International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 527-539
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Neutrosophic Fuzzy Ideals of Near-Rings

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Abstract

In this paper, the notion of Neutrosophic fuzzy ideals of near-rings are introduced and discussed some algebraic properties like union ,intersection , homomorphic image and preimage of neutrosophic fuzzy ideals of near-rings. Further we discuss about the direct product of Neutrosophic fuzzy ideals of near-rings .

AMS Subject Classification: 03E72,13A15,16Y30

Key Words and Phrases: Neutrosophic fuzzy set, Neutrosophic fuzzy ideal of near-rings

1 Introduction

The concept of fuzzy set was introduced by Zadeh [12] in 1965.The notion fuzzy sub near-rings and fuzzy ideals of near-rings was introduced by Abou_zaid[1].This concept is further discussed by Kim [6] and Dutta[5]. V.Chinnadurai and S.Kadalarasi[4] discussed the direct product of n(n=1,2,....,k) fuzzy subnearring, fuzzy ideal and fuzzy R-subgroups. The concept of neutrosophy was introduced by Florentin Smarandache[10]as a new branch of philosophy. Neutrosophy is a base of Neutrosophic logic which is an extension of fuzzy logic in which indeterminancy is included.In Neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F.The theory of neutrosophic set have achieved great success in various fields like medical diagnosis, image processing, decision making problem ,robotics and so on. I.Arockiarani[3] consider the neutrosophic set with value from the subset of [0,1] and etxtended the research in fuzzy neutrosophic set.J.Martina Jency and I.Arockia rani [8] initiate the concept of subgroupoids in fuzzy neutrosophic set. In this paper we introduce the notion of neutrosophic fuzzy sub near-ring, neutrosophic fuzzy ideals of near-rings and discussed some algebraic properties. Also we discussed about the direct product of neutrosophic neutrosophic fuzzy ideals of near-rings.

2 Preliminaries

Definition 1. 1. $(N, +)$ is a group

2. $(N, .)$ is a semigroup

3. $(x + y).z = x.z + y.z$ for $x, y, z \in N$

Definition 2. An ideal of a near-ring N is a subgroup I of N such that

1. $(I, +)$ is a normal subgroup of $(N, +)$
2. $IN \subseteq I$
3. $y(x + i) - -yx \in$ I, for $x, y \in N$ and $i \in I$.

I is a left ideal of N if I satisfies (i) and (ii) and right ideal of N if I satisfies (i) and (iii).

Definition 3. A fuzzy set μ in a near-ring N is said to be fuzzy sub near-ring of N if

1. $\mu(x - y) \geq \min(\mu(x), \mu(y)), forallx, y \in N$
2. $\mu(xy) \geq \min(\mu(x), \mu(y))$, for all $x, y \in N$

Definition 4. A fuzzy set μ in a near-ring N is said to be fuzzy ideal of N if

(i) $\mu(x - y) \geq min(\mu(x), \mu(y))$, for all $x, y \in N$

(ii) $\mu(y + x - y) \geq \mu(x)$, for all $x, y \in N$

(iii) $\mu(xy) \geq \mu(y)$, for all $x, y \in$ N

(iv) $\mu((x + z)y - xy \geq \mu(z)$, for all $x, y \in N$

A fuzzy set with (i)-(iii) is called fuzzy left ideal of N where as fuzzy set with (i),(ii) and (iv) is called a fuzzy right ideal of N.

Definition 5. A Neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, a indeterminacy function I_A(x) and a falsity membership function $F_A(x)$ is defined as

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

where T_A , I_A,$F_A : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition 6. Let A and B be neutrosophic fuzzy sets of X. Then ,

1. $A \cup B = \{< x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x) >\}$, where

   $T_{A\cup B}(x) = max(T_A(x), T_B(x), I_{A\cup B}(x) = min(I_A(x), I_B(x), F_{A\cup B}(x)) = min(F_A(x), F_B(x))$

   ,for all $x \in X$.

2. $A \cap B = (T_{A\cap B}(x), I_{A\cap B}(x)), F_{A\cap B}(x) >\}$, where

   $T_{A\cap B}(x) = min(T_A(x), T_B(x)), I_{A\cap B}(x) = max(I_A(x), I_B(x)), F_{A\cap B}(x) = max(F_A(x), F_B(x))$

Definition 7. A Neutrosophic fuzzy set A in a near-ring N is called neutrosophic fuzzy sub near-ring of N if

1. $T_A(x-y) \geq \min(T_A(x), T_A(y)), I_A(x-y) \leq \max(I_A(x), I_A(y)), F_A(x-y) \leq \max(F_A(x), F_A(y))$
2. $T_A(xy) \geq \min(T_A(x), T_A(y)), I_A(xy) \leq \max(I_A(x), I_A(y)), F_A(xy) \leq \max(F_A(x), F_A(y))$

3 Neutrosophic fuzzy ideals of near-rings

Definition 8. Let N be a near-ring. A Neutrosophic fuzzy set A in a near-ring N is called neutrosophic fuzzy ideal of N if

(i) $T_A(x-y) \geq min(T_A(x), T_A(y)), I_A(x-y) \leq max(I_A(x), I_A(y)), F_A(x-y) \leq max(F_A(x), F_A(y))$

(ii) $T_A(y+x-y) \geq T_A(x), I_A(y+x-y) \leq I_A(x), F_A(y+x-y) \leq F_A(x)$

(iii) $T_A(xy) \geq T_A(y), I_A(xy) \leq I_A(y), F_A(xy) \leq F_A(y)$

(iv) $T_A((x+z)y-xy) \geq T_A(z), I_A((x+z)y-xy) \leq I_A(z), F_A((x+z)y-xy) \leq F_A(z)$.

A Neutrosophic fuzzy subset with the above condition (i)-(iii) is called a neutrosophic fuzzy left ideal of N where as with (i),(ii) and (iv) is called neutrosophic fuzzy right ideal of N.

Theorem 9. *Let A and B be neutrosophic fuzzy ideal of N. If $A \subset B$, then $A \cup B$ is a neutrosophic fuzzy ideal of N.*

Proof. Let A and B are neutrosophic fuzzy ideals of N. Let x,y,z∈ N.

(i)

$$\begin{aligned}
T_{A\cup B}(x-y) &= max(T_A(x-y), T_B(x-y)) \\
&\geq max(min(T_A(x), T_A(y)), min(T_B(x), T_B(y)) \\
&= min\{max(T_A(x), T_B(x)), max(T_A(y), T_B(y))\} \\
&= min(T_{A\cup B}(x), T_{A\cup B}(y)) \\
I_{A\cup B}(x-y) &= min(I_A(x-y), I_B(x-y)) \\
\leq min(max(I_A(x), I_A(y)), max(I_B(x), I_B(y)) & \\
&= max\{min(I_A(x), I_B(x)), max(I_A(y), I_B(y))\} \\
&= max(I_{A\cup B}(x), I_{A\cup B}(y)) \\
F_{A\cup B}(x-y) &= min(F_A(x-y), F_B(x-y)) \\
\leq min(max(F_A(x), F_A(y)), max(F_B(x), F_B(y)) & \\
&= max\{min(F_A(x), F_B(x)), max(F_A(y), F_B(y))\} \\
&= max(F_{A\cup B}(x), F_{A\cup B}(y))
\end{aligned}$$

(ii)

$$\begin{aligned}
T_{A\cup B}(y+x-y) &= max(T_A(y+x-y), T_B(y+x-y)) \\
&\geq max(T_A(x), T_B(x)) \\
&= T_{A\cup B}(x) \\
I_{A\cup B}(y+x-y) &= min(I_A(y+x-y), I_B(y+x-y)) \\
&\leq min(I_A(x), I_B(x)) \\
&= I_{A\cup B}(x) \\
F_{A\cup B}(y+x-y) &= min(F_A(y+x-y), F_B(y+x-y)) \\
&\leq min(F_A(x), F_B(x)) \\
&= F_{A\cup B}(x)
\end{aligned}$$

(iii)

$$\begin{aligned}T_{A\cup B}(xy) &= max(T_A(xy), T_B(xy)) \geq max(T_A(y), T_B(y)) = T_{A\cup B}(y)\\ I_{A\cup B}(xy) &= min(I_A(xy), I_B(xy)) \leq min(I_A(y), I_B(y)) = I_{A\cup B}(y)\\ F_{A\cup B}(xy) &= min(F_A(xy), F_B(xy)) \leq min(F_A(y), F_B(y)) = F_{A\cup B}(y)\end{aligned}$$

(iv)

$$\begin{aligned}T_{A\cup B}((x+z)y-xy) &= max\{T_A((x+z)y-xy), T_B((x+z)y-xy)\}\\ &\geq max(T_A(z), T_B(z)) = T_{A\cup B}(z)\\ I_{A\cup B}((x+z)y-xy) &= min\{I_A((x+z)y-xy), I_B((x+z)y-xy)\}\\ &\leq min(I_A(z), I_B(z)) = I_{A\cup B}\\ T_{A\cup B}((x+z)y-xy) &= min\{F_A((x+z)y-xy), F_B((x+z)y-xy))\}\\ &\leq min(F_A(z), F_B(z)) = F_{A\cup B}(z).\end{aligned}$$

Therefore, $A \cup B$is a neutrosophic fuzzy ideal of N. □

Theorem 10. *Let A and B be neutrosophic fuzzy ideal of N. Then $A \cap B$ is a neutrosophic fuzzy ideal of N.*

Proof. Let A and B are neutrosophic fuzzy ideals of N. Let $x, y, z \in N$

(i)

$$\begin{aligned}T_{A\cap B}(x-y) &= min(T_A(x-y), T_B(x-y))\\ &\geq min(min(T_A(x), T_A(y)), min(T_B(x), T_B(y))\\ &= min\{min(T_A(x), T_B(x)), min(T_A(y), T_B(y))\}\\ &= min(T_{A\cap B}(x), T_{A\cap B}(y))\\ I_{A\cap B}(x-y) &= max(I_A(x-y), I_B(x-y))\\ &\leq max(max(I_A(x), I_A(y)), max(I_B(x), I_B(y))\\ &= max\{max(I_A(x), I_B(x)), max(I_A(y), I_B(y))\}\\ &= max(I_{A\cap B}(x), I_{A\cap B}(y))\\ F_{A\cap B}(x-y) &= max(F_A(x-y), F_B(x-y))\\ &\leq max(max(F_A(x), F_A(y)), max(F_B(x), F_B(y))\\ &= max\{max(F_A(x), F_B(x)), max(F_A(y), F_B(y))\}\\ &= max(F_{A\cap B}(x), F_{A\cap B}(y))\end{aligned}$$

(ii)

$$\begin{aligned}T_{A\cap B}(y+x-y) &= min(T_A(y+x-y), T_B(y+x-y))\\ &\geq min(T_A(x), T_B(x))\\ &= T_{A\cap B}(x)\\ I_{A\cap B}(y+x-y) &= max(I_A(y+x-y), I_B(y+x-y))\\ &\leq max(I_A(x), I_B(x))\\ &= I_{A\cap B}(x)\\ F_{A\cap B}(y+x-y) &= max(F_A(y+x-y), F_B(y+x-y))\\ &\leq max(F_A(x), F_B(x))\\ &= F_{A\cap B}(x)\end{aligned}$$

(iii)

$$T_{A\cap B}(xy) = min(T_A(xy), T_B(xy)) \geq min(T_A(y), T_B(y)) = F_{A\cap B}(y)$$
$$I_{A\cap B}(xy) = max(I_A(xy), I_B(xy)) \leq max(I_A(y), I_B(y)) = I_{A\cap B}(y)$$
$$F_{A\cap B}(xy) = max(F_A(xy), F_B(xy)) \leq max(F_A(y), F_B(y)) = F_{A\cap B}(y)$$

(iv)

$$\begin{aligned} T_{A\cap B}((x+z)y - xy) &= min\{T_A((x+z)y - xy), T_B((x+z)y - xy))\} \\ &\geq min(T_A(z), T_B(z)) = T_{A\cap B}(z) \\ I_{A\cap B}((x+z)y - xy) &= max\{I_A((x+z)y - xy), I_B((x+z)y - xy))\} \\ &\leq max(I_A(z), I_B(z)) = I_{A\cap B}(z) \\ F_{A\cap B}((x+z)y - xy) &= max\{F_A((x+z)y - xy), F_B((x+z)y - xy))\} \\ &\leq max(F_A(z), F_B(z)) = F_{A\cap B}(z) \end{aligned}$$

Therefore, $A \cap B$ is a neutrosophic fuzzy ideal of N.

□

Corollary 11. *If $A_1, A_2, ..., A_n$ are neutrosophic fuzzy ideals of N, then $A = \bigcap_{i=1}^{n} A_i$ is a neutrosophic fuzzy ideal of N.*

Lemma 12. *For all $a, b \in I$ and i is any positive integer, if $a = b$, then*

(i) $a^i \leq b^i$

(ii) $[min(a, b)]^i = min(a^i, b^i)$

(iii) $[max(a, b)]^i = max(a^i, b^i)$

Theorem 13. *Let A be a Neutrosophic fuzzy ideal of N. Then $A^m = \{< x, T_{A^m}(x), I_{A^m}(x), F_{A^m}(x) >: x \in N\}$ is a Neutrosophic fuzzy ideal of N^m, where m is a positive integer and*

$$T_{A^m}(x) = (T_A(x))^m, I_{A^m}(x) = (I_A(x))^m F_{A^m}(x) = (F_A(x))^m.$$

Proof. Let A be a Neutrosophic fuzzy ideal of N. Let $x, y, z \in N$.

(i)

$$\begin{aligned} T_{A^m}(x-y) &= (T_A(x-y))^m \\ &\geq [min(T_A(x), T_A(y))]^m = min((T_A(x))^m, (T_A(y))^m) \\ &= min(T_{A^m}(x), T_{A^m}(y)) \\ I_{A^m}(x-y) &= (I_A(x-y))^m \\ &\leq [max(I_A(x), I_A(y))]^m = max((I_A(x))^m, (I_A(y))^m) \\ &= max(I_{A^m}(x), I_{A^m}(y)) \\ F_{A^m}(x-y) &= (F_A(x-y))^m \\ &\leq [max(F_A(x), F_A(y))]^m = max((F_A(x))^m, (F_A(y))^m) \\ &= max(F_{A^m}(x), F_{A^m}(y)) \end{aligned}$$

(ii)

$$T_{A^m}(y+x-y) = (T_A(y+x-y))^m \geq (T_A(x))^m = T_{A^m}(x)$$
$$I_{A^m}(y+x-y) = (I_A(y+x-y))^m \leq (I_A(x))^m = I_{A^m}(x)$$
$$F_{A^m}(y+x-y) = (F_A(y+x-y))^m \leq (F_A(x))^m = F_{A^m}(x)$$

(iii)

$$T_A^m(xy) = (T_A(xy))^m \geq (T_A(y))^m = T_A^m(y),$$
$$I_A^m(xy) = (I_A(xy))^m \leq (I_A(y))^m = I_A^m(y),$$
$$F_A^m(xy) = (F_A(xy))^m \leq (F_A(y))^m = F_A^m(y).$$

(iv)

$$T_{A^m}((x+z)y-xy) = (T_A((x+z)y-xy))^m \geq (T_A(z))^m = T_{A^m}(z)$$
$$I_{A^m}((x+z)y-xy) = (I_A((x+z)y-xy))^m \leq (I_A(z))^m = I_{A^m}(z)$$
$$F_{A^m}((x+z)y-xy) = (F_A((x+z)y-xy))^m \leq (F_A(z))^m = F_{A^m}(z)$$

Therefore, A^m is a Neutrosophic fuzzy ideal of N^m . □

4 Direct product of Neutorsophic fuzzy ideals of near-rings

Definition 14. Let A and B be neutrosophic fuzzy subsets of near-rings $N_1$and N_2 respectively. Then, the direct product of neutrosophic fuzzy subsets of near-rings is defined by
$A \times B : N_1 \times N_2 \to [0,1]$ such that $A \times B = \{< (x,y), T_{A\times B}(x,y), I_{A\times B}(x,y), F_{A\times B}(x,y) >: x \in N_1, y \in N_2\}$
where,$T_{A\times B}(x,y) = min(T_A(x), T_B(y)), I_{A\times B}(x,y) = max(I_A(x), I_B(y))$
and $F_{A\times B}(x,y) = max(F_A(x), F_B(y))$

Definition 15. Let A and B be neutrosophic fuzzy subsets of near-rings $N_1$and N_2 respectively. Then $A \times B$ is a neutrosophic fuzzy ideal of $N_1 \times N_2$ if it satisfies the following conditions:

(i)

$$T_{A\times B}((x_1,x_2)-(y_1,y_2)) \geq min(T_{A\times B}(x_1,x_2), T_{A\times B}(y_1,y_2)),$$
$$I_{A\times B}((x_1,x_2)-(y_1,y_2)) \leq max(I_{A\times B}(x_1,x_2), I_{A\times B}(y_1,y_2)),$$
$$F_{A\times B}((x_1,x_2)-(y_1,y_2)) \leq max(F_{A\times B}(x_1,x_2), F_{A\times B}(y_1,y_2))$$

(ii)

$$T_{A\times B}((y_1,y_2)+(x_1,x_2)-(y_1,y_2)) \geq min(T_{A\times B}(x_1,x_2)),$$
$$I_{A\times B}((y_1,y_2)+(x_1,x_2)-(y_1,y_2)) \leq max(I_{A\times B}(x_1,x_2)),$$
$$F_{A\times B}((y_1,y_2)+(x_1,x_2)-(y_1,y_2)) \leq max(F_{A\times B}(x_1,x_2))$$

(iii)

$$T_{A\times B}((x_1,x_2)(y_1,y_2)) \geq T_{A\times B}(y_1,y_2),$$
$$I_{A\times B}((x_1,x_2)(y_1,y_2)) \leq I_{A\times B}(y_1,y_2),$$
$$F_{A\times B}((x_1,x_2)(y_1,y_2)) \leq F_{A\times B}(y_1,y_2)$$

(iv)

$$T_{A\times B}([(x_1,x_2)+(z_1,z_2)](y_1,y_2)-(x_1,x_2)(y_1,y_2)) \geq T_{A\times B}(z_1,z_2),$$
$$I_{A\times B}([(x_1,x_2)+(z_1,z_2)](y_1,y_2)-(x_1,x_2)(y_1,y_2)) \leq I_{A\times B}(z_1,z_2)$$
$$F_{A\times B}([(x_1,x_2)+(z_1,z_2)](y_1,y_2)-(x_1,x_2)(y_1,y_2)) \leq T_{A\times B}(z_1,z_2)$$

Theorem 16. *Let A and B be neutrosophic fuzzy ideals of N_1,N_2 respectively. Then $A\times B$ is a neutrosophic fuzzy ideal of $N_1\times N_2$.*

Proof. Let A and B be neutrosophic fuzzy ideals of N_1, N_2 respectively.
Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)\in N_1\times N_2$

(i)

$$\begin{aligned}
T_{A\times B}((x_1,x_2)-(y_1,y_2)) &= T_{A\times B}(x_1-y_1,x_2-y_2)\\
&= min(T_A(x_1-y_1),T_B(x_2-y_2)\\
&\geq min\{min[T_A(x_1),T_A(y_1))],min[T_B(x_2),T_B(y_2))]\\
&= min\{min[T_A(x_1),T_B(x_2)],min[T_A(y_1),T_B(y_2)]\\
&= min(T_{A\times B}(x_1,x_2),T_{A\times B}(y_1,y_2))\\
I_{A\times B}((x_1,x_2)-(y_1,y_2)) &= I_{A\times B}(x_1-y_1,x_2-y_2)\\
&= max(I_A(x_1-y_1),I_B(x_2-y_2)\\
&\leq max\{max[I_A(x_1),I_A(y_1))],max[I_B(x_2),I_B(y_2))]\\
&= max\{max[I_A(x_1),I_B(x_2)],max[I_A(y_1),I_B(y_2)]\\
&= max(I_{A\times B}(x_1,x_2),I_{A\times B}(y_1,y_2))\\
F_{A\times B}((x_1,x_2)-(y_1,y_2)) &= F_{A\times B}(x_1-y_1,x_2-y_2)\\
&= max(F_A(x_1-y_1),F_B(x_2-y_2)\\
&\leq max\{max[F_A(x_1),F_A(y_1))],max[F_B(x_2),F_B(y_2))]\\
&= max\{max[F_A(x_1),F_B(x_2)],max[F_A(y_1),F_B(y_2)]\\
&= max(F_{A\times B}(x_1,x_2),F_{A\times B}(y_1,y_2))
\end{aligned}$$

(ii)

$$\begin{aligned}
T_{A\times B}((y_1,y_2)+((x_1,x_2))-(y_1,y_2)) &= T_{A\times B}B((y_1+x_1-y_1),(y_2+x_2-y_2))\\
&= min[T_A(y_1+x_1-y_1),T_B(y_2+x_2-y_2)]\\
&\geq min(T_A(x_1),T_B(x_2))\\
&= T_{A\times B}(x_1,x_2)I_{A\times B}((y_1,y_2)+((x_1,x_2))-(y_1,y_2))\\
&= I_{A\times B}((y_1+x_1-y_1),(y_2+x_2-y_2))\\
&= max[I_A(y_1+x_1-y_1),I_B(y_2+x_2-y_2)]\\
&\leq max(I_A(x_1),I_B(x_2))\\
&= I_{A\times B}(x_1,x_2)F_{A\times B}((y_1,y_2)+((x_1,x_2))-(y_1,y_2))\\
&= F_{A\times B}((y_1+x_1-y_1),(y_2+x_2-y_2))\\
&= max[F_A(y_1+x_1-y_1),F_B(y_2+x_2-y_2)]\\
&\leq max(F_A(x_1),F_B(x_2))\\
&= F_{A\times B}(x_1,x_2)
\end{aligned}$$

(iii)

$$\begin{aligned}
T_{A\times B}(x_1,x_2)(y_1,y_2) &= T_{A\times B}(x_1y_1,x_2y_2)\\
&= min(T_A(x_1y_1),T_B(x_2y_2))\\
&\geq min(T_A(y_1),T_B(y_2))\\
&= T_{A\times B}((y_1,y_2)\\
I_{A\times B}(x_1,x_2)(y_1,y_2) &\\
&= I_{A\times B}(x_1y_1,x_2y_2)\\
&= max(I_A(x_1y_1),I_B(x_2y_2))\\
&\leq max(I_A(y_1),I_B(y_2))\\
&= I_{A\times B}(y_1,y_2)\\
F_{A\times B}(x_1,x_2)(y_1,y_2) &\\
&= F_{A\times B}(x_1y_1,x_2y_2)\\
&= max(F_A(x_1y_1),F_B(x_2y_2))\\
&\leq max(FI_A(y_1),F_B(y_2))\\
&= F_{A\times B}(y_1,y_2)
\end{aligned}$$

(iv)

$$\begin{aligned}
T_{A\times B}([(x_1,x_2)+(z_1,z_2)-(y_1,y_2)-(x_1,x_2)(y_1,y_2) &= T_{A\times B}([x_1+z_1]y_1-x_1y_1,\\
&[x_2+z_2]y_2-x_2y_2)\\
&= min(T_A[x_1+z_1]y_1-x_1y_1),\\
&T_B[x_2+z_2]y_2-x_2y_2)\\
&\geq min(T_A(z_1),T_B(z_2)\\
&= T_{A\times B}(z_1,z_2)\\
I_{A\times B}([(x_1,x_2)+(z_1,z_2)-(y_1,y_2)-(x_1,x_2)(y_1,y_2) &= I_{A\times B}([x_1+z_1]y_1-x_1y_1,\\
&[x_2+z_2]y_2-x_2y_2)\\
&= max(I_A(x_1+z_1]y_1-x_1y_1),\\
&I_B[x_2+z_2]y_2-x_2y_2)\\
&\leq max(I_A(z_1),I_B(z_2)\\
&= I_{A\times B}(z_1,z_2)\\
F_{A\times B}([(x_1,x_2)+(z_1,z_2)-(y_1,y_2)-(x_1,x_2)(y_1,y_2) &= F_{A\times B}([x_1+z_1]y_1-x_1y_1,\\
&[x_2+z_2]y_2-x_2y_2)\\
&= max(F_A(x_1+z_1]y_1-x_1y_1),\\
&F_B[x_2+z_2]y_2-x_2y_2)\\
&\leq max(F_A(z_1),F_B(z_2)\\
&= F_{A\times B}(z_1,z_2)
\end{aligned}$$

Therefore $A \times B$ is a neutrosophic fuzzy ideal of $N_1 \times N_2$ □

5 Homomorphism of Neutorsophic fuzzy ideals of near-rings

Definition 17. Let R and S be two near- rings. Then the mapping $f : R \to S$ is called near-ring homomorphism if for all $x, y \in R$, the following holds

(i) $f(x+y) = f(x) + f(y)$
(ii) $f(xy) = f(x)f(y)$.

Definition 18. Let X and Y be two non-empty sets and $f : X \to Y$ be a function.

(i) If B is a Neutrosophic fuzzy set in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the Neutrosophic fuzzy set in X defined by

$$f^{-1}(B) = \{< (x), f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x)) >: x \in X\}$$

where $f^{-1}(T_B(x)) = T_B(f(x))$ and so on.

(ii) If Ais a Neutrosophic fuzzy set in X, then the image of A under f,denoted by $f(A)$, is the Neutrosophic fuzzy set in Y defined by
$f(A) = \{(y), f(T)_A(y), f(I_A(y)), f(F)_A(y) \; : y \in Y\}$, where

$$f(T_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} T_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

$$f(I_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} I_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

$$f(F_A(y)) = \begin{cases} \inf_{x \in f^{-1}(y)} F_A(x) & if\ f^{-1}(y) \neq 0_N \\ 1 & otherwise \end{cases}$$

where $f(F_A(y)) = (1 - f(1 - F_A))(y)$

Theorem 19. *Let N and N' be any two near-rings and f be a homomorphism of N on to N'. If A' is a Neutrosophic fuzzy ideal of N', then $f^{-1}(A)$ is a Neutrosophic fuzzy ideal of N.*

Proof. Let x ,y,z$\in$ N.

(i)

$$\begin{aligned} f^{-1}(T_A)(x-y) &= T_A(f(x-y)) \\ &= T_A(f(x) - f(y)) \\ &\geq min(T_A(f(x), T_A(f(y)))) \\ &= min(f^{-1}(T_A)(x), f^{-1}(T_A)(y)) \\ f^{-1}(I_A)(x-y) &= I_A(f(x-y)) \\ &= I_A(f(x) - f(y)) \\ &\leq max(I_A(f(x), I_A(f(y)))) \\ &= max(f^{-1}(I_A)(x), f^{-1}(I_A)(y), f^{-1}(F_A)(x-y)) \\ &= F_A(f(x-y)) \\ &= F_A(f(x) - f(y)) \\ &\leq max(F_A(f(x), F_A(f(y)))) \\ &= max(f^{-1}(F_A)(x), f^{-1}(F_A)(y)) \end{aligned}$$

(ii)

$$\begin{aligned}
f^{-1}(T_A)(y+x-y)) &= T_A(f(y+x-y)) \\
&= T_A(f(y)+f(x)-f(y)) \\
&\geq min(T_A(f(x))) \\
&= min(f^{-1}(T_A)(x)f^{-1}(I_A)(y+x-y)) \\
&= I_A(f(y+x-y)) \\
&= I_A(f(y)+f(x)-f(y)) \\
&\leq max(I_A(f(x))) \\
&= max(f^{-1}(I_A)(x), f^{-1}(F_A)(y+x-y)) \\
&= F_A(f(y+x-y)) \\
&= F_A(f(y)+f(x)-f(y)) \\
&\leq max(F_A(f(x))) \\
&= max(f^{-1}(F_A)(x))
\end{aligned}$$

(iii)

$$\begin{aligned}
f^{-1}(T_A)(xy) &= T_A(f(xy)) \\
&= T_A(f(x)f(y)) \\
&\geq min(T_A(f(y))) \\
&= min(f^{-1}(T_A)(y)) \\
f^{-1}(I_A)(xy) &= I_A(f(xy)) \\
&= I_A(f(x)f(y)) \\
&\leq max(I_A(f(y))) \\
&= max(f^{-1}(I_A)(y)) \\
f^{-1}(F_A)(xy) &= F_A(f(xy)) \\
&= F_A(f(x)f(y)) \\
&\leq max(F_A(f(y))) \\
&= max(f^{-1}(F_A)(y))
\end{aligned}$$

(iv)

$$\begin{aligned}
f^{-1}(T_A)((x+z)y-xy) &= T_A(f[(x+z)y-xy]) \\
&= T_A([f(x)+f(z)]f(y)-f(x)f(y)) \\
&\geq min(T_A(f(z))) \\
&= min(f^{-1}(T_A)(z)f^{-1}(I_A)((x+z)y-xy)) \\
&= I_A(f[(x+z)y-xy]) \\
&= I_A([f(x)+f(z)]f(y)-f(x)f(y)) \\
&\leq max(I_A(f(z)) \\
&= max f^{-1}(I_A)(z), f^{-1}(F_A)((x+z)y-xy)) \\
&= F_A(f[(x+z)y-xy]) \\
&= F_A([f(x)+f(z)]f(y)-f(x)f(y)) \\
&\leq max(F_A(f(z))) \\
&= max f^{-1}(F_A)(z).
\end{aligned}$$

Therefore, $f^{-1}(A)$ is a Neutrosophic fuzzy ideal of N. □

Theorem 20. *Let N_1 and N_2 be any two near-rings and f be a homomorphism of N_1 on to N_2. If A is a Neutrosophic fuzzy ideal of N_1, then f(A) is a Neutrosophic fuzzy ideal of N_2.*

Proof. Let $y_1, y_2, y_3 \in N_2$ and $x_1, x_2, x_3 \in N_1$

(i)

$$\begin{aligned} f(T_A(y_1 - y_2) &= \sup_{x_1,x_2 \in f^{-1}(N_2)} T_A(x_1 - x_2) \\ &\geq \sup_{x_1,x_2 \in f^{-1}(N_2)} \min[(T_A(x_1), T_A(x_2)] \\ &= min(\sup_{x_1 \in f^{-1}(N_2)} T_A(x_1), \sup_{x_2 \in f^{-1}(N_2)} T_A(x_2)) \\ &= min(f(T_A(y_1), f(T_A(y_2)) \end{aligned}$$

(ii)

$$\begin{aligned} f(T_A(y_1 + y_2 - y_1) &= \sup_{x_1,x_2 \in f^{-1}(N_2)} T_A(x_1 + x_2 - x_1) \\ &\geq \sup_{x_1 \in f^{-1}(N_2)} T_A(x_1) \\ &= f(T_A(y_1)) \end{aligned}$$

(iii)

$$\begin{aligned} f(T_A(y_1 y_2) &= \sup_{x_1,x_2 \in f^{-1}(N_2)} T_A(x_1 x_2) \\ &\geq \sup_{x_1 \in f^{-1}(N_2)} T_A(x_2) \\ &= f(T_A(y_2)) \end{aligned}$$

(iv)

$$\begin{aligned} f(T_A)[(y_1 + y_3)y_2 - y_1 y_2] &= \sup_{x_1,x_2,x_3 \in f^{-1}(N_2)} T_A[(\mathrm{x}_1+\mathrm{x}_3)\mathrm{x}_2-\mathrm{x}_1\mathrm{x}_2] \\ &\geq \sup_{x_3 \in f^{-1}(N_2)} T_A(x_3) = f(T_A(y_3) \end{aligned}$$

Similarly we can prove that

$$\begin{aligned} &f(I_A(y_1 - y_2) \geq min(f(I_A(y_1), f(I_A(y_2)), \\ &f(I_A(y_1 + y_2 - y_1) \geq f(I_A(y_1)), \\ &f(I_A(y_1 y_2) \geq f(I_A(y_2)), f(I_A[(y_1 + y_3)y_2 - y_1 y_2)] \\ &\geq f(I_A(y_3) \end{aligned}$$

. Also, we can prove that

$$\begin{aligned} &f(F_A(y_1 - y_2) \leq max(f(F_A(y_1), f(F_A(y_2)), \\ &f(F_A(y_1 + y_2 - y_1) \leq f(F_A(y_1)), \\ &f(F_A(y_1 y_2) \leq f(F_A(y_2)), \\ &f(F_A[(y_1 + y_3)(y_2 - y_1 y_2)] \\ &\leq f(F_A(y_3). \end{aligned}$$

Hence, $f(A)$ is a Neutrosophic fuzzy ideal of N_2. □

6 Conclusion:

In this paper we have studied the notion of neutrosophic fuzzy sub near-ring, neutrosophic fuzzy ideals of near-rings and discussed some algebraic properties. We have proved that union of two neutrosophic fuzzy ideals of near-ring is a neutrosophic fuzzy ideal of that near-ring. Also we have proved that the positive integral powers of neutrosophic fuzzy ideal of near-ring is a neutrosophic fuzzy ideal. We have defined the direct product on neutrosophic fuzzy ideals of near-rings and proved that the direct product of any two neutrosophic fuzzy ideals of near-rings is a neutrosophic fuzzy ideal.Also, we can extend the result for finite number of neutrosophic fuzzy ideals of near-rings.

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