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Goal Programming Approach in Multi-objective Intuitionistic Fuzzy Linear Fractional Programming Problem

S. Rukmani[1], R. Sophia Porchelvi[2]

[1]*Department of Mathematics,*
STET Women's College,
Mannargudi, Tamilnadu, India.
rukuthigu@gmail.com
[2]*Department of Mathematics,*
A.D.M. College for Women (Autonomous),
Nagapattinam, Tamilnadu, India.
sophiaporchelvi@gmail.com

Abstract

In this Paper the solution through a novel procedure for solving the Intuitionistic Fuzzy Multi-objective Linear Fractional Program related problems through the goal programming approach has been proposed. A linearization procedure is presented to overcome the computational difficulties. In order to avoid decision deadlock, the Intuitionistic fuzzy goals are determined by finding the best and worst solution of each objective functioning. Solution procedure of solving Intuitionistic Fuzzy Multi-objective Linear Fractional Programming problems via min-sum goal programming is explained to solve a particular problem rather easily. An attempt has been made to provide a better numerical example and a financial application to demonstrate the efficiency of the proposed approach.

Key Words and Phrases: Multi-objective Linear Fractional Program, Intuitionistic Fuzzy Set, Goal Program, Membership/ non-membership function.

1 Introduction

The Goal programming approach to fuzzy programming problems introduced by Mohamed [4] was extended to fuzzy multi-objective linear fractional programming problems by B.B.Pal et al.[6]. In this paper it is extended to solve IFMOLFP problems. Here, the Intuitionistic fuzzy goals of the objectives are determined by finding best and worst solutions for each objective function. They are then characterized by G.S.Mahapatra [3] the associated membership and non-membership function. These functions are transformed into intuitionistic fuzzy membership and non-membership goals by introducing under and over deviational variables and assigning highest membership value unity and lowest non-membership zero as aspiration level to each of them.

The Paper is organized as follows: Preliminaries and basic definitions are given in Section 2. In this section the formulation of goal programming and intuitionistic fuzzy optimization are discussed and the procedure for linearization of the membership and non-membership function are also explained at the end of this section. The min-sum GP problem was formulated in Section 3. The solution procedure for solving MOLFPP is given in Section 4. A practical application is discussed in Section 5. Summary of the paper has been given at the end.

2 Preliminaries

2.1 Formulation of membership and non-membership function

Consider the Multi objective linear fractional programming problem
Optimize $Z_i(X) = \dfrac{c_i X + \alpha_i}{d_i X + \beta_i}, i = 1, 2, \ldots\ldots .k$

$$\text{Subject to } X \in S = \{X \in R^n | AX\,(\leq, = \text{ or } \geq)\, b, X \geq 0, b \in R^m\} \tag{1}$$

Where $c_i, d_i \in R^n; \alpha_i, \beta_i$ are constants and $S \neq \varnothing$

Sometimes, the decision maker may have the decision deadlock to fix the goal, in order to avoid such circumstances, individual best (maximum) and worst (minimum) values of each objective function under the given constraints should be found and take the solutions as $U_i^{acc} = \max\{Z_i(x)\}$ and $L_i^{acc} = \min\{Z_i(x)\}$ for membership function.

For maximization problem, one can take the upper and lower bound for the non-membership function as $U_i^{reg} = U_i^{acc} - \varepsilon_i$ where $\varepsilon_i = (U_i^{acc} - U_i^{reg})$ for $i = 1, \ldots, k$ based on the decision maker's choice and assume $L_i^{reg} = L_i^{acc}$.

For of minimization problem, one can take the upper and lower bound for the non-membership function as $U_i^{reg} = U_i^{acc}$ and $L_i^{reg} = L_i^{acc} + \varepsilon_i$ where $\varepsilon_i = (L_i^{reg} - L_i^{acc})$ for $i = 1, \ldots, k$ based on the decision makers choice.

Now, the membership and non-membership function of maximization problem can be defined as follows:

$$\mu_i(Z_i(x)) = \begin{cases} 0 & \text{if} \quad Z_i(x) \leq L_i^{acc} \\ \dfrac{Z_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} & \text{if} \quad L_i^{acc} \leq Z_i(x) \leq U_i^{acc} \\ 1 & \text{if} \quad Z_i(x) \geq U_i^{acc} \end{cases} \tag{2}$$

$$\nu_i(Z_i(x)) = \begin{cases} 1 & \text{if} \quad Z_i(x) \leq L_i^{reg} \\ \dfrac{U_i^{reg} - Z_i(x)}{U_i^{reg} - L_i^{reg}} & \text{if} \quad L_i^{reg} \leq Z_i(x) \leq U_i^{reg} \\ 0 & \text{if} \quad Z_i(x) \geq U_i^{reg} \end{cases} \tag{3}$$

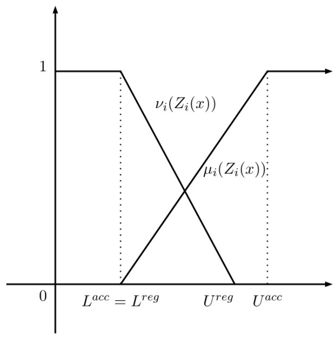

Figure 1: Membership and non-membership functions of maximization problem

The membership and non-membership function of minimization problem can be defined as follows :

$$\mu_i(Z_i(x)) = \begin{cases} 1 & \text{if} \quad Z_i(x) \leq L_i^{acc} \\ \dfrac{U_i^{acc} - Z_i(x)}{U_i^{acc} - L_i^{acc}} & \text{if} \quad L_i^{acc} \leq Z_i(x) \leq U_i^{acc} \\ 0 & \text{if} \quad Z_i(x) \geq U_i^{acc} \end{cases} \tag{4}$$

$$\nu_i(Z_i(x)) = \begin{cases} 0 & \text{if} \quad Z_i(x) \leq L_i^{reg} \\ \dfrac{Z_i(x) - L_i^{reg}}{U_i^{reg} - L_i^{reg}} & \text{if} \quad L_i^{reg} \leq Z_i(x) \leq U_i^{reg} \\ 1 & \text{if} \quad Z_i(x) \geq U_i^{reg} \end{cases} \tag{5}$$

2.2 Goal Programming formulation

To get the best achievement for maximization problem, the membership should be maximized and non-membership should be minimized and in intuitionistic fuzzy Programming approach, the highest degree of membership is 1 and the lowest degree of non-membership is 0.
Therefore, Achieve: $\{Z_i\,(x)\,/\,(\max\ \mu_i\,(Z_i\,(x)) \quad \& \ \min \nu_i(Z_i\,(x))\}\,,\ i = 1, 2, \ldots..k.$
(i.e)Achieve : $\left\{Z_i\,(x)\,/\,\left(\mu_i\,(Z_i\,(x)) + d_{i1}^- - d_{i1}^+ = 1 \quad \& \quad \nu_i\,(Z_i\,(x)) + d_{i2}^- - d_{i2}^+ = 0\right)\right\},$

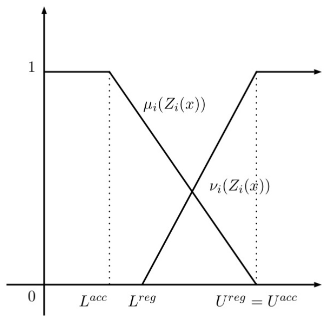

Figure 2: Membership and non-membership functions of minimization problem

$1 \leq i \leq j$

In which, d_{i1}^{-}, d_{i2}^{-} and d_{i1}^{+}, d_{i1}^{+} are the under attainment and over attainment, respectively of the ith goal. The deviational variables d_{i1}^{+} and d_{i2}^{-} can be removed from the goal programming because the over achievement of membership function and underachievement of non-membership function are acceptable.
Therefore, the membership and non-membership function can be defined as,

$$\text{Achieve } \left\{Z_i(x)/\mu_i\left(Z_i\left(x\right)\right) + d_{i1}^{-} \geq 1 \text{ and } \vartheta_i\left(Z_i\left(x\right)\right) - d_{i2}^{+} \leq 0\right\}, \quad 1 \leq i \leq j \quad (6)$$

The following linearization procedure is used to overcome the complexity in solving the multi-objective intuitionistic fuzzy linear programmig problem.

2.3 Linearization of membership and non-membership goals

Using (2) and (6), the membership function for maximization problem can be represented as,

$$\frac{Z_i(x) - L_i{}^{acc}}{U_i{}^{acc} - L_i{}^{acc}} + d_{i1}{}^{-} \geq 1$$

$$K_i Z_i(x) - K_i L_i{}^{acc} + d_{i1}{}^{-} \geq 1, \text{ where } K_i = \frac{1}{U_i{}^{acc} - L_i{}^{acc}}$$

Introducing the linear fractional expression of $Z_i\left(x\right)$, then

$$K_i(c_i X + \alpha_i) + d_{i1}{}^{-}(d_i X + \beta_i) \geq K_i{}^{1}(d_i X + \beta_i), \text{ where } K_i{}^{1} = 1 + K_i L_i{}^{acc}$$

(i.e) $C_i X + d_{i1}{}^{-}(d_i X + \beta_i) \geq G_i$, where $C_i = K_i c_i - K_i{}^{1} d_i, G_i = K_i{}^{1}\beta_i - K_i\alpha_i$

$$C_i X + D_{i1}{}^{-} \geq G_i, \text{ where } D_{i1}{}^{-} = d_{i1}{}^{-}(d_i X + \beta_i)_i, \quad (7)$$

Since, $d_{i1}{}^{-} \leq 1, \dfrac{D_{i1}{}^{-}}{d_i X + \beta_i} \leq 1,$

$$-d_i X + D_{i1}{}^{-} \leq \beta_i \quad (8)$$

Using (4) and (6), the non-membership function can be represented as,

$$\frac{U_i{}^{reg} - Z_i(x)}{U_i{}^{reg} - L_i{}^{reg}} - d_{i2}{}^{+} \leq 0$$

And proceed as above, we obtain

$$C_i{}^1X + D_{i2}{}^+ \geq G_i{}^1 \tag{9}$$

Where $C_i{}^1 = A_ic_i - A_i{}^1d_i, G_i{}^1 = A_i{}^1\beta_i - A_i\alpha_i$, and $D_{i2}{}^+ = d_{i2}{}^+(d_iX + \beta_i)$,
Since, $d_{i2}{}^+ \leq 1$,
Therefore, $\dfrac{D_{i2}{}^+}{d_iX + \beta_i} \leq 1$,

$$-d_iX + D_{i2}{}^+ \leq \beta_i \tag{10}$$

3 Problem formulation

Thus the min-sum goal programming model formulation for MIIFLP problem becomes,

$$\begin{aligned} \text{Min } Z &= \sum_i D_{i1}{}^- + D_{i2}{}^+ \\ C_iX + D_{i1}{}^- &\geq G_i \\ C_i{}^1X + D_{i2}{}^+ &\geq G_i{}^1 \\ -d_iX + D_{i1}{}^- &\leq \beta_i \\ -d_iX + D_{i2}{}^+ &\leq \beta_i \\ AX &\begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \\ X &\geq 0 \\ D_{i1}{}^-, D_{i2}{}^+ &\geq 0 \end{aligned} \tag{11}$$

To obtain the efficient solution, the following solution procedure paves a way to solve the Multi-objective intuitionistic fuzzy linear programming problem unambiguously.

4 Solution Procedure

Step: 1 Calculate the best $(\max\{Z_i(x)\})$ and worst $(\min\{Z_i(x)\})$ values for each objective function under the given constraints.

Step: 2 Set $U_i^{acc} = \max\{Z_i(x)\}$ and $L_i^{acc} = \min\{Z_i(x)\}$ for the upper and lower bound for membership function.

Step: 3 (i) If the objective function is to be maximized, set the upper and lower bound for non-membership function as $U_i^{reg} = U_i^{acc} - \varepsilon_i$ where $\varepsilon_i = (U_i^{acc} - U_i^{reg})$ for $i = 1, \ldots, k$ and $L_i^{reg} = L_i^{acc}$

(or)

(ii) If the objective function is to be minimized, set $U_i^{reg} = U_i^{acc}$ and $L_i^{reg} = L_i^{acc} + \varepsilon_i$ where $\varepsilon_i = (U_i^{acc} - U_i^{reg})$ for $i = 1, \ldots, k$.

In both cases, the values of ε_i are based on the decision maker's choice.

Step: 4 Elicit the membership $\mu_i(Z_i(X))$ and non-membership $\nu_i(Z_i(X))$ for each objective function.

Step: 5 Formulate the min-sum GP model.

Step: 6 Solve the formulated problem by using TORA software for different values of ε_i to obtain the best optimal solution.

The following financial optimization problem has been adopted from Tunjo Peric et al.,(2012) is used for better understanding of the above solution procedure. They have solved this problem using Taylor's formula and goal programming techniques. They transformed the non-linear programming model into linear programming models with the use of Taylor's formula and obtained the solution from various linear formulations. Here, the obtained solution through the above proposed method is identical with one of the model proposed by them.

5 Practical Application: Financial Planning

Consider a firm which is expected to achieve a target of US$ 60.0 million of capital in the forthcoming year. In order to increase the value of the firm, the financial condition is to be reconstructed. Some financial ratios is designed in order to satisfy the manager of the firm in anticipation of the sales in the next year. The Table 1 in Tunjo Peric et al.,(2012) shows the variables which are considered. There are four conflicting fractional goals which include: (1) minimization of the current ratio, (2) minimization of the debt ratio, (3) maximization of the turnover ratio and (4) maximization of the profitability ratio.

Mathematical formulation

$$\begin{aligned}
\min z_1(x) &= \frac{x_{11}}{x_{21}} && \{\text{Current ratio}\} \\
\min z_2(x) &= \frac{x_{21}+x_{22}}{x_{23}+x_{24}} && \{\text{Debt ratio}\} \\
\min z_3(x) &= \frac{60}{x_{11}+x_{12}} && \{\text{Turnover ratio}\} \\
\min z_4(x) &= \frac{x_{24}}{60} && \{\text{Profitability ratio}\}
\end{aligned}$$

Subject to

$$\begin{aligned}
&x_{11}+x_{12} = x_{21}+x_{22}+x_{23}+x_{24}, \\
&150 \le x_{11} \le 250, \quad x_{12} \le 300, \quad x_{11}+x_{12} \ge 350, \\
&75 \le x_{21} \le 300, \quad 100 \le x_{22} \le 300, \quad x_{21}+x_{22} \ge 250, \\
&75 \le x_{23} \le 125, \quad 100 \le x_{24} \le 140, \quad x_{11}, x_{12}, x_{21}, x_{22}, x_{23}, x_{24} \ge 0.
\end{aligned}$$

Model Solving

The lower and upper acceptance are obtained by minimizing and maximizing each of the four objective functions individually on a given set of constraints

$$\begin{aligned}
&U_1^{acc} = 3.3333,\ L_1^{acc} = 0.8571\ ;\quad U_2^{acc} = 1.5714,\ L_2^{acc} = 0.9434; \\
&U_3^{acc} = 0.1412,\ L_3^{acc} = 0.1091\ ;\quad U_4^{acc} = 2.3333,\ L_2^{acc} = 1.6667;
\end{aligned}$$

The membership and non-membership can be defined as in Section 2.1 and problem has been formulated as

$$\min(D_{11}^- + D_{21}^- + D_{31}^- + D_{41}^- + D_{12}^+ + D_{22}^+ + D_{32}^+ + D_{42}^+)$$

$$0.8571x_{21} - x_{11} + D_{11}^- \geq 0; \quad D_{11}^- - 2.4762_21 \leq 0;$$
$$x_{11} - 0.8771x_{21} - D_{12}^+ \leq 0; \quad D_{12}^+ - 2.4562x_{21} \leq 0;$$
$$0.9434x_{23} + 0.9434x_{24} - x_{21} - x_{22} + D_{21}^- \geq 0; \quad D_{21}^- - 0.628x_{23} - 0.628x_{24} \leq 0;$$
$$x_{21} + x_{22} - 0.9734x_{23} - 0.9734x_{24} - D_{22}^+ \leq 0; \quad D_{22}^+ - 0.598x_{23} - 0.598x_{24} \leq 0;$$
$$0.1412x_{11} + 0.1412x_{12} - D_{31}^- \leq 60; \quad D_{31}^- - 0.0321x_{11} - 0.0321x_{12} \leq 0;$$
$$0.138x_{11} + 0.138x_{12} - D_{32}^+ \leq 60; \quad D_{32}^+ - 0.029x_{11} - 0.029x_{12} \leq 0;$$
$$x_{24} + D_{41}^- \geq 140; \quad D_{41}^- \leq 39.996; \quad x_{24} + D_{42}^+ \geq 127.998; \quad D_{42}^+ \leq 27.996;$$
$$x_{11} + x_{12} = x_{21} + x_{22} + x_{23} + x_{24}; \quad 150 \leq x_{11} \leq 250; \quad x_{12} \leq 300;$$
$$x_{11} + x_{12} \geq 350; \quad 75 \leq x_{21} \leq 300; \quad 100 \leq x_{22} \leq 300; \quad x_{21} + x_{22} \geq 250;$$
$$75 \leq x_{23} \leq 125; \quad 100 \leq x_{24} \leq 140; \quad x_{11}, x_{12}, x_{21}, x_{22}, x_{23}, x_{24} \geq 0.$$

The formulated problem has been solved as per the solution procedure given in Section 4 for various values of ε_i by using TORA software and the best optimal solution is,
$x_{11} = 165, x_{12} = 300, x_{21} = 150, x_{22} = 100, x23 = 75, x24 = 140$,
$d_1^- = 36.435,\ d_1^+ = 0,\ d_2^- = 47.169,\ d_2^+ = 0,\ d_3^- = 5.658,\ d_3^+ = 4.2630, d_4^- = 0,\ d_4^+ = 0$
$\mu_1 = 90\%,\ \nu_1 = 0\%,\ \mu_2 = 65\%,\ \nu_2 = 0\%,\ \mu_3 = 62\%,\ \nu_3 = 32\%,\ \mu_4 = 100\%,\ \nu_4 = 0\%$

Here, the profitability ratio is fully achieved and the current ratio is 90% achieved. The other ratios are achieved by more than 60%. This optimal solution is identical with the solution given in Tunjo Peric (2012).

Assets	Variable	Expected Values	Liabilities and equality	Variable	Expected Values
Current assets	X_{11}	$150 \leq x_{11} \leq 250$	Current liabilities	X_{21}	$75 \leq x_{21} \leq 300$
Fixed assets	X_{12}	$X_{12} \leq 300$	Long-term liabilities	X_{22}	$X_{21} + x_{22} \geq 250$ $100 \leq x_{22} \leq 300$
Total assets	$X_{11} + x_{12}$	$X_{11} + x_{12} \geq 350$	Shareholders equity	X_{23}	$75 \leq x_{23} \leq 125$
			Retained earning added	X_{24}	$100 \leq x_{24} \leq 140$
			Total liabilities and equity	$X_{21} + X_{22}$ $+X_{23} + X_{24}$	

Table 1: Definition of variables in the balance sheet (B/S)

6 Conclusion

In this paper, Multi-objective intuitionistic fuzzy linear fractional programming problem has been solved through Goal programming approach. To overcome the decision deadlock of the decision maker, the aspiration levels of the objective functions are not imprecise, but they are calculated by finding the best and worst values for each objective function and are used for acceptance level of membership functions. The rejection tolerance is calculated from the acceptance tolerance for different values of ε_i based on the decision maker choice to get the satisfactory solution. The values of ε_i can be changed by the decision maker to get the best solution. We hope this paper will be very useful for finding the better solution for linear fractional programming problem.

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