International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 499-507
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

An Economic Order Quantity Model for Weibull Distribution Deterioration with Selling Price Sensitive Demand and Varying Hold Cost

W.Ritha[1] and I.Antonitte Vinoline[2]

[1,2]*Department of Mathematics,*
Holy Cross College (Autonomous),Trichy -2.
[1]*ritha_prakash@yahoo.co.in*

Abstract

In the present business world, the selling price assumes a vital part to expanding or diminishing the demand rates of any items. Therefore, the demands of numerous items are subject to offering cost. For the results the researchers are established, many inventory models with price dependent demand rates. The holding cost is assumed as a time dependent, i.e. varying with time. The researchers established the demand is the linear decreasing function of price or the demand rate is a negative power function of price. In this proposed model we assumed an inventory model for deteriorating items with non-linear price dependent demand rate. The deterioration rate of the proposed model is assumed as three parameter Weibull distribution deterioration. The demand rate is price sensitive that means considered as a quadratic selling price dependent demand rate. Shortages are allowed. Mathematical formulation with the solution procedure of the introduced model is developed. Some numerical examples are recommended to substantiate the solution procedure.
Keywords: Inventory model, selling price dependent demand, three parameter Weibull deterioration, partial backlogging

1 Introduction

chain management (SCM) [21] is the outline of, products, merchandise, raw materials, finance, information, and etc. from supplier to manufacturer, manufacturer to wholesaler, wholesaler to retailers, and retailers to consumers. The eventual focus of any intensive supply chain management system is to control the inventory problems. Inventory management is a part of supply chain management, adjusted the amount of supply merchandise.

Inventory models depend on the minimizing the aggregate important stock expenses or boosting the aggregate benefits, i.e. minimizing the total relevant inventory costs or maximizing the total profits.
Deterioration is characterized as damage, harms, spoilage, or decay. Sustenance things, drugs, pharmaceutics, electronic parts and radioactive substances are a few illustrations of things in which adequate deterioration might happen, amid the typical stockpiling period. This loss must be considered while analyzing the inventory problems of those products.
In the classical EOQ model, i.e. Harris [8] (1913) Model, deterioration rate was considered as constant. The fundamental EOQ model was extended by Ghare and Schrader [6] (1963) considering the deterioration rate as non-constant. Then many authors work in deteriorating items say, [16]-[18], etc. Covert and Philip (1973) [5] further extended the Ghare and Schrader's [6] model and evolved an EOQ model for a variable rate assuming deterioration of a two-parameter Weibull distribution. The mathematical statement that portrays the model is given by

$$\frac{dI(t)}{dt} + g(t)I(t) = -d \tag{1}$$

where, d= demand rate, I(t) = inventory level, and g(t) $=\alpha\beta t^{\beta}$; α = shape parameter, β = scale parameter. The two-parameter Weibull distribution, is proper for those things which have diminished rate of deterioration, if just at the beginning, the rate of deterioration is to very high and comparable, the expanding rate of deterioration if just at the starting is roughly zero. However, said conditions are not satisfied in the case of numerous items, to eliminate this situation, Philip (1974) [10] expanded the model [6] by considering a three-parameter Weibull dissemination and supplant $g(t)$ with

$$g(t) = \alpha\beta(t-\gamma)^{\beta} \tag{2}$$

where,α = shape parameter, β = scale parameter, and γ = location parameter of Weibull distribution deterioration. Next, Tadikamalla [14], Chakrabarty et al. [3] considered an inventory model with three parameter Weibull distribution rate. For more details one can refer to ([1], [7], [12], and the references therein.) Offering the costs of a stock is a vital part in order to attend the shopkeepers in a business association. Generally, they pay reasonable costs of stock, whatever might be the costs, on the basis of its quality and lifespan. The price of any product is effected by its demand. The expanding costs show the diminishing demand and diminishing demands is the reason for expanding the cost. The first inventory model for price dependent demands was given by Whitin [20] in 1955. After that, many researcher works on these aspects such as [2], [4], Ladany and Sternlieb [9], [11], Urban [15], Wee [19], and the references therein.
Later, in the EOQ (Economic Order Quantity) models for price dependent demands the researchers have used two types of demand functions. First, in which where the demand is the linear decreasing function of price and in the other, demand rate is a negative power function of price. In 2011, Sana [13] developed an EOQ model for deteriorating items by considering the finite time horizon and demand is assumed as a quadratic decreasing function of price and divides the time horizon into finite (n) equal periods.
In this paper, we propose an inventory model for deteriorating items with non-linear price dependent demand rate. The deterioration rate of the proposed model is assumed as three parameter Weibull distribution deterioration. The demand rate is price sensitive that means considered as a quadratic selling price dependent demand rate. Shortages is allowed and which is partial backlogged.

The rest of the paper is organized as follows: assumptions and notation are given in section 2. The mathematical formulation is established in section 3. Numerical examples are given in section 4. Finally, the conclusion of the introduced model is presented in section 5.

2 Notations and Assumptions

The notation which are used throughout the paper ,given as below:

Notation

p the selling price per unit

$D(p)$ the selling price dependent demand rate

θ (t) the time dependent three parameter Weibull deterioration rate

A the ordering cost per unit

h the holding cost per unit

c the deterioration cost per unit

I_0 the initial inventory level

T the fixed time length of inventory cycle when the inventory level reaches zero

Assumptions

1. The deterioration rate is considered as a time dependent, i.e., $\theta(t)$ =$\alpha\beta$(t-γ)$^($$\beta - 1$)
2. Demand rate D(p) is considered as a price sensitive, i.e. D(p) = a - bp- c p^2.
3. The replenishment rate is instantaneous, and time horizon is infinite.
4. Lead time is considered as zero.
5. Shortages are allowed.
6. Holding cost h(t) per unit is assumed as time dependent,
   i.e. h(t) = h +αt, where α; h > 0.

3 Mathematical Formulation

The demand rate [13] decreases quadratically with respect to price p, such that
D(p) = a − b p − cp^2; where a > 0, b > 0, and c > 0
Assume that, the supplier, purchase the seasonal goods with I_0 units and has the authority to change the prices as mark up and mark down on selling price p. In the time period [0, T] the inventory level decreases due to demand and deterioration both. Thus, the differential equation of inventory level during time period $[0, T]$ is given as below:

$$\frac{dI(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1}I(t) = -D(p); 0 \le t \le T \tag{3}$$

Now, we solve the above equation with boundary condition I(0) = I_0, and I(T) = 0. We have

$$I(t) = e^{(-\alpha(t-\gamma)\beta)}\left[D(P)\left((T-t)+\frac{\alpha}{(\beta+1)}\left((T-\gamma)^{(\beta+1)}-(t-\gamma)^{(\beta+1)}\right)\right)\right] \quad (4)$$

$$I_0 = e^{(-\alpha(-\gamma)\beta)}\left[D(p)\left((T+\frac{\alpha}{(\beta+1)}\left((T-\gamma)^{(\beta+1)}-(-\gamma)^{(\beta+1)}\right)\right)\right] \quad (5)$$

Now, to find the total relevant inventory cost, we calculate the following costs:

1.Ordering Cost (OC) $= \frac{A}{T}$

2. Holding Cost (HC)

HC $=\frac{h}{T}\left(\int_0^T I(t)dt\right)$

$$= \frac{h}{T}\left(\int_0^T e^{(-\alpha(-\gamma)\beta)}\left[D(p)\left((T-t)+\frac{\alpha}{(\beta+1)}\left((T-\gamma)^{(\beta+1)}-(t-\gamma)^{(\beta+1)}\right)\right)\right]dt\right) \quad (6)$$

3. Deterioration Cost (DC)

$$DC = \frac{cI_0}{T} = \frac{ce^{(-\alpha(t-\gamma)\beta)}}{T}\left[D(p)\left(\left(T+\frac{\alpha}{(\beta+1)}\right)\left((T-\gamma)^{(\beta+1)}-(-\gamma)^{\beta+1}\right)\right)\right] \quad (7)$$

Hence, the total cost is given as,

TC = OC + HC + DC

$$= \frac{A}{T}+\frac{h}{T}\left(\int_0^T e^{(-\alpha(t-\gamma)\beta)}\left[D(p)\left((T-t)+\frac{\alpha}{(\beta+1)}\left((T-\gamma)^{(\beta+1)}-(t-\gamma)^{(\beta+1)}\right)\right)\right]dt\right)$$
$$+\frac{ce^{(-\alpha(-\gamma)\beta)}}{T}\left[D(p)\left(\left(T+\frac{\alpha}{(\beta+1)}\right)\left((T-\gamma)^{(\beta+1)}-(-\gamma)^{\beta+1}\right)\right)\right] \quad (8)$$

Next, to find the total profit (TP), we calculate the sales revenue, i.e.,
Sales Revenue = p(α - βp - γp^2)
Thus, the total profit function (TP) is given as below

TP = Sales Revenue - total cost
TP = p(α - β p - γ p^2) - TC

= p(α - βp - γ p^2)

$$= \frac{-A}{T} - \frac{h}{T}\left(\int_0^T e^{(-\alpha(t-\gamma)\beta)}\left[D(p)\left((T-t)+\frac{\alpha}{(\beta+1)}\left((T-\gamma)^{(\beta+1)}-(t-\gamma)^{(\beta+1)}\right)\right)\right]dt\right) - \frac{ce^{(-\alpha(-\gamma)\beta)}}{T}\left[D(p)\left(\left(T+\frac{\alpha}{(\beta+1)}\right)\left((T-\gamma)^{(\beta+1)}-(-\gamma)^{\beta+1}\right)\right)\right] \quad (9)$$

The above Equation (9) is a function of two variables T and p, thus for the necessary condition, we differentiate TP with respect to T and p and equate it to be 0. i.e.,

$$\frac{\partial \text{TP}}{\partial \text{T}} = 0 \text{ and } \frac{\partial \text{TP}}{\partial \text{p}} = 0... \quad (10)$$

and finds the values of T and p. For the convexity of TC(T, p), we must have

$$\frac{\partial^2 TP}{\partial T^2} < 0, \frac{\partial^2 TP}{\partial P^2} < 0 and \frac{\partial^2 TP}{\partial T^2}\frac{\partial^2 TP}{\partial p^2} - \left[\frac{\partial TP}{\partial T}\frac{\partial TP}{\partial P}^2\right] > 0$$

4 Numerical Example

To verify the proposed model we substitute theappropriate values of the parameter, i.e., h = 1, c = 25,γ = 0.002, β = 2, γ = 3, α_1= 0.001 per units per years; a =100; b = 1.8; c = 0.25; A = 150, in Equation (10), and find the optimal values of T and p. Then, the optimum values are T = 19.79 weeks and p = \$39.42 per unit per weeks and total profit is TP= \$5998.01 per unit per weeks.
Table 1: Effect of changes in the parameters of Example

Parameters	$Value$	T^*	p^*	TP
α	0.01	25.3796	43.21	8436.64
	0.02	19.7867	39.42	5998.01
	0.03	17.0643	35.88	4148.23
	0.04	15.2953	32.38	2689.35
β	1	100.329	61.23	2770.5
	2	57.78	39.23	2823.9
	3	34.89	11.554	2899.7
	4	0.3592	1.861	2941.9
γ	1	17.9533	45.20	9919
	2	18.953	43.01	8297.02
	3	19.786	39.42	5998.01
	4	20.43	34.65	3596.56
a	50	19.786	37.928	6870.34
	75	19.786	38.685	6424.86
	100	19.786	39.42	5998.01
	125	19.78	40.128	5589.2
b	1.4	19.7867	39.68	5735.76
	1.6	19.7867	39.55	5866.58
	1.8	19.7867	39.418	5998.01
	2.0	19.7867	39.288	6130.04
c	0.15	19.7867	40.46	3414.04
	0.20	19.7867	39.83	4702.27
	0.25	19.7867	39.42	5998.01
	0.30	19.7867	39.13	7297.92
A	50	19.78	39.42	6003.07
	150	19.78	39.42	5998.01
	200	19.78	39.42	5995.49
	250	19.78	39.42	5992.96
α_1	0.0005	19.82	39.47	6028.3
	0.0010	19.786	39.42	5998.01
	0.0125	19.099	38.40	5450.51
	0.0150	18.974	38.28	5360.46

The impact of analysis of some significantparameters is given in Table (1). Taking into account thegiven after effect of Table 1, we give the sensitivity ofabove model as below:

1. Increase in the value of the scale parameter, i.e.,α,we observe the cycle time T increases with high sensitivity, and the total profit function TP and selling price p decrease with high sensitivity for scale parameter.

2. Increase in the value of the shape parameter, i.e.,β, we observe the cycle time T, the total profit functions TP,and the selling price p decreases with high sensitivity

for scale parameter.

3. Increase in the value of location parameter, i.e.,γ, we observe the cycle time T increases slowly and the total profit function TP decreases fast. Also, the selling price p decreases moderately.

4. Increase in the value of a, b, and c, we observe the cycle time T is unchanged, i.e. insensible, but, the selling price p and the total profit function TP are moderately sensitive.

5. When the value of ordering cost, i.e. A is changing,the selling price p, cycle time T is unchanged, i.e.insensitive, but the total profit function TP decreases lowly, that is the total profit is lowly sensitive.

6. For α, the cycle time T, the selling price p, and the total profit function TP are less changeable, i.e. lowly sensitive.

5 Conclusion

In the present business world, the selling price assumes a vital part to expanding or diminishing the demand rates of any items. Therefore, the demands of numerous items are subject to offering cost. For the results the researchers are established, many inventory models with price dependent demand rates. The holding cost is assumed as a time dependent, i.e. varying with time. The researchers established the demand is the linear decreasing function of price or the demand rate is a negative power function of price. Sana [13] developed a quadratic type price dependent demand rates. In this proposed model we assumed as quadratic type price depending demand rates with varying holding cost dependent on times. For future research, the model will be reached out by considering the finite rate of replenishment, time dependent deterioration, trade credit or permissible delay in payments, inflation rate, quantity discounts etc.

References

[1] M. Bakker, J. Riezebos, and R.H. Teunter,*Review of inventory systems with deterioration since 2001, European Journal of Operational Research*, 221 (2), 275-284 (2012).

[2] T.H. Burwell, D.S. Dave, K.E. Fitzpatrick, and M.R.Roy, *Economic lot size model for price-dependent demand under quantity and freight discounts, International Journal Production Economics*, 48, 141-155 (1997).

[3] T. Chakrabarty, B.C. Giri, and K.S. Chaudhuri,*An EOQ model for items with Weibull distribution deterioration, shortages and trended demand: An extension of Philip's Model, Computers Ops. Res.*, 25(7/8), 649 - 657 (1998).

[4] H. Chen, Y. Chen, C.H. Chiu, T.M. Choi, and S. Sethi, *Coordination mechanism for the supply chain with lead time consideration and price-dependent demand, European Journal of Operational Research*, 203, 70 – 80 (2010).

[5] R.P. Covert and G.C. Philip, *An EOQ Model for Items with Weibull Distribution Deterioration, AIIE Transactions*, 5, 323-326 (1973).

[6] P.M. Ghare and G.F. Shrader, *A model for exponentially decaying inventories, Journal of Industrial Engineering*, 14, 238-243 (1963).

[7] S.K. Goyal and B.C. Giri,*Recent trends in modeling of deteriorating inventory, European Journal of Operational Research*134(1), 1-16 (2001).

[8] F.W. Harris, *How many stock to keep on hand, Factory, The Magazine Management*, 10(3), 240-241 and 280-284, (1913).

[9] S.P. Ladany and A. Sternlieb, *The interaction of economic order quantities and marketing policies, AIIE Transactions*, 6, 35-40 (1974).

[10] G.C. Philip, *A generalized EOQ model for items with Weibull distribution deterioration, AIIE Transaction*, 6, 159- 162 (1974).

[11] H. Polatoglu and I. Sahin,*Optimal procurement policies underprice-dependent demand, Inernational Journal of Production Economics*, 65, 141-171 (2000).

[12] F. Rafaat, *Survey of literature on continuously deteriorating inventory models, Journal of Operational Research Society*, 42, 27-37 (1991).

[13] S.S. Sana, *Price-sensitive demand for perishable items-an EOQ model, Applied Mathematics and Computations*, 217, 6248-6259 (2011).

[14] P.R. Tadikamalla,*An EOQ inventory model for items with gamma distributed deterioration, AIIE Transactions*, 10, 100- 103 (1977).

[15] T.L. Urban,*Computational performance and efficiency of lower-bound procedures for the dynamic facility layout problem*, Journal of Operational Research, 57, 271-279 (1992).

[16] Vandana and B.K. Shama, *EPQ inventory model for non-instantaneous deteriorating item sunder trade credit policy, International Journal of Mathematical Science and EnggineeringAppllications*, 9(I), 179-188 (2015).

[17] Vandana and B.K. Shama, *An EOQ model for retailers partial permissible delay in payment linked to order quantity with shortages*, Math. Comput. Simulation. 125, 99-112(2016).

[18] Vandana and B.K. Sharma, *An inventory model for non-instantaneous deteriorating items with quadratic demand rate and shortages under trade-credit policy, Journal of Applied Analysis and Computation*, 6(3), 720-737 (2016).

[19] H.M. Wee,*A replenishment policy for items with a pricedependent demand and a varying rate of deterioration, Production Planning & Control*,8, 494-499 (1997).

[20] T.M. Whitin,*Inventory control and price theory, Management Science*, **2**, 6168 (1955).

[21] T.M. Whitin, *Theory of Inventory Management, Princeton University Press, Princeton, NJ*, (1957).