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Estimation of Mean Time to Recruitment for a Two Graded Manpower System With Two Thresholds, Different Epochs for Exits and Correlated Inter-Decisions Under Correlated Wastage Using Bivariate Policy of Recruitment

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Abstract

In this paper, an organization with two grades, subjected to exit of personnel due to policy decisions taken by the organization is considered. As the exit of personnel is unpredictable, a bivariate recruitment policy involving two thresholds is suggested to enable the organization to plan its decision on recruitment. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the mean time to recruitment is obtained when thresholds follows independent and identically distributed exponential random variables, the inter-policy decision times are identically distributed constantly correlated and exchangeable exponential random variables under correlated wastages.

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Key Words and Phrases: Two graded manpower system, decision and exit epochs, constantly correlated and exchangeable exponential random variable, ordinary renewal process, bivariate policy of recruitment with two thresholds, mean time to recruitment.

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1 Introduction

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1], [3] and [5] the authors have discussed the manpower planning models by Markovian and renewal theoretic approach. In [5] the author has studied the problem of time to recruitment for a two graded manpower system and obtained the variance of the time to recruitment. In [2] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory threshold for the cumulative loss of manpower in this manpower system.

In the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account in [6] and [7] the author has studied the problem of meantime to recruitment by considering optional and mandatory thresholds for a two graded manpower system which has non-instantaneous exits at decision epochs.

In the present paper, for a two graded manpower system, a mathematical model is constructed in which attrition due to policy decision take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A bivariate policy of recruitment based on shock model approach is used to determine the expected time to recruitment when the system has different epochs for policy decisions and exits and the inter-decision times are identically distributed constantly correlated and exchangeable exponential random variables and loss of manpower follows constantly correlated exchangeable and exponential random variable.

2 Model Description

Consider an organization taking decisions at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative.

Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th} exit point. $X_i's$ are identically distributed

and constantly correlated exchangeable and exponential random variable with density function $m(.)$, distribution function $M(.)$ Mean $1/a; a > 0$.
Let S_k be the total loss of manpower up to the first k exit points.
Let ρ be the correlation between X_i and X_j where $i \neq j$. and $b = \alpha(1-\rho)$
Let U_j be the continuous random variable representing the time between $(j-1)^{th}$ and j^{th} policy decisions. It is assumed that $U_j^?s$ are identically distributed constantly correlated and exchangeable exponential random variables with probability density function $f(.)$, distribution function $F(.)$ and mean u.
Let R be the correlation between U_i and U_j where $i \neq j$. and $v = u(1-R)$
Let W_k be the continuous random variable representing the time between the $(k-1)^{th}$ and k^{th} exit times. It is assumed that $W_k^?$s are independent and identically distributed random variables with probability density function $g(.)$, probability distribution function $G(.)$
Let $N_e(t)$ be the number of exits points lying in $(0, t]$
Let d be a non negative constant representing threshold for number of decisions
Let $Y_A, Y_B(Z_A, Z_B)$ are the exponential random variable denoting the optional thresholds for grade A and B with distribution function H(.), and density function h(.) and mean $\frac{1}{\lambda_A}, \frac{1}{\lambda_B}, (\frac{1}{\mu_A}, \frac{1}{\mu_B})$ respectively, where $\lambda_A, \lambda_B, \mu_A, \mu_B$ are positive. Assume that $Y_A < Z_A \& Y_B < Z_B$.
Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower.
Let q be the probability that every policy decision has exit of personnel. $(q \neq 0)$.
Let T be the random variable denoting the time to recruitment with distribution function $L(.)$,density function $l(.)$, mean $E(T)$
$A^*(.), a(.)$ be the Laplace-Stieltjes transform and Laplace transform of $A(.)$ and $a(.)$ respectively.

The bivariate CUM policy of recruitment employed in is paper is stated as follows. Recruitment is done whenever the cumulative loss of man power crosses the mandatory threshold or the number of decisions crosses the corresponding threshold whichever is earlier. However, the organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

3 Main Result

$P(T > t) = \sum_{k=0}^{d-1} P$ { there are exactly k exits are taken in $(0, t)$, $k = 0, 1, 2, \ldots,$ x Probability that the total number of exits in these k decisions does not cross the optional level Y or the total number of exits in these k decisions crosses the optional level Y but lies below the mandatory level Z and the organization is not making recruitment}

$$\begin{aligned} P(T > t) &= \sum_{k=0}^{d-1}\{G_k(t) - G_{k+1}(t)\}P\{S_k \leq Y\} \\ &+p\sum_{k=0}^{d-1}\{G_k(t) - G_{k+1}(t)\}P\{S_k > Y\}P\{S_k \leq Z\} \end{aligned} \tag{1}$$

Since $X_i^; s$ are assumed to be identical constantly correlated and exchangeable exponential random variable with parameter α, Cumulative distribution of the partial sum is given in 1955, Gurland [4],

$$G_k(t) = (1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i\phi(k+i,y/b)}{(1-\rho+k\rho)^{i+1}(k+i-1)!} \tag{2}$$

where $\phi(k+i,y/b) = \int_0^{y/b} e^{-z}z^{k+i-1}dz,\ b = a(1-\rho)$ and ρ is the constant correlation between X_i and X_j $i \neq j$;

Case-(i): $Y = \max\{Y_A, Y_B\}$& $Z = \max\{Z_A, Z_B\}$

$$h(y) = \lambda_A e^{-\lambda_A y} + \lambda_B e^{-\lambda_B y} - (\lambda_A+\lambda_B)e^{-(\lambda_A+\lambda_B)y} \tag{3}$$

$$P(S_k < Y) = (1-\rho)[A_{1k} + A_{2k} - A_{3k}] \tag{4}$$

$$P(S_k < Z) = (1-\rho)[A_{4k} + A_{5k} - A_{6k}] \tag{5}$$

where

$$\begin{aligned}
A_{1k} &= \frac{1}{(b\lambda_A+1)^{k-1}[(1-\rho+k\rho)(b\lambda_A+1)-k\rho]}\ ,\\
A_{2k} &= \frac{1}{(b\lambda_B+1)^{k-1}[(1-\rho+k\rho)(b\lambda_B+1)-k\rho]}\ ,\\
A_{3k} &= \frac{1}{(b(\lambda_A+\lambda_B)+1)^{k-1}[(1-\rho+k\rho)(b(\lambda_A+\lambda_B)+1)-k\rho]}\ ,\\
A_{4k} &= \frac{1}{(b\mu_A+1)^{k-1}[(1-\rho+k\rho)(b\mu_A+1)-k\rho]}\ ,\\
A_{5k} &= \frac{1}{(b\mu_B+1)^{k-1}[(1-\rho+k\rho)(b\mu_B+1)-k\rho]}\ ,\\
A_{6k} &= \frac{1}{(b(\mu_A+\mu_B)+1)^{k-1}[(1-\rho+k\rho)(b(\mu_A+\mu_B)+1)-k\rho]}\ .
\end{aligned} \tag{6}$$

using (4),(5)&(6) in (1) becomes,

$$\begin{aligned}
P(T>t) &= \sum_{k=1}^{d-1}\{G_k(t)-G_{k+1}(t)\}(1-\rho)[A_{1k}+A_{2k}-A_{3k}]\\
&+p\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}(t)\}\left[1-\{(1-\rho)[A_{1k}+A_{2k}-A_{3k}]\}\right]\left[(1-\rho)[A_{4k}+A_{5k}-A_{6k}]\right]
\end{aligned} \tag{7}$$

$$\begin{aligned}
\bar{l}(s) = &-(1-\rho)\{\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{1k}+A_{2k}-A_{3k}]\}\\
&+p\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{4k}+A_{5k}-A_{6k}]\\
&-p(1-\rho)\{\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{1k}+A_{2k}-A_{3k}][A_{4k}+A_{5k}-A_{6k}]\}
\end{aligned} \tag{8}$$

It can be shown that distribution function $G(.)$ of the inter exit times satisfy the relation

$$2G(x) = \sum_{n=1}^{\infty}(1-q_A)^{n-1}q_A F_n(x) + \sum_{n=1}^{\infty}(1-q_B)^{n-1}q_B F_n(x) \tag{9}$$

$$2\overline{g}(s) = q_A\sum_{n=1}^{\infty}(1-q_A)^{n-1}F_n^*(s) + q_B\sum_{n=1}^{\infty}(1-q_B)^{n-1}F_n^*(s) \tag{10}$$

where $F_n^*(s) = \frac{(1-R)(1+vs)^{1-n}}{(1-R)(1+vs)+nRvs}$

$$\overline{g}(0) = 1 \text{ and } \overline{g}^{"}(0) = \frac{-v}{2(1-R)}(\frac{1}{q_A}+\frac{1}{q_B}) \tag{11}$$

note that

$$E(T^r) = (-1)^r[\frac{d^r}{ds^r}\bar{l}(s)]_{s=0}, \quad r = 1,2,\ldots \tag{12}$$

Using (8) and (11) in (12)

$$\begin{aligned} E(T) = \frac{(1-\rho)v}{2(1-R)}(\frac{1}{q_A}+\frac{1}{q_B})\{\sum_{k=1}^{d-1}[A_{1k}+A_{2k}-A_{3k}] + p\sum_{k=1}^{d-1}[A_{4k}+A_{5k}-A_{6k}] \\ -p(1-\rho)\sum_{k=1}^{d-1}[A_{1k}+A_{2k}-A_{3k}][A_{4k}+A_{5k}-A_{6k}]\} \end{aligned} \tag{13}$$

Equation (13) gives the mean time to recruitment for case-I
Case-II:$Y = \min(Y_A, Y_B)\&Z = \min(Z_A, Z_B)$

$$hy = (\lambda_A+\lambda_B)e^{-(\lambda_A+\lambda_B)y}$$

$$P(T>t) = \sum_{k=1}^{d-1}\{G_k(t)-G_{k+1}\}(1-\rho)A_{3k}+p\sum_{k=1}^{d-1}\{G_k(t)-G_{k+1}\}[1-\{(1-\rho)A_{3k}\}][(1-\rho)A_{6k}] \tag{14}$$

$$\begin{aligned} \bar{l}(s) = -(1-\rho)\{\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{3k}] + p\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{6k}] \\ -p(1-\rho)\{\sum_{k=1}^{d-1}\{(\overline{g}(s))^k-(\overline{g}(s))^{k+1}\}[A_{3k}.A_{6k}]\}\} \end{aligned} \tag{15}$$

Using (11) and (15) in (12), we get

$$E(T) = \frac{(1-\rho)v}{2(1-R)}(\frac{1}{q_A}+\frac{1}{q_B})\{\sum_{k=1}^{d-1}[A_{3k}] + p\sum_{k=1}^{d-1}[A_{6k}] - p(1-\rho)\sum_{k=1}^{d-1}A_{3k}A_{6k}\} \tag{16}$$

Equation (16) gives the means the time to recruitment for case-II
Case-III: $Y = Y_A + Y_B$ & $Z = Z_A + Z_B$

$$P(Y_A+Y_B>x) = \left(\frac{\lambda_A}{\lambda_A-\lambda_B}\right)e^{-\lambda_B y} - \left(\frac{\lambda_B}{\lambda_A-\lambda_B}\right)e^{-\lambda_A y}$$

$$P(T>t)=(1-\rho)\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}\}(B_{2k}-B_{1k})$$
$$+p(1-\rho)\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}\}[1-\{(1-\rho)(B_{2k}-B_{1k})[B_{5k}-B_{4k}]\} \tag{17}$$

where $B_{1k}=\frac{\lambda_B}{\lambda_A(\lambda_A-\lambda_B)}A_{1k}$, $B_{2k}=\frac{\lambda_A}{\lambda_B(\lambda_A-\lambda_B)}A_{2k}$, $B_{4k}=\frac{\mu_B}{\mu_A(\mu_A-\mu_B)}A_{4k}$, $B_{5k}=\frac{\mu_A}{\mu_B(\mu_A-\mu_B)}A_{5k}$

$$\bar{l}(s)=-(1-\rho)\{\sum_{k=0}^{d-1}\{(\bar{g}(s))^k-(\bar{g}(s))^{k+1}\}[B_{2k}-B_{1k}]+p\sum_{k=0}^{d-1}\{(\bar{g}(s))^k-(\bar{g}(s))^{k+1}\}[B_{5k}-B_{4k}]$$
$$-p(1-\rho)\{\sum_{k=0}^{d-1}\{(\bar{g}(s))^k-(\bar{g}(s))^{k+1}\}[B_{2k}-B_{1k}][B_{5k}-B_{4k}]\}\} \tag{18}$$

Using (11) and (18) in (12),

$$E(T)=\frac{(1-\rho)v}{2(1-R)}(\frac{1}{q_A}+\frac{1}{q_B})\{\sum_{k=1}^{d-1}[B_{2k}-B_{1k}]+p\sum_{k=1}^{d-1}[B_{5k}-B_{4k}]$$
$$-p(1-\rho)\sum_{k=1}^{d-1}[B_{2k}-B_{1k}][B_{5k}-B_{4k}]\} \tag{19}$$

Equation (19) gives the mean time to recruitment for case-III
Remark:

1. Computation of $E(T)$ for extended exponential and $SCBZ$ property possessing thresholds is similar for both the cases as their distribution will have just additional terms.

2. The result on mean time to recruitment by using a univariate policy of recruitment is deduced by letting $t\to\infty$ in all the above three cases.

4 Findings

From the above tables the following observations are presented which agree with reality.
1. As α increases, on the average, the inter-decision time decreases and consequently the mean of time to recruitment decreases when the other parameters are fixed.

5 Conclusion

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points (iii)provision of optional and mandatory thresholds. From the organization?s point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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