that $\lim_{t\to\infty} N^*(f, g, 0_+) = 0$ for every $f \in F(X)$ hence $f = 0$. □

Theorem 6. *Let $\{f_k\}$ be a sequence in 2- fuzzy 2- anti normed linear space $(F(X), N^*)$. If for every $t > 0$, there exist a constant $\lambda \in (0,1)$ such that $N^*(f_k - f_{k+1}, g, t) \leq N^*(f_{k-1} - f_k, g, \frac{t}{\lambda})$ for all $f \in F(X)$ then $\{f_k\}$ is a cauchy sequence in 2- fuzzy 2- anti normed linear space.*

Proof. Let $t > 0$ and $\lambda \in (0,1)$ then for $m\ geqk$

$$\begin{aligned} N^*(f_k - f_m, g, t) &\leq N^*(f_k - f_{k+1} + f_{k+1} - f_m, g, t) \\ &\leq max\{N^*(f_k - f_{k+1}, g, (1-\lambda)t), N^*(f_{k+1} - f_m, g, \lambda t)\} \\ &\dots\dots\dots\dots\dots\dots \\ &\leq max\{N^*(f_0 - f_1, g, \frac{(1-\lambda)\ t}{\lambda^k}), N^*(f_{k+1} - f_m, g, \lambda t)\}. \end{aligned}$$

Also

$$\begin{aligned} N^*(f_{k+1} - f_m, g, \lambda t) &\leq N^*(f_{k+1} - f_{k+2} + f_{k+2} - f_m, g, t) \\ &= max\{N^*(f_{k+1} - f_{k+2}, g, \frac{(1-\lambda)\ t}{\lambda^k}), N^*(f_{k+2} - f_m, g, \lambda_2 t)\} \\ &\dots\dots\dots\dots\dots\dots \\ &= max\{N^*(f_0 - f_1, g, \frac{(1-\lambda)\ t}{\lambda^k}), N^*(f_{k+2} - f_m, g, \lambda_2 t)\} \end{aligned}$$

On repeating the argument

$$N^*(f_k - f_m, g, t) \leq max\{N^*(f_0 - f_1, g, \frac{(1-\lambda)\ t}{\lambda^k}), N^*(f_{m-1} - f_m, g, \lambda_{m-n-1}t)\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\leq max\{N^*(f_0 - f_1, g, \frac{(1-\lambda)\,t}{\lambda^k}), N^*(f_0 - f_1, g, \frac{t}{\lambda^k})\}$$

Since $(1-\lambda)\frac{t}{\lambda^k} \leq \frac{t}{\lambda^k}$ we get $N^*(f_k - f_m, g, t) \leq N*(f_0 - f_1, g, \frac{(1-\lambda)t}{\lambda^k}$

Letting $m \geq k \to \infty lim_{m,k\to\infty} N^*\,(f_k - f_m, g, t) = 0$ for every $f_k, f_m, g \in F(X), t > 0$
Therefore $\{f_n\}$ is a cauchy sequence. □

Theorem 7. *Let $(X, N^*, \diamond)$ be a 2- fuzzy 2- anti normed linear space with t-conorm $\diamond$ satisfying the following condition $S, T : F(X) \to F(X)$ such that*

$$N^*(f, Tg_1{-}Sg_2, t) \leq N^*(f, g_1{-}Sg_2, 2t) \diamond N^*(f, g_2{-}Tg_1, t) \diamond N^*(f, Tg_1{-}g_2, t) \diamond N*(f, g_1{-}Sg_2, 2t) \quad (2)$$

holds for all $g_1, g_2 \in F(X)$. Then T and S have a common fixed point.

Proof. Construct the sequence $\{h_n\}$ by taking

$$h_{n+1} = Th_n, h_{n+2} = Sh_{n+1} \quad (3)$$

We shall prove, $N^*(f, h_{n+1} - h_{n+2}, 2t) \leq N^*(f, h_{n+1} - h_{n+2}, t)$
As a contrary suppose $N^*(f, h_{n+1} - h_{n+2}, 2t) > N^*(f, h_n - h_{n+1}, t)$
Using(2),

$$\begin{aligned} N^*(f, Tg_1 - Sg_2, t) &\leq N^*(f, g_1 - Sg_2, 2t) \diamond N^*(f, g_2 - Tg_1, t) \diamond N^*(f, Tg_1 - g_2, t) \\ &\diamond N^*(f, g_1 - Sg_2, 2t) \\ &\leq N^*(f, h_n - Sh_{n+1}, 2t) \diamond N^*(f, h_{n+1} - Th_n, 2t) \diamond N^*(f, Th_n - h_{n+1}, t) \\ &\diamond N^*(f, h_n - Sh_{n+1}, 2t) \\ &= N*(f, h_n - h_{n+2}, 2t) \diamond N^*(f, h_{n+2} - h_{n+1}, 2t) \diamond N^*(f, h_{n+1} - h_{n+1}, t) \\ &\diamond N^*(f, h_n - Sh_{n+2}, 2t) \\ &= N^*(f, h_n - h_{n+1} + h_{n+1} - h_{n+2}, 2t) \diamond N^*(f, h_{n+1} - h_{n+1}, t) \\ &\diamond N^*(f, h_{n+1} - h_{n+1}, t) \diamond N^*(f, h_n - h_{n+1} + h_{n+1}, -h_{n+2}, 2t) \\ &< N^*(f, h_{n+1} - h_{n+2}, kt) \diamond N^*(f, h_{n+1} - h_{n+2}, kt) \diamond N^*(f, h_{n+1} - h_{n+2}, kt) \\ &\diamond N^*(f, h_{n+1} - h_{n+2}, kt) \\ &\Longrightarrow N^*(f, h_{n+1} - h_{n+2}, kt) \\ .&< N^*(f, h_{n+1} - h_{n+2}, kt). \end{aligned}$$

which is a contradiction so it must be $N^*(f, h_{n+1}?h_{n+2}, kt) \leq N^*(f, h_{n+1} - h_{n+2}, t)$
Similarly we can prove $N^*(f, h_{n+2}?h_{n+3}, kt) \leq N^*(f, h_{n+1} - h_{n+2}, t)$ In general $N^*(f, h_{n+1}?h_{n+2}, kt) \leq N*(f, h_n?h_{n+2}, kt)\{h_n\}$ is a cauchy sequence and converges to a point $\boldsymbol{l}$, Now prove that $S(g) = \boldsymbol{l}$. Let $B(f, S(g), \varepsilon, r)$ be the neighborhood of

$T(f)$ Since, $h_{n\to}\boldsymbol{l}$, for $\varepsilon, r > 0$ there is an integer N such that $N^*(f, h_n?h_{n+1}\varepsilon) < 1?\ r$ and $N^*(f, \boldsymbol{l}?h_{n+1}, \varepsilon) < 1?\ r$ for all $n \geq N(\varepsilon, r)$ By (2) and (3) consider

$$\begin{aligned} N^*(f, h_{n+1} - Sg, \varepsilon) &= N^*(f, h_n - Sg, \varepsilon) \\ &\leq N^*(f, h_n - Sg, 2\varepsilon) \diamond N^*(f, \boldsymbol{l} - Th, \varepsilon) \\ &\diamond N^*(f, Th_n - \boldsymbol{l}, \varepsilon) \diamond N^*(f, h_n - Sg, 2\varepsilon) \\ &= N^*(f, h_n - Sg, 2\varepsilon) \diamond N^*(f, \boldsymbol{l} - h_{n+1}, \varepsilon) \\ &\diamond N^*(f, h_{n+1} - \boldsymbol{l}, \varepsilon) \diamond N^*(f, h_n - Sg, 2\varepsilon) \\ &= N^*(f, h_n - h_{n+1} + h_{n+1}, -Sg, 2\varepsilon) \\ &\diamond N^*(f, \boldsymbol{l} - h_{n+1}, \varepsilon) \diamond N^*(f, h_{n+1} - \boldsymbol{l}, \varepsilon) \\ &\diamond N^*(f, h_n - h_{n+1} + h_{n+1} - Sg, 2\varepsilon) \\ &\leq N^*(f, h_n - h_{n+1}, \varepsilon) \diamond N^*(f, \boldsymbol{l} - h_{n+1}, \varepsilon) \\ &\diamond N^*(f, h_{n+1} - Sg, \varepsilon) \diamond N^*(f, \boldsymbol{l} - h_{n+1}, \varepsilon) \\ &\diamond N^*(f, h_{n+1} - \boldsymbol{l}, \varepsilon) \diamond N^*(f, h_n - h_{n+1}, \varepsilon) \\ &\diamond N^*(f, h_{n+1} - \boldsymbol{l}, \varepsilon) \diamond N^*(f, h_{n+1} - Sg, \varepsilon) \\ &< 1 - r. \end{aligned}$$

This implies that,$N^*(f, h_{n+1} - Sg, \varepsilon) < 1?r$. since $h_{n+1\to}\boldsymbol{l}, Sg \to \boldsymbol{l}$, similarly $Tg = \boldsymbol{l}$ Now to prove the uniqueness of $\boldsymbol{l}$ as a common fixed point of T and S Let m be another common fixed point then by (1) for some $t > 0$,

$$\begin{aligned} N^*(f, Tg_1 - Sg_2, kt) &\leq N^*(f, g_1 - Sg_2, 2t) \diamond N^*(f, g_2 - Tg_1, t) \diamond N^*(f, Tg_1 - g_2, t) \\ &\diamond N^*(f, g_1 - Sg_2, 2t) \\ N^*(f, \boldsymbol{l} - m, kt) &= N^*(f, \boldsymbol{l} - m, 2t) \diamond N^*(f, m - \boldsymbol{l}, t) \diamond N^*(f, \boldsymbol{l} - m, t) \\ &\diamond N^*(f, \boldsymbol{l} - m, 2t) \\ &= N^*(f, \boldsymbol{l} - m, t) \diamond N^*(f, \boldsymbol{l} - m, t) \diamond N^*(f, m - \boldsymbol{l}, t) \\ &\diamond N^*(f, \boldsymbol{l} - m, t) \diamond N^*(f, \boldsymbol{l} - m, t) \diamond N^*(f, \boldsymbol{l} - m, t) \\ N^*(f, \boldsymbol{l} - m, kt) &\leq N^*(f, \boldsymbol{l} - m, t) \\ &= N^*(f, \boldsymbol{l} - m, \frac{t}{k}) \leq \ldots\ldots \leq N^*(f, \boldsymbol{l} - m, \frac{t}{k}) \\ &= \mathbf{0} \ as \ n \to \infty \end{aligned}$$

This proves $\boldsymbol{l} \ = m$. □

Theorem 8. *Let $(X, N^*, \diamond)$ be a complete 2- fuzzy 2- anti normed linear space. Let B and C be a continuous mapping of F(X) then B and C have a common fixed point in F(X). If there exist a continuous mapping A of F(X) into B(F(X)∩ C(F(X)) which commute weakly with B and C and*

$$N^*(Af - Ag, h, qt) \leq max\{N^*(Bf - Cg, h, t), \frac{N^*(Bf - Cg, h, t)}{N^*(Af - Cg, h, t)}, \frac{N^*(Bf - Cg, h, t)}{N^*(Af - Bg, h, t)}\} \tag{4}$$

For all f, g , h ∈ F(X) and $t > 0$ and $0 < q < 1$ and $lim_{n\to\infty} N^*(f - g, h', t) = 0$ *for all f, g, h' in F(X). Then B , C and A have unique common fixed point.*

Proof. Define a sequence $\{f_n\}$ such that $Af_{2n} = Bf_{2n+1}$ and $Af_{2n-1} = Cf_{2n}$ for n= 1,2, ...Let us prove that $\{Af_n\}$ is a Cauchy sequence for this suppose $f = f_{2n}$ and $g = f$
Using(4),

$$N^*(Af_{2n} - Af_{2n+1}, h, qt) \leq max\{N^*(Bf_{2n} - Cf_{2n+1}, h, t), \frac{N^*(Bf_{2n} - Cf_{2n+1}, h, t)}{N^*(Af_{2n} - Cf_{2n+1}, h, t)}, \frac{N^*(Bf_{2n} - Cf_{2n+1}, h, t)}{N^*(Af_{2n} - Bf_{2n+1}, h, t)}\}$$

$$\begin{aligned} N^*(Af_{2n} - Af_{2n+1}, h, qt) &\leq max\{N^*(Af_{2n-1} - Af_{2n}, h, t), \frac{N^*(Af_{2n-1} - Af_{2n}, h, t)}{N^*(Af_{2n} - Af_{2n}, h, t)}, \frac{N^*(Af_{2n-1} - Af_{2n}, h, t)}{N^*(Af_{2n} - Af_{2n}, h, t)}\} \\ &\leq N^*(Af_{2n-1} - Af_{2n}, h, t) \\ &\leq N^*(Af_{2n-2} - Af_{2n-1}, h, \frac{t}{q}) \end{aligned}$$

By induction
$N^*(Af_{2k} - Af_{2m+1}, h, qt) \leq \{N^*(Af_{2k-1} - Af_{2m}, h, t)$ for every k and m in N
Further if $2m + 1 > 2k$ then,

$$\begin{aligned} N^*(Af_{2k} - Af_{2m+1}, h, qt) &\leq \{N * (Af_{2k-1} - Af_{2m}, h, t) \\ &\leq N^*(Af_{2k-2} - Af_{2m-1}, h, \frac{t}{q}) \\ &\cdots\cdots\cdots \\ &\leq N^*(Af_0 - Af_{2m+2k}, h, \frac{t}{q^{2k-1}} \ldots\ldots (5) \end{aligned}$$

If $2k > 2m$ then

$$\begin{aligned} N^*(Af_{2k} - Af_{2m+1}, h, qt) &\leq \{N^*(Af_{2k-1} - Af_{2m}, h, t) \\ &\leq N^*(Af_{2k-2} - Af_{2m-1}, h, \frac{t}{q}) \\ &\cdots\cdots\cdots \\ &\leq N^*(Af_{2k-2m} - Af_0, h, \frac{t}{q^{2m-1}}) \ldots\ldots (6) \end{aligned}$$

By simple induction we get from (5) and (6)
$N^*(Af_n - Af_{n+p}, h, qt) \leq N^*(Af_0 - Af_p, h, \frac{t}{q^{n-1}})$
For $n = 2k, p = 2m + 1$ or $n = 2k + 1, p = 2m + 1$
$N^*(Af_n - Af_{n+p}, h, qt) \leq$ Max $\{N^*(Af_0 - Af_1, h, \frac{t}{2q^{n-1}}), N^*(Af_1 - Af_{n+p}, h, \frac{t}{2q^{n-1}})\}$
For $n = 2k, p = 2m$ or $n = 2k + 1, p = 2m$ and for every positive integer p and n in N
$N^*(Af_0 - Af_p, h, \frac{t}{q^{n-1}}) \to 1$ as $n \to \infty$
Thus $\{Af_n\}$ is a Cauchy sequence there exist $h' \in F(X)$,such that $\lim_{n\to\infty} Af_n = \lim_{n\to\infty} Bf_{2n+1} = lim_{n\to\infty} Cf_{2n} = h'$

It follows that $Ah^{'} = Bh^{'} = Ch^{'}$ Again using (4),

$$N^*(Ah^{'} - AAh^{'}, h, qt) \leq max\{N^*(Bh^{'} - CAh^{'}, h, t), \frac{N^*\left(Bh^{'} - CAh^{'}, h, t\right)}{N^*(Ah' - CAh', h, t)}, \frac{N^*\left(Bh^{'} - CAh^{'}, h, t\right)}{N^*(Ah' - BAh', h, t)}\}$$

$$N^*(Ah^{'} - A^2h^{'}, h, qt) \leq max\{N^*(Bh^{'} - CAh^{'}, h, t), \frac{N^*\left(Bh^{'} - CAh^{'}, h, t\right)}{N^*(Ah' - Ch', h, t)}, \frac{N^*\left(Bh^{'} - CAh^{'}, h, t\right)}{N^*(Ah' - Bh', h, t)}\}$$

$$\leq N^*(Bh^{'} - CAh^{'}, h, t)$$

$$\leq N^*(Bh^{'} - CAh^{'}, h, t)$$

$$\leq N^*(Ah^{'} - A^2h^{'}, h, t) \qquad (\because Ah^{'} = Bh^{'} = Ch^{'})$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\leq N^*(Ah^{'} - A^2h^{'}, h, \frac{t}{q^n}) \to 1 asn \to \infty.$$

Therefore $Ah^{'} = A^2h^{'}$.Thus $h^{'}$ is the common fixed point of A, B and C.Let us prove the uniqueness.Suppose $g^{'}$ be another common fixed point of A, B, C. From (6)

$$N^*(Ah^{'} - Ag^{'}, h, qt) \leq max\{N^*(Bh^{'} - Cg^{'}, h, t), \frac{N^*\left(Bh^{'} - Cg^{'}, h, t\right)}{N^*(Ah' - Cg', h, t)}, \frac{N^*\left(Bh^{'} - Cg^{'}, h, t\right)}{N^*(Ah' - Bg', h, t)}\}$$

$$\leq max\{N^*(h^{'} - g^{'}, h, t), \frac{N^*\left(h^{'} - g^{'}, h, t\right)}{N^*(h' - g', h, t)}, \frac{N^*\left(h^{'} - g^{'}, h, t\right)}{N^*(h' - g', h, t)}\}$$

Therefore
$N^*(Ah^{'} - Ag^{'}, h, qt) \leq N^*(h^{'} - g^{'}, h, t) \Longrightarrow N^*(h^{'} - g^{'}, h, qt) \leq N^*(h^{'} - g^{'}, h, t)$
Hence it follows that $h^{'} = g^{'}$. □

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