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On Solving Multi-Objective Fractional Linear Programming Problem by Using Intuitionstic Fuzzy Numbers

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Abstract

In this paper, a new approach for solving multi-objective fractional linear programming problem by using Intuitionstic fuzzy numbers. Fuzzy arithmetic operations and simplex algorithm is proposed for solving multi-objective Fractional linear programming problem with parameters represented by Intuitionstic fuzzy numbers. Using the proposed method the fuzzy optimal solution of fuzzy multi-objective Fractional linear programming problem can be easily obtained. Finally a numerical example is provided to check the efficiency of the proposed method.

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1 Introduction

Most of the real life problems exhibit properties of multi-objectivity and fuzziness in nature. In real life situations, linear fractional models arise in decision making such as construction planning, health care and hospital planning. Several methods have been to solve LFP Problems. Isbell and Marlow first identified an example of LFP Problems and sequence of linear programming problems.In fuzzy decision making problems,the idea of maximizing decision was anticipated by Bellman and Zadeh.Fuzzy set theory introduced by Zadeh is generalization of conventional set theory to represent vagueness or imprecision in everyday life. There are many kinds of formulation to the objective function of problems,may be linear programming ,quadratic programming,multi objective and fractional linear programming all this kind it is possible.In this work multi-objective fractional linear programming problem with symmetric trapezoidal fuzzy numbers. In Section 2 some basic definitions and arithmetic operations of symmetric trapezoidal fuzzy numbers. In Section 3 a new method is proposed for fractional fuzzy multi-objective linear programming problem and in Section 4 a numerical example is solved. Finally conclusions are drawn in Section 5.

2 Preliminaries

In this section, some basic definitions and arithmetic operations of symmetric trapezoidal fuzzy numbers and fractional linear programming problems.

2.1 Fractional linear programming Problem

A Fractional linear programming problem is defined as
$\max Z = \dfrac{cx+p}{dx+q}$, subject to $AX \leq B, X \geq 0$
Where $c = (c_1, c_2, \ldots c_n)$, $d = (d_1, d_2, \ldots d_n), B = (b_1, b_2, \ldots b_n)^T, X \in R^n, x \in X, p$ and q are scalar.

2.2 Multi objective Fractional linear programming Problem

A Multi objective Fractional linear programming problem is defined as
$\max Z_i = \dfrac{cx+p}{dx+q}, \quad i = 1, 2, \ldots, K.$

Definition 1. A fuzzy number $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is said to be a symmetric trapezoidal fuzzy number if its membership function is given as follows,

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-(a^L-\alpha)}{\alpha}, & a^L - \alpha \leq x \leq a^L \\ 1, & a^L \leq x \leq a^U \\ \dfrac{(a^U+\alpha)-x}{\alpha}, & a^U \leq x \leq a^U + \alpha \\ 0 & \end{cases}$$

Definition 2. A Symmetric trapezoidal number is defined as

$$\mu_1(x) = \begin{cases} \dfrac{x-(a_1-\alpha)}{\alpha}, & \text{for } x \in [a_1 - a, a_1] \\ 1, & \text{for } x \in [a_1, a_2] \\ \dfrac{a_2 - \beta - x}{\beta}, & \text{for } x \in [a_1, a_2 + \beta] \\ 0, & \text{Otherwise} \end{cases}$$

$$\nu_1(x) = \begin{cases} \dfrac{(a_1 - x)}{\alpha}, & \text{for } x \in [a_1 - \alpha, a_1] \\ 0, & \text{for } x \in [a_1, a_2] \\ \dfrac{x - a_2}{\beta}, & \text{for } x \in [a_2, a_2 + \beta] \\ 1, & \text{Otherwise} \end{cases}$$

3 Complementary development method to solve multiobjective fractional linear programming problem

The method proposed here is used to transform the fractional linear programming problem into linear programming problem is developed here. An algorithm for solving a multi objective fractional linear programming problem.

3.1 Algorithm of complementary development method for solving multi objective intuitionstic fuzzy number fractional linear programming problem

Step 1: Dividing the first objective function into linear function in which the first function represents the numerator function and second one is the denominator function. The value of the objective function is taken as maximum for the numerator and minimum for the denominator function.

Step 2: Reclamation a function $\max \tilde{z}(x)$ from subtracting the denominator function from the numerator function using the arithmetic operation of symmetric trapezoidal fuzzy numbers and this function is putting in mathematical module made up of original restriction of problem in addition to non negative conditions and to solve this linear system go to Step 4

Step 3: Reclamation a function $\max \tilde{z}(x)$ from adding the denominator function from the numerator function using the arithmetic operation of symmetric trapezoidal fuzzy numbers and this function is putting in mathematical module made up of original restriction of problem in addition to non negative conditions and to solve this linear system go to Step 4.

Step 4: By adding slack variable and then solve the system to obtain the solution for x_j by using simplex method and also using the ranking function of symmetric trapezoidal intuitionstic fuzzy numbers.

Step 5: The same procedure is repeated for the second objective function.

Step 6: By using preemptive optimization method the same procedure is repeated until all the objective functions are optimized.

4 Numerical Example

Consider a multi objective fractional linear programming problem

$$\max Z_1 = \frac{(2,4,2,2)(3,3,1,1)\tilde{x}_1 + (2,2,1,1)(1,3,2,2)\tilde{x}_2}{(1,1,2,2)(1,1,3,3)\tilde{x}_1 + (1,4,2,2)(2,3,1,1)}$$
$$\max Z_2 = \frac{(1,2,2,2)(2,1,1,1)\tilde{x}_1 + (2,3,1,1)(1,4,2,2)\tilde{x}_2}{(.5,.5,2,2)(.5,.5,3,3)\tilde{x}_1 + (1,4,2,2)(2,3,1,1)}$$

Subject to constraints

$$x_1 + 2x_2 \leq 2$$
$$3x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

$$\max Z_1 = (1,3,2,2)(3,1,2,2)\tilde{x}_1 + (1,3,2,2)(3,1,2,2)\tilde{x}_2 - (2,82,2)(5,5,3,3)$$

Subject to constraints

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + 2x_2 + x_4 = 4$$

By using preemptive optimization method using TORO software $x_1 = 1, x_2 = 5$, $\max Z_1 = 1$,

$$\max Z_2 = (1,2,2,2)(2,1,2,2)\tilde{x}_1 + (1,3,2,2)(2,2,4,4)\tilde{x}_2 - (2,32,2)(1,4,1,1)$$

Subject to constraints

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + 2x_2 + x_4 = 4$$
$$x_1 + .5x_2 + x_5 = 1$$

By using preemptive optimization method using TORO software $x_1 = 0.67, x_2 = 0.67$, $\max Z_2 = 0.33$
In addition

$$\max Z_1 = (5,3,2,2)(4,4,3,3)\tilde{x}_1 + (1,3,2,2)(2,2,3,3)\tilde{x}_2 - (2,3,2,2)(1,4,1,1)$$

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + 2x_2 + x_4 = 4$$

By using preemptive optimization method using TORO software $x_1 = 1.33, x_2 = 0$, $\max Z_1 = 15.67$

$$\max Z_2 = (1,2,3,3)(2,1,2,2)\tilde{x}_1 + (3,3,2,2)(1,5,3,3)\tilde{x}_2 + (2,32,2)(1,4,1,1)$$

Subject to constraints

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + 2x_2 + x_4 = 4$$
$$1.33x_1 + x_5 = 15.67$$

By using preemptive optimization method using TORO software $x_1 = 0, x_2 = 1$, max $Z_2 = 11$.

5 Conclusion

In this paper, two methods of solving Multi objective intuitionstic fuzzy linear programming problems are provided in which subtraction and addition of denominator and numerator. have been carried out. The new method proposed here gives better results as compared with the complementary developed method exist earlier. This method extended to non linear programming problem in future.

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