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A Joint Economic Lot Size Model with Screening Errors and Investment Opportunity for Quality Improvement of the Production

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Abstract

Building up a long haul connection between the members of the supply chain is essential for the healthier and competitive business world. Vendor plays a vital role in a supply chain to retaining the relationship between the buyers and they assume that every shipment to the buyer contains a good quality of the products. Buyer leading the screening procedure to analyze the item provided by the vendor and recognizes the defective quality things and these items are replaced or repaired by the buyer. To overcome these issues vendor has the option to increase the capital investment to improve the quality of the product. So that this paper presents an integrated vendor buyer inventory model with screening errors and investment cost to maximize the total profit of the supply chain. Also we have considered the emission cost to curb the carbon emission due to the transportation. Mathematical Model and Numerical example are presented and discussed the proposed model.

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Keywords: Two echelon inventory model, screening errors, investment, carbon emission.

1 Introduction

Inventory Management is about the knowledge of what we have in our storehouse and where the supply is placed. Vendor and Buyer are the important supply chain partners in manufacturing and delivering high quality products to the customers. This is often called as an integrated inventory model. This model is based on the concept of combined optimization of vendor and buyer costs instead of finding individual optimization of the supply chain members. In every production the vendor has assumed that the manufacturing products are in a good condition, and they supplied that product to the buyers in a single shipment. But every shipment does not contain a 100% defect free products this fact is known to the buyer during the screening process and the buyer makes the screening error of accepting the non confirming items and rejecting the confirming items. The buyer makes the decision of either repairing defective items or purchases the new products for replacing the defective items. To avoid defective items the vendor has to develop their production capabilities and its performances ([12]). The best way of developing the production of a company the vendor has to invest in the production either in the form of physical investment like factories, machines, equipment etc or increasing the manpower of the company by conducting the training to the workers and giving incentives to them. And also the environment concerns to reduce the carbon emission which is also necessary for a company in a globalized world. The contribution of this proposed model is to evaluate the integrated inventory model with the screening errors and investment opportunity for quality improvement of the production. Investment opportunity may help to improve the quality of the product and also create the better understanding between the supply chain members. The remaining paper is organized as follows, Section 2 presents a literature review of the presented model, and Section 3 illustrates the mathematical model of proposed model. Section 4 presents the numerical example and finally the conclusion of the paper is presented in Section 5.

2 Literature review

The widely accepted an inventory model is an economic order quantity (EOQ) model. It was developed by ([3] in the year (1913) and this model was encouraged and established by many authors as well as the research scholars. This model was mainly based on the assumption of the perfect quality items. ([15]) was the first person who studied the concept of defective items in a basic EOQ model and he concluded that the every production process contains a fraction of defective items. Under this concept many authors developed their researches.([17]) studied a defective items scenario but assumed that the time between the in-control and the out-of-control state of the process followed an exponential distribution. Similarly, ([13]) produced an inspection policy for EOQ model with a known fraction of defective items.

([18]) present an imperfect inventory model with error-free screening process in the lot-sizing decisions. ([10]) extended the above model with the concept of error in screening processes and studied the impact of screening errors on lot-sizing decisions and supply chain costs.([16]) studied a human error in inspection planning and suggested a repeat inspection plan for misclassifications of multi-characteristic critical components. ([2]) and Khan generalized this inspection plan for critical components by incorporating inspection error and misclassification costs of non-defective, repairable, or scrap.

([4]) et al, simplified the model in ([18]), while ([5]) et al, used it to develop an inventory policy for items with imperfect quality and suggested that deliveries in smaller lots may be beneficial. ([1]) extended the ([18]) model and suggested the concept of reworking of defective items when backlogging is permitted.([6]) used a similar approach to derive an analytic solution for the optimal order quantity and the number of deliveries for a vendor-buyer-supply chain. [14] and Konstantaras developed the model with the concept of study the timing of withdrawing and selling defective lots and proposed an alternative to the model in ([18]). ([10])introduced inspection errors into this modified lot-sizing model. Recently, the new idea was developed by many researches that are either repairing of the defective items in buyer's facility or replacing the defectives through local buying this concept was developed by ([7]) The above reviews show that the different concept of imperfect quality items with the inventory model. But producing imperfect quality items are the major drawbacks for the vendor and the production of imperfect quality items are the reason for decreasing the optimal order size or lot size. To strengthen the production processes vendor has to increase the capital investment. It will create a better coordination among the production and quality of the product as well as supply chain members. In a Green environment, reducing the carbon emission is also necessary for the firms due to transportation activities. So the proposed model studies about the concept of inventory model with screening error and investment opportunity for the production along with the carbon emission cost.

2.1. Notations

This section extends the model of [11] et al., (2017) by introducing the vendor's capital investment cost in repairing or purchasing the non- conforming items model. The following notations are used for developing the mathematical model.

A : Cost of internal transportation to and from the repair shop

α : Fraction of items classified as non- defectives after screening

B_1: Number of items classified as defectives in one cycle

B_2 : Number of defective items returned from the market in one cycle

β : Fraction of items classified as defective after screening

c : Unit procurement cost

C_a : Cost of accepting defective items

C_r : Cost of rejecting non defective items

C_R : Cost of repairing a defective items

D : Number of units demanded per year

d : Unit Screening cost

γ : Fraction of defectives classified by the inspector

$$\gamma' : 1 - \gamma$$

h_g : Unit holding cost for non-defective items per year

h_d : Unit holding cost for defective items per year

h_R : Unit holding cost for repaired item

h_v : Vendor's holding cost per item per unit time

K : Ordering cost per cycle

m_1 : Probability of committing type I error
m_2 : Probability of committing type II error

ρ : Actual fraction of defectives
R : Repair rate of defective items

S': Setup cost in the repair shop for every cycle
S : Vendor's setup cost per setup
s : Unit selling price

T : Cycle length
t_I : Inspection time in a cycle

t_R : Time to repair and transport defective items in a cycle

t_t: Time to transport defective items to the repair shop
x : Screening rate
y : Optimal order size

y_r : Optimal order size for the repair model

y_p : Optimal order size for the purchase model
i : Vendor's fractional opportunity cost of capital per unit time

m : Number of shipment from vendor to the buyer in one production run
θ : Probability of the vendor's production process that can go out of control

θ_0 : Original probability of the vendor's production process that can go out of control
$q(\theta)$: Vendor's capital investments require reducing the out of control probability from
θ_0 to θ
d_1 : Distance travelled per km

β' : Social cost from vehicle emission (mu/h)

v : average velocity (km /h)

3 Mathematical Model

The vendor assumes that the manufacturing products are in a good condition and they supplied that product to the buyer in a single shipment so the vendor's cost functions are defined as follows:

Vendor's setup cost per unit time is = $\frac{SD}{my}$

Vendor's holding cost per unit time is = $\frac{h_v y}{2}\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)$

Opportunity cost for process quality is = $iq\ln\left(\frac{\theta_0}{\theta}\right)$

Buyer's cost functions in a repair model including screening errors as follows:

Buyer receives the supplied products and identifies that the shipment contains defective items during the inspection time t_I and send them to repair in time t_R. Screening process leads to both type I and type II errors. The defective lot B_1 is sent to the repair shop as a single batch at time t_I. Falsely accepted defective items are used to fulfill the demand. However these defective items are returned from the market and sent to the repair shop as a single batch (B_2) at the end of the next screening cycle. With the help of repair option the required demand is to be satisfied and there are no shortages, which indicates

$y(1-\gamma)\geq$ DT

Total revenue for the buyer is = sy

Procurement cost per cycle is PC(y) = K+cy

Here K is the fixed ordering cost and c is the variable cost.

The screening cost per cycle involves cost of inspection and screening errors and they are denoted as follows,

Inspection Cost IC(y) = dy + $C_a\rho y m_2$+ $C_r(1-\rho)ym_1$

The total cost at the repair shop is the sum of setup cost, repair cost, holding cost, and the transportation cost

RC(y) = S'+ 2A+ $\gamma y(C_R+h_R t_R)$

Holding cost of the buyer is defined as follows,

HC(y) = $h_g\left[\frac{y^2}{2x}+y(1-\alpha)t_R+\frac{T}{2}\left\{y\left\{1-\frac{D}{X}\right\}-Dt_R\right\}-\frac{yt_R}{2}\right]+h_d\left(\frac{\beta yT}{2}\right)$

Carbon emission cost due to the vehicles CE(y) = $2\beta'\frac{d_1}{v}$

The total cost of the supply chain is expressed as follows,

Total cost = Buyer's cost + Vendor's cost

TC(y) = K+cy+ dy + $C_a\rho y m_2$+ $C_r(1-\rho)ym_1$+S'+ 2A+ $\gamma y(C_R+h_R t_R)$+ $2\beta'\frac{d_1}{v}$

$$h_g\left[\frac{y^2}{2x}+y(1-\alpha)t_R+\frac{T}{2}\left\{y\left\{1-\frac{D}{X}\right\}-Dt_R\right\}-\frac{yt_R}{2}\right]+h_d\left(\frac{\beta yT}{2}\right)+\frac{SD}{my}+iq\ln\left(\frac{\theta_0}{\theta}\right)$$
$$+\frac{h_v y}{2}\left(m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right) \quad (1)$$

Using the cycle time the total profit per unit time of a repair model is,

Total revenue – buyer's total cost- vendor's total cost

TPU(y) = $\frac{sD}{\gamma'}-\frac{KD}{\gamma' y}-\frac{cD}{\gamma'}-\frac{dD}{\gamma'}-\frac{S'D}{\gamma' y}-\frac{C_a\rho y m_2 D}{\gamma'}-\frac{C_r(1-\rho)ym_1 D}{\gamma'}-\frac{2AD}{\gamma' y}-\frac{\gamma D(C_R+h_R t_R)}{\gamma'}$

$$-\frac{h_g D}{\gamma'}\left\{\frac{y}{2x}+\left(\frac{1}{2}-\alpha\right)t_R\right\}-\frac{h_g}{2}\left\{y\left(1-\frac{D}{x}\right)-Dt_R\right\}-h_d\left(\frac{\beta y}{2}\right)-\frac{SD\gamma'}{my\gamma'}-$$
$$\frac{h_v y}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]-2\beta'\frac{d_1 D}{v\gamma' y}-iq\ln\left(\frac{\theta_0}{\theta}\right) \quad (2)$$

It can be simplified as follows,

TPU(y) = $\phi_1-\frac{\pi_1 y}{2}-\mu_1\left(\frac{\gamma y}{R}+2t_t\right)-\frac{\left(K+S'+2A+\frac{S\gamma'}{m}+2\frac{\beta' d_1}{v}\right)D}{\gamma' y}-iq\ln\left(\frac{\theta_0}{\theta}\right)$

Where,

$$\phi_1=\frac{D\left(s-c-d-C_a\rho m_2-C_r(1-\rho)m_1-\gamma C_R\right)}{\gamma'}$$

$$\mu_1 = h_g \left\{ \left(\frac{1}{2} - \alpha \right) + \gamma h_R \right\} \left(\frac{D}{\gamma'} \right)$$

$$\pi_1 = h_g \left[1 + \frac{D}{\gamma' x} - \frac{D}{x} - \frac{2D\gamma}{R} \right] + h_d \beta + h_v \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right]$$

Differentiate with respect to 'y' we get the optimal order quantity of repair model as follows,

$$y_r* = \sqrt{\frac{2D\left(K + S' + 2A + \frac{S\gamma'}{m} + 2\frac{\beta' d_1}{v}\right)}{\gamma' \left[\pi_1 + \frac{2\mu_1 \gamma}{R}\right]}} \tag{3}$$

Buyer's cost functions in replacing the defective items by purchasing new items
in a local market.

In this model, the buyer consider the non conforming items as waste so disposing that items at a valuable disposal cost and the buyer's make the decision of purchasing new items in a local market for replacing the defective ones. So the required holding cost of this model is defined as follows,

Holding cost of the buyer is HC(y) $= h_g y^2 \left[\frac{1}{2D} \left\{ (1-\alpha)^2 + \alpha^2 \right\} + \frac{\alpha}{x} \right] + h_d \left(\frac{\beta y T}{2} \right)$

The expected total cost is,

TC(y) = K+cy+ dy + $C_a \rho y m_2$+ $C_r(1-\rho) y m_1$+ $2\beta' \frac{d_1}{v}$+$C_P \alpha y$

$$h_g y^2 \left[\frac{1}{2D} \left\{ (1-\alpha)^2 + \alpha^2 \right\} + \frac{\alpha}{x} \right] + h_d \left(\frac{\beta y T}{2} \right) + \frac{SD}{my} + iq \ln \left(\frac{\theta_0}{\theta} \right)$$
$$+ \frac{h_v y}{2} \left(m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right) + C_d \gamma y \tag{4}$$

Using the cycle time the total profit per unit time of a purchase model is,

TPU(y) = Sales revenue − buyer's cost − vendor's cost

$$= \frac{sD}{\gamma'} - \frac{KD}{\gamma' y} - \frac{cD}{\gamma'} - \frac{dD}{\gamma'} - \frac{C_a \rho D m_2}{\gamma'} - \frac{C_r (1-\rho) D m_1}{\gamma'} - \frac{C_p \alpha D}{\gamma'} - \frac{C_d \gamma D}{\gamma'} - \frac{SD\gamma'}{my\gamma'} -$$
$$\frac{h_g y}{2\gamma'} \left\{ (1-\alpha)^2 + \alpha^2 + \frac{2\alpha D}{x} \right\} - h_d \left(\frac{\beta y}{2} \right) - \frac{h_v y}{2} \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] - \frac{2\beta' d_1 D}{\gamma' v y}$$
$$- iq \ln \left(\frac{\theta_0}{\theta} \right) \tag{5}$$

The above equation can be written as a simplified manner as follows,

TPU(y) $= \phi_2 - \frac{\pi_2 y}{2} - \frac{\left(K + \frac{S\gamma'}{m} + 2\frac{\beta' d_1}{v}\right) D}{\gamma'}$-$iq \ln \left(\frac{\theta_0}{\theta} \right)$

Where, $\phi_2 = \frac{D(s - c - d - C_P \alpha - C_d \gamma - C_a \rho m_2 - C_r (1-\rho) m_1)}{\gamma'}$

$$\pi_2 = \frac{h_g}{\gamma'} \left\{ (1-\alpha)^2 + \alpha^2 + \frac{2\alpha D}{x} \right\} + h_d \beta + h_v \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right]$$

Differentiate with respect to y we get the optimal order quantity of purchase model,

$$y_P^* = \sqrt{\frac{2D\left(K + \frac{S\gamma'}{m} + \frac{2\beta' d_1}{v}\right)}{\pi^2 \gamma'}}$$

4 Numerical Example

To demonstrate the applications and usefulness of the above proposed model, we consider the following parameters.

D= 50,000, x= 175200, R= 50000, t_t = 0.0045, K = 100, h_R = 5, h_g=5, h_d=2, d= 0.5, c=25, s=50, C_a = 500, S=100, S'=100, A=200, m_1 = 0.02, m_2=0.02, C_r = 5, ρ=0.02, d_1 =250, v=180,β'=0.5, i=0.1, q=400, θ_0=0.002, θ=0.000058, m=1, P= 150000, C_p= 40,C_d=2, h_v=4, α= 0.02.

The optimal order quantity of a repair model is y_r^*= 3406, and Total profit of the supply chain is Rs. 1192908. The optimal order quantity of a purchase model is y_p^*= 1807, and the Total profit of the supply chain is Rs. 1203783.

5 Conclusion

In this proposed model we have study about the inventory model with imperfect quality items. Due to the imperfect quality items the optimal order size or lot size of the system is reduced. To develop the performance of the production the vendor has to increase the capital investment it would leads to improve quality of the production as well as it is useful for the manufacturer to retain their customers.

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