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Bounds on the 2−domination number of a fuzzy graph

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Abstract

In this paper, the upper and lower bounds of $\gamma_2(G)$, the 2−domination number of a fuzzy graph G are given. A condition for a node to be in every 2−dominating set of a fuzzy graph is given. We also prove some results on 2−domination number of a fuzzy graph.

AMS Subject Classification: 03E72, 05C72
Key Words and Phrases: Strong neighbours, 2−dominating set, 2−domination number, Strong adjacency nodes of matrix.

1 Introduction

In graphs, the study of domination was started by Ore and Berge [1, 10]. The domination number and the independent domination number were introduced by Cockayne and Hedetniemi [2]. In the year 1985, the $n-$ domination in graphs was introduced by Fink and Jacobson [3]. The concept of fuzzy relation was introduced by Zadeh [13] in his classical paper in 1965. Rosenfeld [11] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness.
Using effective edges, A.Somasundram and S.Somasundram [12] discussed domination in fuzzy graphs. Nagoor Gani and Chandrasekaran [4] discussed domination in fuzzy graph using strong arcs. Nagoor Gani and Vadivel [8][9] discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. Nagoor Gani and Prasanna Devi [7] discussed edge domination and edge independence in fuzzy graphs. Nagoor Gani and Prasanna Devi [6] also discussed about 2−domination in fuzzy graphs in the year 2015. 2− bondage number of a fuzzy graph was discussed by Nagoor Gani and Prasanna Devi [5].

2 Preliminaries

A *fuzzy graph* G $= \langle \sigma, \mu \rangle$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all $x, y \in V$, we have $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$. A fuzzy graph H $= \langle \tau, \rho \rangle$ is called a *fuzzy subgraph* of G if $\tau(v_i) \leq \sigma(v_i)$ for all $v_i \in V$ and $\rho(v_i, v_j) \leq \mu(v_i, v_j)$ for all $v_i, v_j \in V$. The *underlying crisp graph* of a fuzzy graph G= $\langle \sigma, \mu \rangle$ is denoted by $G^* = \langle \sigma^*, \mu^* \rangle$, where $\sigma^* = \{v_i \in V/\sigma(v_i) > 0\}$ and $\mu^* = \{(v_i, v_j) \in V \times V/\mu(v_i, v_j) > 0\}$. An edge in G is called an *isolated edge* if it is not adjacent to any edge in G. A node in G is called an *isolated node* if it is not adjacent to any node in G.

An arc (x, y) in a fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be *strong* if $\mu^\infty(x,y) = \mu(x,y)$ then x, y are called *strong neighbours*. The *strong neighbourhood* of the node u is defined as $N_S(u) = \{v \in V : (u,v) \text{is a strong arc}\}$. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in V - D$, there exist $u \in D$ such that u dominates v. The *domination number*, $\gamma(G)$, of a fuzzy graph G, is the smallest number of nodes in any dominating set of G.

A subset D of V is called a *2−dominating set* of a fuzzy graph G if for every node $v \in V - D$ there exist atleast two strong neighbours in D. The *2−domination number* of a fuzzy graph G denoted by $\gamma_2(G)$, is the minimum cardinality of a 2−dominating set of G. Let G be a fuzzy graph with n nodes. Then the *strong adjacency nodes of matrix* B is a n×n matrix whose entries are defined as

$$b_{ij} = 1, \qquad if \quad i = j$$

$$b_{ij} = \begin{cases} 1, & if \;\; v_i, v_j \;\; are \;\; strong \;\; neighbours \\ 0, & otherwise \;\; if \;\; i \neq j \end{cases}$$

In this paper we discuss about the upper and lower bounds of $\gamma_2(G)$, the 2− domination number of a fuzzy graph G and also some results on 2−domination number of a fuzzy graph.

3 Results on 2-domination number

In this section we discuss about the upper and lower bounds of the 2−domination number of a fuzzy graph G. Also a condition for a node to be in every 2−dominating set of a fuzzy graph is given.

Theorem 1. *Let G be a fuzzy graph and B be the strong adjacency nodes of matrix of G. If the number of 1′s in a row is atmost 2 in B then the corresponding node belongs to every 2−dominating set of G.*

Proof. Let G be a fuzzy graph and B be its strong adjacency nodes of matrix.
Case (i): Let the number of $1'$s in a row be one.

Since the number of $1'$s in a row is one then the node corresponding to that row is an isolated node. An isolated node is always dominated by itself only. That is every isolated node belongs to every $2-$dominating set.

Thus if the number of $1'$s in a row is one then the corresponding node belongs to every $2-$ dominating set of G.

Case (ii): Let the number of $1'$s in a row be two.

Since the number of $1'$s in a row is two then the node $'u'$ corresponding to this row has $\mid N_S(u) \mid = 1$. And we know that if $\mid N_S(v) \mid \leq 1$ then v belongs to every $2-$dominating set of G. Therefore u belongs to every $2-$dominating set of G.

Thus if the number of $1'$s in a row is two then the corresponding node belongs to every $2-$dominating set of G.

Thus if number of $1'$s in a row is atmost 2 in B then the corresponding node belongs to every $2-$dominating set of G. □

Theorem 2. *For any fuzzy graph G with n nodes,* $\gamma_2(G) \geq n/(1 + \triangle_S(G))$.

Proof. For any fuzzy graph G, we know that $\gamma_2(G) \geq \gamma(G)$ and the lower bound of the domination number $\gamma(G)$ is $n/(1 + \triangle_S(G))$. i.e., $\gamma(G) \geq n/(1 + \triangle_S(G))$

$\Rightarrow \gamma_2(G) \geq \gamma(G) \geq n/(1 + \triangle_S(G))$

$\Rightarrow \gamma_2(G) \geq n/(1 + \triangle_S(G))$

□

Remark 3. *For any fuzzy graph G with n nodes then* $n/(1+\triangle_S(G)) \leq \gamma_2(G) \leq n$.

Proof. From the above theorem, $n/(1 + \triangle_S(G)) \leq \gamma_2(G)$ and since there are n nodes in the fuzzy graph G, $\gamma_2(G) \leq n$.

$\Rightarrow n/(1 + \triangle_S(G)) \leq \gamma_2(G) \leq n$. □

Theorem 4. *For any fuzzy graph G with n$\geq$ 2 nodes,* $\gamma_2(G) = 2$ *iff G must have 2 nodes which are strong neighbours to every other nodes in G.*

Proof. Let G be a fuzzy graph with $n \geq 2$ nodes.

Now let us assume that $\gamma_2(G) = 2$.

Let $D = \{u, v\}$ be the minimum $2-$dominating set of the given fuzzy graph G. Let $z \in V - D$, then z should be dominated by both u and v i.e., Both u and v are strong neighbours to z.

$\Rightarrow$ u and v are strong neighbours to every other nodes of G.

$\Rightarrow$ G have 2 nodes which are strong neighbours to every other nodes of G.

Conversely assume that G have 2 nodes which are strong neighbours to every other nodes of G.

Then these 2 nodes will form a $2-$dominating set of G which is also a minimum $2-$dominating set.

$\Rightarrow \gamma_2(G) = 2$. □

Theorem 5. *For any fuzzy graph G with n$\geq$ 2 nodes then* $\gamma_2(G) \geq 2$.

Proof. Let G be fuzzy graph with n$\geq$ 2 nodes.

Let D be a minimum $2-$dominating set of G then $\mid D \mid = \gamma_2(G)$. For every node $u \in V - D$ there exist atleast 2 strong neighbours in D.

$\Rightarrow$ D should have atleast 2 nodes.

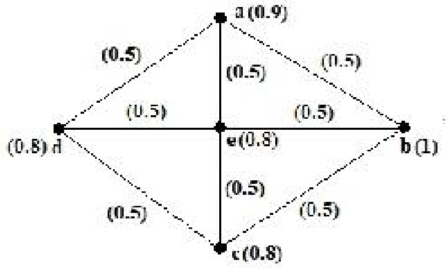
$\Rightarrow \mid D \mid \geq 2$
$\Rightarrow \gamma_2(G) \geq 2.$ □

Theorem 6. *If there are p nodes with number of strong neighbours less than or equal to 1 then $\gamma_2(G) \geq p$.*

Proof. Let G be a fuzzy graph with n nodes. Let $u \in V$ with $\mid N_S(u) \mid \leq 1$ and G has p such nodes.

We know that if $\mid N_S(u) \mid \leq 1$ then u belongs to every 2−dominating set of G. These p nodes belongs to every 2−dominating set of the given fuzzy graph G. i.e., these p nodes belongs to every minimum 2−dominaing set of G.
$\Rightarrow \gamma_2(G) \geq p.$ □

Remark 7. *Let G be a fuzzy graph with n nodes. If every node in G has atleast 2 strong neighbours, then the domination number $\gamma(G)$ need not be equal to the 2−domination number $\gamma_2(G)$ i.e., If $N_S(u) \geq 2$ then every dominating set in G need not be a 2−dominating set in G i.e., $\Rightarrow \gamma_2(G)$ need not be equal to $\gamma(G)$. Let us discuss this with examples.*

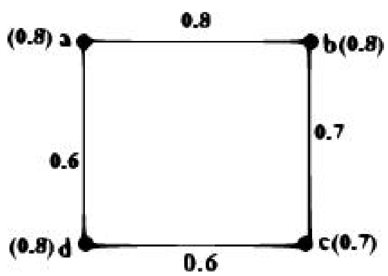


(G)
Fig.1

Here in G, all arcs are strong arcs and
$N_S(a) = N_S(b) = N_S(c) = N_S(d) = 3, N_S(e) = 4$
Therefore $N_S(u) \geq 2, \forall u \in V$, where $V = \{a, b, c, d, e\}$.

$\{e\}$ is a minimum dominating set of G but it is not a 2−dominating set of G and $\gamma(G) = 1$.

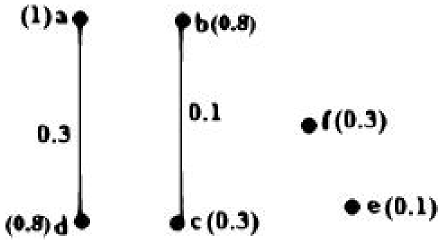
$\{a, c\}, \{b, d\}$ are minimum 2−dominating set of G and also a dominating set of G. Here $\gamma_2(G) = 2$.
Thus we have $\gamma(G) \neq \gamma_2(G)$



(G)
Fig.2

Here in G, all arcs are strong arcs and
$N_S(a) = N_S(b) = N_S(c) = N_S(d) = 2$.
$\{a, c\}, \{b, d\}$ are all minimum dominating set of G which is also a minimum $2-$dominating set of G. Thus $\gamma(G) = \gamma_2(G)$.

Remark 8. *If G is a totally disconnected fuzzy graph with n nodes then $\gamma_2(G) = n$. But the converse is not true i.e., if $\gamma_2(G) = n$ then all the nodes of G are need not to be isolated and hence G is need not to be a totally disconnected fuzzy graph. Let us discuss with an example.*



(G)
Fig.3

Here $\gamma_2(G) = n = 6$.
But all the nodes of are not isolated and G is not a totally disconnected fuzzy graph.

4 Conclusion

We have given the upper and lower bounds of the $2-$domination number $\gamma_2(G)$ of a fuzzy graph G. A condition for a node to be in every $2-$dominating set of a fuzzy graph is also given. We have proved some results on $2-$domination number of a fuzzy graph using some suitable examples.

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