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# $4-$Remainder Cordial Labeling of Some Special Graphs

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**Abstract**

Let $G$ be a $(p,q)$ graph. Let $f : V(G) \to \{1,2,3,4\}$ be a $1-1$ map. For each edge $uv$ assign the label $r$ where $r$ is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function $f$ is called a remainder cordial labeling of $G$ if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with even integers and number of edges labelled with odd integers. A graph $G$ with admits a remainder cordial labeling is called a remainder cordial graph. In this paper we present the $4-$remainder cordial labeling behavior of Flower graph, Sunflower graph and Subdivision of Ladder graph,etc,.

**AMS Subject Classification:** 05C78

**Key Words and Phrases:** Flower graph, Sunflower graph, Ladder and Subdivision of Ladder graph

## 1 Introduction

Graphs considered here are finite and simple. Let $G_1$ and $G_2$ be two graphs with vertex sets $V_1$ and $V_2$ and edge sets $E_1$ and $E_2$ respectively. Let $G_1$ and $G_2$ be two graphs with vertex sets $V_1$ and $V_2$ and edge sets $E_1$ and $E_2$ respectively. Then their join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$. The graph $W_n = C_n + K_1$ is called a wheel. In a wheel, a vertex of degree 3 is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with the rim and the other incident with the central vertex are called spokes. Any graph derived from a graph $G$ by a sequence of edge subdivisions is called a subdivision of $G$ or a $G$-subdivision. Ponraj et al. [4], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of path, cycle, star, bistar, complete graph, etc, in [6]. and also the concept of $k$-remainder cordial labeling introduced in [6] and investigate the 4-remainder cordial labeling behaviour of certain graphs. In this paper we present the $4-$remainder cordial labeling behavior of Flower graph, Sunflower graph, and Subdivision of Ladder graph. Terms are not defined here follows from Harary [3] and Gallian [2].

## 2 Remainder cordial labeling

**Definition 1.** Let $G$ be a $(p, q)$ graph. Let $f : V(G) \rightarrow \{1, 2, 3, 4\}$ be an injective map. For each edge $uv$ assign the label $r$ where $r$ is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function $f$ is called a remainder cordial labeling of $G$ if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with even integers and number of edges labelled with odd integers. A graph $G$ with a remainder cordial labeling is called a remainder cordial graph.

First we investigate the $4-$remainder cordial labeling behaviour of the Flower graph.

A flower graph $Fl_n$ as the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

**Theorem 2.** *If the flower graph $Fl_n$ is $4-$remainder cordial for all $n$.*

*Proof.* We consider the central vertex $v_0$ of $Fl_n$ graph and assign the label 3 to the central vertex $v_0$ in the following two cases.

**Case(i).** $n$ is even.

Assign the labels $2, 3$ alternatively to the vertices $v_1, v_2, \ldots, v_n$. Next consider the vertices $w_i$ of degree two. Assign the labels $4, 1$ alternatively to the first two vertices

$w_1$ and $w_2$. Next assign the labels $4, 1$ alternatively to the next two vertices $w_3$ and $w_4$. Proceeding like this until we reach the vertex $w_n$. Clearly in this process the vertex $w_n$ received the label 1.

**Case(ii).** $n$ is odd.

As in case(i), assign the labels to the vertices $v_i$ , $w_i, (1 \leq i \leq n-1)$. Next assign the labels $2, 4$ respectively to the vertices $v_n$ and $w_n$. Thus the vertex labeling $f$ is 4-remainder cordial labeling follows from the table 1.

| Nature of $n$ | $v_f(1)$ | $v_f(2)$ | $v_f(3)$ | $v_f(4)$ | $e_f(0)$ | $e_f(1)$ |
|---|---|---|---|---|---|---|
| $n$ is even | $\frac{n}{2}$ | $\frac{n}{2}$ | $\frac{n}{2}+1$ | $\frac{n}{2}$ | $2n$ | $2n$ |
| $n$ is odd | $\frac{n-1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $2n$ | $2n$ |

Table 1:

□

Next we investigate the $4-$remainder cordial labeling behavior of the Sunflower graph.

The Sunflower graph $S_n$ is a graph which is obtained by taking a wheel $W_n$ with central vertex $v_0$ and the cycle $C_n : v_1v_2 \ldots v_nv_1$ and new vertices $w_1, w_2, \ldots, w_n$ where $w_i$ is joined by vertices $v_i, v_{i+1}(modn)$.

**Theorem 3.** *The sunflower graph $S_n$ is $4-$remainder cordial for all $n$.*

*Proof.* **Case(i).** $n$ is even.

Consider the central vertex $v_0$ $S_n$ graph and assign the label 3 to the central vertex $v_0$. We now move to the rim vertices $v_i$. Assign the labels $2, 3$ alternatively to the vertices $v_1, v_2, \ldots, v_n$. Next assign the label 1 to the $\frac{n}{2}$ vertices $w_1, w_2, \ldots, w_{\frac{n}{2}}$. Next assign the labels 4 to the remaining non-labelled vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \ldots, w_n$.

**Case(ii).** $n$ is odd.

In this case assign the label 4 to the central vertex $v_0$. Next assign the labels to the vertices $v_i$ , $w_i, (1 \leq i \leq n-1)$ as in case(i). Finally assign the labels 2 and 3 respectively to the vertices $v_n$ and $w_n$. Thus the table 2 given below, establish that this vertex labeling $f$ is 4-remainder cordial labeling.

□

We now investigate the $4-$remainder cordial cordial labeling behaviour of the lotus inside a circle graph.

| Nature of $n$ | $v_f(1)$ | $v_f(2)$ | $v_f(3)$ | $v_f(4)$ | $e_f(0)$ | $e_f(1)$ |
|---|---|---|---|---|---|---|
| $n$ is even | $\frac{n}{2}$ | $\frac{n}{2}$ | $\frac{n}{2}+1$ | $\frac{n}{2}$ | $2n$ | $2n$ |
| $n$ is odd | $\frac{n-1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $2n$ | $2n$ |

Table 2:

The lotus inside a circle graph $LC_n$ is a graph obtained from the cycle $C_n : u_1u_2 \dots u_nu_1$ and a star $K_{1,n}$ with central vertex $v_0$ and the end vertices $v_1, v_2, \dots, v_n$ by joining each $v_i$ to the vertices $u_i$ and $u_{i+1}(modn)$.

**Theorem 4.** *The $LC_n$ graph is $4-$remainder cordial for all $n$.*

*Proof.* Assign the label 3 to the vertex $v_0$ . The proof is divided into two cases.

**Case(i).** $n$ is even.

Assign the label 2 to the vertices $v_1, v_3, \dots, v_{n-1}$ and assign the label 1 to the vertices $v_2, v_4, \dots, v_n$. Next consider the vertices $u_i$. Assign the label 3 to the vertices $u_1, u_3, \dots, u_{n-1}$ and assign the label 4 to the vertices $u_2, u_4, \dots, u_n$.

**Case(ii).** $n$ is odd.

As in case(i), assign the labels to the vertices $v_i$ , $u_i, (1 \le i \le n-1)$. Next assign the labels 2 and 4 to the vertices $v_n$ and $u_n$ respectively. Thus the table 3 establish that this vertex labeling $f$ is 4-remainder cordial labeling.

| Nature of $n$ | $v_f(1)$ | $v_f(2)$ | $v_f(3)$ | $v_f(4)$ | $e_f(0)$ | $e_f(1)$ |
|---|---|---|---|---|---|---|
| $n$ is even | $\frac{n}{2}$ | $\frac{n}{2}$ | $\frac{n}{2}+1$ | $\frac{n}{2}$ | $2n$ | $2n$ |
| $n$ is odd | $\frac{n-1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $\frac{n+1}{2}$ | $2n$ | $2n$ |

Table 3:

For illustration, 4-remainder cordial labeling of $LC_6$ is shown in Figure **??**.

□

Next we investigate the $4-$remainder cordial labeling behavior of the subdivision of ladder $S(L_n)$ graph.

**Theorem 5.** *The subdivision of ladder is $4-$remainder cordial.*

*Proof.* Let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_iv_i : 1 \le i \le n\}$. Let $x_i, y_i$ and $z_i$ be the vertices which are subdivide the edges $u_iu_{i+1}, v_iv_{i+1}$ and $u_iv_i$. We now give a $4-$remainder cordial labeling as given below. The proof is divided into four cases depends on the nature of $n$.

**Case(i).** $n \equiv 0 \pmod 4$
Assign the labels $1, 2, 4, 3$ respectively to the vertices $u_1, u_2, u_3$ and $u_4$. Next assign the labels $1, 2, 4, 3$ respectively to the next four vertices $u_5, u_6, u_7$ and $u_8$. Continue in this pattern until reach the vertex $u_n$. We now consider the vertices $v_1, v_2, \ldots, v_n$. Assign the labels to the vertices $v_i$ as in $u_i (1 \leq i \leq n)$. Next assign the labels $1, 3, 2, 4$ respectively to the vertices $x_1, x_2, x_3$ and $x_4$. Similarly assign the labels $1, 3, 2, 4$ to the next four vertices $x_5, x_6, x_7$ and $x_8$ respectively. Proceeding like this until reach the vertex $x_{n-4}$. Finally assign the labels $1, 3, 2$ respectively to the vertices $x_{n-3}, x_{n-2}, x_{n-1}$. Assign the labels to the vertices $y_i$ as in $x_i (1 \leq i \leq n-4)$ and assign the labels $4, 3$ and $2$ respectively to the vertices $y_{n-3}, y_{n-2}$ and $y_{n-1}$. Finally assign the labels to the vertices $z_1, z_2, \ldots, z_n$ as in the pattern $4, 3, 1, 2; 4, 3, 1, 2; \ldots; 4, 3, 1, 2$.

**Case(ii).** $n \equiv 1 \pmod 4$
As in Case(i), assign the labels to the vertices $u_i, v_i, z_i : (1 \leq i \leq n-1)$ and $x_i : (1 \leq i \leq n-2)$. Next assign the labels to the vertices $y_1, y_2, \ldots, y_n$ as in the pattern $1, 3, 2, 4; 1, 3, 2, 4; \ldots; 3, 3, 2, 4$. Finally assign the labels $1, 1, 4, 4$ respectively to the vertices $u_n, v_n, z_n, x_{n-1}$.

**Case(iii).** $n \equiv 2 \pmod 4$
As in the case(i), assign the labels to the vertices $u_i, v_i, z_i : (1 \leq i \leq n-2)$ and $x_i, y_i : (1 \leq i \leq n-3)$. Finally assign the labels $1, 3; 1, 3; 4, 2$ to the vertices $u_{n-1}, u_n; v_{n-1}, v_{n; z_{n-1}, z_n}$. Assign the labels $4, 4; 4, 2$ respectively to the vertices $x_{n-2}, x_{n-1}; y_{n-2}, y_{n-1}$.

**Case(iv).** $n \equiv 3 \pmod 4$
As in the case(ii), assign the labels to the vertices $u_i, v_i, z_i : (1 \leq i \leq n-2)$ and $x_i : (1 \leq i \leq n-3)$. Finally assign the labels $2, 4; 2, 4; 3, 1$ to the vertices $u_{n-1}, u_n; v_{n-1}, v_{n; z_{n-1}, z_n}$. Next assign the labels $1, 3, 2, 4$ respectively to the vertices $y_1, y_2, y_3$ and $y_4$. Similarly assign the labels $1, 3, 2, 4$ to the next four vertices $y_5, y_6, y_7$ and $y_8$ respectively. Proceeding like this until reach the vertex $y_{n-7}$. Finally assign the labels $3, 3, 2, 4, 2, 3$ respectively to the vertices $y_{n-6}, y_{n-5}, y_{n-4}, y_{n-3}$, $y_{n-2}, y_{n-1}$ and $y_n$.Clearly this vertex labeling is a $4-$ remainder cordial labeling.

□

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