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Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits Having Two Types of Policy Decisions

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Abstract

In this paper, the problem of time to recruitment is studied for a single grade manpower system in which attrition takes place due to two types of policy decisions where this classification is done according to the intensity of attrition. It is assumed that policy decisions and exits occur at different epochs and the exit time process is an ordinary renewal process. A stochastic model is constructed and the system characteristics mean and variance of time to recruitment are obtained using an univariate policy of recruitment when the policy decision times form an ordinary renewal process or a geometric process or an order statistics. Employing a different method, analytical results in closed form for system characteristics are derived by assuming specific distribution for loss of manpower and its breakdown threshold.

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Key Words: Single grade manpower system, decision and exit epochs, intensity of attrition, ordinary renewal process, geometric process, order statistics, univariate cum policy of recruitment and variance of time to recruitment.

1 Introduction

Attrition is a common phenomenon in many marketing organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the breakdown threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. Many researchers have studied several problems in manpower planning using different methods. In [1] and [2], the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [18], the authors have initiated the study on the problem of time to recruitment for the single grade manpower system which is subject to attrition, using the univariate policy of recruitment based on shock model approach for replacement of systems in reliability theory. In [15], the author has obtained the system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequences. In [17], the author has studied this problem assuming geometric process and order statistics for inter-decision times. In [14], the authors have studied the problem of time to recruitment by assuming that the attrition is generated by a geometric process of inter- decision times using a different probabilistic analysis. In [12], the author has initiated the study of the problem of time to recruitment by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. While in one stochastic model described in [12], the author has obtained the variance of time to recruitment when the inter-decision times form a geometric process, in [20], the authors have studied the problem when the inter-decision times form an order statistics. Assuming that the decision epochs and the exit points at which the actual attrition takes place are different, the authors in [3], [5], [9] and [10] have respectively obtained the variance of the time to recruitment using Laplace transform technique according as the inter-decision times form an ordinary renewal process or a sequence of exchangeable and constantly correlated exponential random variables or a geometric process or an order statistics. In [4], [6], [16] and [7] the authors have studied the research work in [3], [5], [9] and [10] using a different probabilistic analysis. In [8], the authors have studied the results of [4], [6], [16], and [7] using univariate Max policy of recruitment. Recently, in [11] the authors have studied the work in [8] when the breakdown threshold level for the maximum loss of manpower is the sum of the exponential breakdown threshold levels of wastage and frequent breaks of existing workers. The present paper extends the research work in [4], [16] and [7] when the policy decisions are of two types, one with low attrition rate and the other with high rate of attrition.

2 Model Description and Analysis

Consider an organization taking policy decisions at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization.

There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For $i = 1, 2, 3$, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} exit point with probability distribution function $M(.)$, and mean $\frac{1}{\alpha}$ $(\alpha > 0)$. Let Si be the cumulative loss of manpower in the first i exit points and m_i be its probability density function. The policy decisions which produce depletion of manpower are classified into two types depending upon the intensity of attrition. It is assumed that the first type of policy decisions has high attrition rate $\lambda_1(\lambda_1 > 0)$ and the second type has low attrition rate $\lambda_2(\lambda_2 > 0)$. Let a_1 and $(1 - a_1)$ be the proportion of decisions with high and low attrition rate respectively. Let A_i be the time between $(i-1)^{th}$ and i^{th} policy decisions, forming a sequence of independent random variables. Let B_i be the time between $(i-1)^{th}$ and i^{th} exit times, forming a sequence of independent and identically distributed random variables with probability distribution function $G(.)$ and density function g(.). Let D_{i+1} be the waiting time upto $(i+1)$ exits. Let Y be the independent exponential threshold level for the depletion of manpower in the organization with probability distribution function $H(.)$ and density function $h(.)$. Let q be the probability that every policy decision has exit of personnel. As $q = 0$ corresponds to the case where exits are impossible, it is assumed that $q \neq 0$. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean $E(T)$ and variance $V(T)$. The univariate cum policy of recruitment employed in this paper is stated as follows: *Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the threshold for the loss of man hours in this organization. Proceeding with the arguments discussed in [4], it is clear that*

$$T = \sum_{i=0}^{\infty} D_{i+1}\chi(S_i \leq Y < S_{i+1}) \tag{1}$$

From (1) *and from the definition of* D_{i+1}, *we get*

$$E(T) = \sum_{i=0}^{\infty}(i+1)E(B)P(S_i \leq Y < S_{i+1}) \tag{2}$$

and

$$E(T^2) = \sum_{i=0}^{\infty}(i+1)[V(B) + (i+1)E^2(B)]P(S_i \leq Y < S_{i+1}) \tag{3}$$

By using law of total probability we get

$$P(S_i \leq Y < S_{i+1}) = \int_0^{\infty}\int_0^{t} \bar{M}(t-x)m_i(x)h(t)dxdt \tag{4}$$

We now obtain explicit analytical expressions for $E(T)$ *and* $V(T)$ *by considering different cases for inter-decision times.*

Case (i): *The hyper-exponential distribution* $F(x) = a1(e^{-\lambda_2 x} - e^{-\lambda_1 x}) + (1 - e^{-\lambda_2 x})$,

$\lambda_1, \lambda_2 > 0,\ x > 0$ *be the common distribution of* $A_i,\ i = 1, 2, ...$
From (2), (3) *and* (4) *we get*

$$E(T) = \frac{(\alpha + \theta)(a_1(\lambda_2 - \lambda_1) + \lambda_1)}{\lambda_1\lambda_2\theta q} \tag{5}$$

and

$$V(T) = \frac{\theta(\alpha+\theta)[2q(a_1(\lambda_2^2-\lambda_1^2)+\lambda_1^2)+(1-2q)(a_1(\lambda_2-\lambda_1)+\lambda_1)^2]}{(\lambda_1\lambda_2\theta q)^2} + \frac{(\alpha+\theta)(2\alpha+\theta)[a_1(\lambda_2-\lambda_1)+\lambda_1]^2}{(\lambda_1\lambda_2\theta q)^2} - \frac{(\alpha+\theta)^2[a_1(\lambda_2-\lambda_1)+\lambda_1]^2}{(\lambda_1\lambda_2\theta q)^2} \tag{6}$$

(5) *and* (6) *give the mean and variance of the time to recruitment for the present case.*

Case (ii): $\{A_i\}_{i=1}^{\infty}$*form a geometric process with rate c,* $(c > 0)$ *and the distribution of* A_1 *is the hyper-exponential distribution mentioned in case (i).*
In this case from (2),(3),(4) *and on simplification we get*

$$E(T) = \frac{(\alpha + \theta)(c[a_1(\lambda_2 - \lambda_1) + \lambda_1])}{\lambda_1\lambda_2\theta(c - 1 + q)} \tag{7}$$

and

$$V(T) = \left[\frac{(\alpha + \theta)}{\theta}\right]\left[\frac{[c^2(2(a_1(\lambda_2^2-\lambda_1^2)+\lambda_1^2))-(a_1(\lambda_2-\lambda_1)+\lambda_1)^2]}{(c^2-1+q)(\lambda_1\lambda_2)^2} + \frac{c^2 q\bar{q}(a_1(\lambda_2-\lambda_1)+\lambda_1)^2}{(c-1+q)^2(c^2-1+q)(\lambda_1\lambda_2)^2}\right] + \left[\frac{\alpha(\alpha+\theta)}{\theta^2}\right]\left[\frac{c^2(a_1(\lambda_2-\lambda_1)+\lambda_1)^2}{(c-1+q)^2(\lambda_1\lambda_2)^2}\right] \tag{8}$$

(7) *and* (8) *give the mean and variance of the time to recruitment for the present case.*

Case (iii): *Inter-policy decision times form an hyper-exponential population.*
A sample of size r is selected from the population. Let $F_{a(j)}(.)$ *and* $f_{a(j)}(.)$ *be the distribution and the probability density function of the* j^{th} *order statistics* $(j = 1, 2, \ldots, r)$ *selected from the sample of size r from the hyper-exponential population* $\{A_{i=1}\}^{\infty}$.
From the theory of order statistics [19], it is known that
$f_{a(j)}(t) = j[F(t)]^{j-1} f(t), j = 1, .., r.$
Suppose

$$f(t) = f_{a(1)}(t) = r[1-F(t)]^{r-1} f(t), where f(t) = a_1(\lambda_1 e^{-\lambda_1 x} - \lambda_2 e^{-\lambda_2 x}) + \lambda_2 e^{-\lambda_2 x} \tag{9}$$

Then from (2), (3) *and* (4) *and on simplification it is found that*

$$E(T) = \left(\frac{r(\alpha + \theta)}{\theta q}\right) K_1 + r(1 - r)\left(\frac{\alpha + \theta}{\theta q}\right) K_2 \tag{10}$$

where

$$K_1 = \left(\frac{a_1(\lambda_2 - \lambda_1) + \lambda_1}{\lambda_1\lambda_2}\right),\ K_2 = \frac{\lambda_1^2\lambda_2(3a_1^2+6a_1-2)-\lambda_1\lambda_2^2(2a_1^2-5a_1-1)+\lambda_1^3(a_1-1)-a_1^2\lambda_2^2+4a_1\lambda_2^3}{4\lambda_1\lambda_2(\lambda_1\lambda_2)^2} \tag{11}$$

and

$$V(T) = \frac{\begin{array}{r} r(\alpha+\theta)\theta q(K_3+K_4) - r^2\theta q(\alpha+\theta)[K_4 + K_1^2K_5^2(1-2r+r^2)] \\ +r^2K_1^2[4\alpha\theta - \alpha\theta q + \theta^2(2-q) + 2\alpha - (\alpha+\theta)]^2 \\ +r^2(1-r)^2K_2^2[4\alpha\theta - \alpha\theta q + \theta^2(2-q) \\ +2\alpha^2 - (\alpha+\theta)^2] + [2r^2(1-r)K_1K_2][4\alpha\theta - \alpha\theta q + \theta^2(2-q) + 2\alpha^2] \end{array}}{\theta^2q^2} \quad (12)$$

where K_1, K_2 *give* (11), $K_3 = \dfrac{2a_1(\lambda_2^2-\lambda_1^2)+\lambda_1^2}{(\lambda_1\lambda_2)^2}$,

$K_4 = \dfrac{8a_1^2\lambda_1^3\lambda_2^2 + [8a_1\lambda_2^2 - a_1^2\lambda_2^2 + a_1\lambda_1^2 - \lambda_1^2](\lambda_1+\lambda_2)^3 - 8a_1\lambda_1^2\lambda_2^2(\lambda_1+\lambda_2)(1-\lambda_2)}{4(\lambda_1\lambda_2)^2(\lambda_1+\lambda_2)^3}$ *and*

$$K_5 = \frac{\lambda_1^2\lambda_2[3a_1^2+4a_1+2(a_1-1)] - \lambda_1\lambda_2^2[1+2a_1^2-5a_1] + a_1\lambda_2^2[4\lambda_2-a_1] + \lambda_1^3(a_1-1)}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2} \quad (13)$$

(10) *and* (12) *give the mean and variance of the time to recruitment when* $f(t) = f_{a(1)}(t)$.
Suppose

$$f(t) = f_{a(r)}(t) = r[F(t)]^{r-1}f(t), \text{ where } f(t) = a_1(\lambda_1e^{-\lambda_1x} - \lambda_2e^{-\lambda_2x}) + \lambda_2e^{-\lambda_2x} \quad (14)$$

Then from (2), (3), (4) *and on simplification it is found that*

$$E(T) = \left(\frac{r(\alpha+\theta)}{\theta q}\right)K_1 + \left(\frac{r(1-r)(\alpha+\theta)}{\theta q}\right)K_6 \quad (15)$$

where

$$K_1 = \frac{a_1(\lambda_2-\lambda_1)+\lambda_1}{\lambda_1\lambda_2} \text{ and } K_6 = \frac{4a_1\lambda_1\lambda_2(1+a_1) - a_1\lambda_2(\lambda_1+\lambda_2)^2 - (1+3a_1)\lambda_1(\lambda_1+\lambda_2)^2}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2} \quad (16)$$

and

$$V(T) = \frac{\begin{array}{r} r(\alpha+\theta)\theta q(2K_6+K_7) - r^2(\alpha+\theta)\theta q[K_7 - (K_1)^2 - (K_6)^2(1+r^2-2r)] \\ +r^2(K_1)^2[4\alpha\theta - \alpha\theta q + 2\theta^2 - \theta^2q + 2\alpha^2 - (\alpha+\theta)^2] \\ +r^2(1-r)^2(K_6)^2[4\alpha\theta - \alpha\theta q + 2\theta^2 - \theta^2q + 2\alpha^2 - (\alpha+\theta)^2] \\ +2r^2(1-r)K_1K_6[4\alpha\theta - \alpha\theta q + 2\theta^2 - \theta^2q + 2\alpha^2] \end{array}}{\theta^2q^2} \quad (17)$$

where K_1, K_2, K_3, K_4, K_5, K_6 *give* (11), (13) *and* (16) *and*

$$K7 = \frac{16a_1(1+a_1)\lambda_1^3\lambda_2^2 - 4a_1\lambda_2^2(\lambda_1+\lambda_2)^3 - 4\lambda_1^2(\lambda_1+\lambda_2)^3(a_1+1)}{(\lambda_1+\lambda_2)^3(2\lambda_1)^2(2\lambda_2)^2} \quad (18)$$

(15) *and* (17) *give the mean and variance of the time to recruitment when* $f(t) = f_{a(r)}(t)$.

Note:

(i) When $a_1 = 0,\ \lambda_2 = \lambda$, *our results agree with the results in [4] and our results are also consistent with those in [13] when* $q = 1$ *for case(i).*

(ii) When $a_1 = 0,\ \lambda_2 = \lambda$, *our results agree with the results in [16]. When* $a_1 = 0, \lambda_2 = \lambda,\ c = 1$ *our results agree with those in [4]. Our results are also consistent with those in [14] when* $a_1 = 0,\ \lambda_2 = \lambda,\ q = 1$. *When* $q = 1, c = 1$, *our results agree with the results in [13] for case(ii).*

(iii) When $a_1 = 0,\ \lambda_2 = \lambda$, *our results agree with the results in [7]. When* $a_1 = 0, \lambda_2 = \lambda,\ r = 1$, *our results agree with those in [4]. Our results are also consistent with those in [17] when* $a_1 = 0,\ \lambda_2 = \lambda,\ q = 1$ *using Laplace transform technique for case(iii). Computation of* $E(T)$ *and* $V(T)$ *for extended exponential and* $SCBZ$ *property possessing thresholds is similar as their distribution will have just additional exponential terms.*

3 Conclusion

The models discussed in this paper are found to be more realistic in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. (iii) Considering the policy decisions are classified into two types according to their intensity of attrition, one with high rate of attrition and the other with low rate of attrition. In the context of attrition, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. Further, the observations on the performance measures given in this paper will be useful to enhance the facilitation of the assessment of the manpower profile in future manpower development prediction, not only in industry but also in a broader domain.

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