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# On Solving Multi Objective Interval Transportation Problem Using Fuzzy Programming Technique

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**Abstract**

In this paper, a new algorithm is developed to solve multi objective transportation problem, where the cost coefficients of the objective functions, and the source and destination parameters are expressed as interval values. Here, fuzzy programming technique is used with linear membership function for different costs to solve MOTP. Numerical examples are provided to illustrate its feasibility.

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**Key Words and Phrases:** Multi -Objective Interval Transportation problem, Parallel Method, Fuzzy linear membership function.

## 1 Introduction

Transportation problem of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. In mathematical interval programming models deal with uncertainty and interval coefficients [8]. An interval transportation problem constructs the data of supply, demand and objective functions such as cost, time etc in some intervals. This problem can be converted into a classical MOTP by using the concept of right limit, half-width, left limit, and centre of an interval [1].

Other than this, many researchers are working in this field. Das at al. (1999) used fuzzy programming technique to solve MOITP in which cost coefficient, destination and source parameters are in interval form. For better solution of MOITP Sengupta and pal (2003) developed fuzzy techniques based solution in which midpoint and width of the interval they consider and provide efficient solution. P. Pandian and G. Natarajan (2010) utilized separation method and obtained compromise solution of integer transportation problems in which transportation cost, supply and demand are in uncertain form. S. K. Roy and D. R. Mahapatra (2011) developed a method based on weighted sum and solved multi-objective stochastic transportation problem in which parameters and lognormal random variables and the coefficients of the objectives are interval numbers with inequality type of constraints. Arpita Panda and Chandan Bikash Das (2013) [2] focused on the solution of Cost Varying Interval Transportation Problem under Two Vechicles by applying fuzzy programming technique. Abdusalam mohmed khalifa, E.E. Ammar (2014) have used the concept of UUIT, ULIT, LUIT and lower interval LLIT to solve Rough Interval Multi-Objective Transportation problems (RITP). Vincent F. Yu, Kuo-Jen Hu& An -Yuan Chang (2015) [8] obtained compromise solution for the MOITP by applying an Interactive approach. Dr. M. S. Annie Christi, and I. Kalpana (2016) [7] obtained compromise solutions of Multi-Objective Fuzzy transportation problems to solve with Non-linear membership functions. Yeola M. C. and Jahav V.A (2016) [3] have proposed a method to solve Multi-Objective transportation problem using fuzzy programming technique is used with fuzzy linear membership function for different cost to solve MOTP. Jignasha G. Patel, Jayesh M. Dhodiya (2017) [9] presents the compromise solution of Multi-objective interval transportation problem obtained using grey situation decision making theory based method objective weights.

This paper is organized as follows. In Section 2 Preliminaries definitions are given. Section 3 represents interval transportation problem, Multi-objective interval transportation problem and fuzzy programming technique Multi-objective interval transportation problem. Numerical examples are solved in Section 4. The last section draws some concluding and remarks.

## 2 Preliminaries

**Definition 1** (Interval). A closed interval is defined by an order pair of brackets as:

$$A = [a_L, a_R] = \{a : a_L \leq a \leq a_R,\ a \in R\}$$

where $a_L$ and $a_R$ are, respectively, the left and right limits of $A$.
The interval is also denoted by its centre and half width as

$$A =< a_c, a_w >= \{a : a_c - a_w \leq a \leq a_c + a_w,\ a \in R\}$$

where $a_c = \dfrac{a_R + a_L}{2}$ and $a_w = \dfrac{a_R - a_L}{2}$ are respectively, the centre and half width of $A$.

**Definition 2** (Operators). If $A$ and $B$ are two closed intervals, and $*$ be a binary operation on the set of real number, then $A * B = \{a * b : a \in A, b \in B\}$ is defined a binary operation.

According to the above definition interval operations are defined as:

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L,\ a_R + b_R]$$
$$A + B =< a_c, a_w > + < b_c, b_w >=< a_c + b_c, a_w + b_w >$$
$$kA = k[a_L, a_R] = [ka_L, ka_R] \text{ if } k \geq 0$$
$$kA = k[a_L, a_R] = [ka_R, ka_L] \text{ if } k \leq 0$$

where $k$ is a real number.

**Definition 3** (Order relation $\leq L_R$)**.** The order relation $\leq L_R$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as:

$$A \leq L_R B \text{ iff } a_L \leq b_L \text{ and } a_R \leq b_R$$
$$A < L_R B \text{ iff } A \leq LRB \text{ and } A \neq B.$$

**Definition 4** (Order relation $\leq_{cw}$)**.** The order relation $\leq_{cw}$ between $A =< a_c, a_w >$ and $B =< b_c, b_w > is$ defined as

$$A \leq_{cw} B \text{ iff } a_c \leq b_c \text{ and } a_w \leq b_w$$
$$A <_{cw} B \text{ iff } A \leq_{cw} B \text{ and } A \neq B.$$

## 3 Interval Transportation Problem (ITP)

The formulation of ITP is the problem of minimizing interval valued objective function with interval costs, interval sources and interval demands parameters, is given in the following Model 1.

**Model 1:**

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to $C_{ij} \in [D_{Lij},\ D_{Rij}]$

$$\sum_{j=1}^{n} X_{ij} \in [a_{Li}, a_{Ri}],\ i = 1, 2, \ldots m$$
$$\sum_{i=1}^{m} X_{ij} \in [b_{Lj}, b_{Rj}],\ j = 1, 2, \ldots n$$
$$\sum_{i=1}^{m} a_{Li=} \sum_{j=1} b_{Lj}$$
$$\sum_{i=1}^{m} a_{Ri=} \sum_{j=1} b_{Rj}$$
$$X_{ij} \geq 0, \quad \forall i, j.$$

where $c_{ij} \in [D_{Lij}, D_{Rij}]$ is an interval representing the uncertain cost for transportation problem. The sources parameter lies in $[a_{Li}, a_{Ri}]$ and destination parameter lies in $[b_{Lj}, b_{Rj}]$. Depending on $[a_{Li}, a_{Ri}]$ and $[b_{Lj}, b_{Rj}]$. We determine $c_{Lij}$ and $c_{Rij}$ which is discussed in the following subsection. Then we define $D_{Lij} = \min\{c_{Lij}, c_{Rij}\}$ and $D_{Rij} = \max\{c_{Lij}, c_{Rij}\}$

### 3.1 Multi Objective Interval transportation problem

The formulation of MITP is the problem of minimizing $k$ interval valued objective functions with interval supply and interval destination parameters is given and an efficient algorithm is presented to find the optimal solution ifMITP.

The mathematical model of MITP when all the cost co-efficient, supply and demand are interval valued is given by:

Minimize $Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[C^k_{Lij},\ C^k_{Rij}]x_{ij}$, where $k = 1, 2, 3, \ldots K$,

Subject to

$\sum_{j=1}^{n} Xij = [a_{Li},\ a_{Ri}], i = 1, 2, 3 \ldots m,$

$\sum_{i=1}^{m} Xij = [b_{Li},\ b_{Ri}], i = 1, 2, 3 \ldots n,$

$xij \geq 0, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n,$

with

$\sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{Lj}$ and $\sum_{i=1}^{m} a_{Ri} = \sum_{j=1}^{n} b_{Rj}$

where the source parameter lies between left limit $a_{Li}$ and right limit $a_{Ri}$. Similarly, destination parameter lies between left limit $b_{Lj}$ and right limit $b_{Rj}$ and $[C^kL_{ij},\ C^kR_{ij}]$, $(k = 1, 2, 3, \ldots, K)$ is an interval indicating the uncertain cost for the transportation problem; it can exemplify delivery time, quality of goods delivered, under used capacity, etc.

### 3.2 Fuzzy programming technique to solve Multi Objective Interval Transportation Problem

In this paper, penalities (transportation cost, delivery time etc.) as membership value through defined membership function. As membership value is higher it is closer to the optimal solution. The decision maker would like to minimize the set of $P$ objectives. Here linear membership function is used and defined as:

$$\mu_1(Z_R) = \begin{bmatrix} 1, & \text{if } Z_R \leq L_R \\ 1 - \dfrac{Z_R - L_R}{U_R - L_R}, & \text{if } L_R \leq Z_R \leq U_R \\ 0, & \text{if } U_R \leq Z_R \end{bmatrix}$$
$$\mu_2(Z_R) = \begin{bmatrix} 1, & \text{if } Z_R \leq L_R \\ 1 - \dfrac{Z_R - L_R}{U_R - L_R}, & \text{if } L_R \leq Z_R \leq U_R \\ 0, & \text{if } U_R \leq Z_R \end{bmatrix} \tag{1}$$

#### 4 Algorithm for solving MOITP

For solving MITP, the proposed method is summarized in the following steps:

Step 1: Construct an Interval Transportation problem (IP).

Step 2: Check whether it is balanced or unbalanced. If it is balanced go to Step 4 otherwise go to next step.

Step 3: Convert the unbalanced Transportation problem into balanced Transportation problem.

Step 4: Construct the Upper bound transportation problem (UP) of the problem (IP) and solve the problem (UP) by Parallel Method [3]. Let { $Y$, for all $i$ and $j$} be an optimal solution to the problem (UP).

Step 5: Construct the lower bound transportation problem (LP) of the given the problem (IP) and solve the problem (LP) with the upper bound constraints, $X_{ij} \leq Y^{0_{ij}}$ for all $i$ and $j$ by the parallel Method (3). Let { $X^{0_{ij}}$ for all $i$ andj} be the optimal solution to the problem (LP) with $X^{0_{ij}} \leq Y^{0_{ij}}$, for all $i$ andj.

Step 6: The optimal solution to the problem (IP) is { $[X^{0_{ij}} \leq Y^{0_{ij}}]$ for all $i$ and $j$}. By the optimal objective value ofthe problem (IP) is $[U_L,\ U_U]$

## 5 Illustration Examples

**Numerical Illustration 1:**
To illustrate the above method, consider the following examples of Multi-Objective transportation problem.

A company has three production facilities (origins) $A_1, A_2$, and $A_3$ with production capacity of 8, 19, 17 units of a product respectively. These units are to be shipped four warehouses $B_1, B_2 B_3$ and $B_4$ with requirement of 11, 3, 14 and 16 units respectively. The transportation cost and time between companies to warehouses are given below:

$$U^{(1)} = \begin{bmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{bmatrix} \quad U^{(2)} = \begin{bmatrix} [3,5] & [2,6] & [5,9] & [4,8] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{bmatrix}$$

Now, the UBITP and LBITP of the given problem is given below:

$$U^{(1)L} = \begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix} \quad U^{(1)U} = \begin{bmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{bmatrix}$$

$$U^{(2)L} = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{bmatrix} \quad U^{(2)U} = \begin{bmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{bmatrix}$$

As the first step we calculate membership value for time. Here $U_R = 7$ and $L_R = 1$ then membership values are as follows:

$$U^{(1)L} = \begin{bmatrix} 1 & 1 & 0.33 & 0.5 \\ 1 & 0.11 & 0.56 & 0.67 \\ 0 & 0 & 0.67 & 0.33 \end{bmatrix}$$

Here $U_R = 9$ and $L_R = 1$ then membership values are as follows:

$$U^{(2)L} = \begin{bmatrix} 0.75 & 0.88 & 0.88 & 1 \\ 0.62 & 0.25 & 0.25 & 10 \\ 0.62 & 1 & 0.75 & 1 \end{bmatrix}$$

Now we calculate minimum membership value

| Destinations/Sources | $B_1$ | $B_2$ | $B_3$ | $B_4$ | |
|---|---|---|---|---|---|
| $A_1$ | 0.75 | 0.88 | 0.33 | 0.5 | 8 |
| $A_2$ | 0.62 | 0 | 0.25 | 0 | 19 |
| $A_3$ | 0 | 0 | 0.67 | 0.33 | 17 |
| | 11 | 13 | 14 | 16 | |

After applying the proposed method we get the solution as follows:
The optimal solution to the LBITP is
$X_{11} = 5, X_{12} = 3, X_{21} = 6, X_{23} = 13, X_{33} = 1, X_{34} = 16$
Next we calculate membership value for cost,
Here $U_R = 11$ and $L_R = 2$ then membership values are as follows:

$$U^{(1)U} = \begin{bmatrix} 1 & 0.89 & 0.22 & 0.33 \\ 1 & 0.11 & 0.56 & 0.67 \\ 0.22 & 0 & 0.67 & 0.45 \end{bmatrix}$$

Here $U_R = 11$ and $L_R = 2$ then membership values are as follows:

$$U^{(2)U} = \begin{bmatrix} 0.67 & 0.56 & 0.78 & 0.67 \\ 0.56 & 0.22 & 0.11 & 0 \\ 0.33 & 0.89 & 0.56 & 1 \end{bmatrix}$$

Now we calculate minimum membership value

| Destinations/Sources | $B_1$ | $B_2$ | $B_3$ | $B_4$ | |
|---|---|---|---|---|---|
| $A_1$ | 0.67 | 0.56 | 0.22 | 0.33 | 8 |
| $A_2$ | 0.56 | 0.11 | 0.11 | 0 | 19 |
| $A_3$ | 0.22 | 0 | 0.56 | 0.45 | 17 |
| | 11 | 13 | 14 | 16 | |

After applying the proposed method we get the solution as follows:
The optimal solution to the UBITP is

$$X_{11} = 8, \quad X_{21} = 3, \quad X_{22} = 3, \quad X_{23} = 13, \quad X_{33} = 1, \quad X_{34} = 112$$

$$U^{(1)L} = 5 \times 1 + 3 \times 1 + 6 \times 1 + 13 \times 2 + 1 \times 3 + 16 \times 5 = 123$$

$$U^{(2)L} = 5 \times 3 + 3 \times 2 + 6 \times 4 + 13 \times 7 + 1 \times 3 + 16 \times 1 = 155$$

$$U^{(1)U} = 8 \times 2 + 3 \times 2 + 3 \times 10 + 13 \times 6 + 1 \times 5 + 16 \times 7 = 247$$

$$U^{(2)U} = 8 \times 5 + 3 \times 6 + 3 \times 9 + 13 \times 10 + 1 \times 6 + 16 \times 2 = 253$$

**Comparison**

| Deepika Rani [4] (Linear membership function) | Deepika Rani [4] (Exponential membership function) | Deepika Rani [4] (Hyperbolic membership function) | Proposed method |
|---|---|---|---|
| $[172.2, 222.55]$ | $[171.50, 221.63]$ | $[172.2, 222.55]$ | $[123, 247]$ |
| $[206.1, 252.75]$ | $[207.54, 254.36]$ | $[206.1, 252.75]$ | $[155, 253]$ |

**Numerical Illustration 2:**
A company has three production facilities (origins) $A_1, A_2$, and $A_3$ with production capacity of 8, 19, 17 units of a product respectively. These units are to be shipped four warehouses $B_1, B_2, B_3$ and $B_4$ with requirement of 11, 3, 14 and 16 units respectively. The transportation cost and time between companies to warehouses are given below:

$$U^{(1)} = \begin{bmatrix} [16,18] & [19,12] & [12,14] & [20,22] \\ [22,24] & [13,15] & [19,21] & [8,10] \\ [14,16] & [28,30] & [8,10] & [5,7] \end{bmatrix} \quad U^{(2)} = \begin{bmatrix} [9,11] & [14,16] & [12,14] & [4,8] \\ [16,18] & [10,12] & [14,18] & [3,5] \\ [8,10] & [20,22] & [6,8] & [5,7] \end{bmatrix}$$

After applying the proposed method we get the solution as follows:
The optimal solution to the LBITP and UBITP is

$$\begin{aligned} X_{11} &= 8, \quad X_{22} = 3, \quad X_{24} = 16, \quad X_{31} = 3, \quad X_{33} = 14 \\ X_{11} &= 8, \quad X_{22} = 3, \quad X_{24} = 16, \quad X_{31} = 3, \quad X_{33} = 14 \\ U^{(1)L} &= 8 \times 16 + 3 \times 13 + 16 \times 8 + 3 \times 14 + 14 \times 8 = 449 \\ U^{(2)L} &= 8 \times 9 + 3 \times 10 + 16 \times 3 + 3 \times 8 + 14 \times 6 = 258 \\ U^{(1)U} &= 8 \times 18 + 3 \times 15 + 16 \times 10 + 3x16 + 14 \times 10 = 537 \\ U^{(2)U} &= 8 \times 11 + 3 \times 12 + 16 \times 5 + 3 \times 10 + 14 \times 8 = 346 \end{aligned}$$

**Comparison**

| Dr. M. S. Annie Christi [7] (Linear membership function) | Dr. M. S. Annie Christi [7] (Exponential membership function) | Dr. M. S. Annie Christi [7] (Hyperbolic membership function) | Proposed method |
|---|---|---|---|
| $[586, 354]$ | $[586, 354]$ | $[586, 354]$ | $[442, 354]$ |

## 6 Conclusion

In this paper an algorithm is developed for a MOITP and this approach is used to find the compromise solution of Multi-objective interval transportation problem obtained using fuzzy programming technique with linear membership functions. Two Numerical examples are illustrated and obtained results compared with some of the methods in literature. The comparison shows that the compromise solution is better and acceptable in real life situation when more than one objective available in transporting a product.

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