International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 389-397
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Fixed Point Theorems in Strong Fuzzy Metric Spaces Using Control Function

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Abstract

In this paper we establish some results on fixed point theorems in strong fuzzy metric spaces by using control function, which are the generalization of some existing results in the literature.

AMS Subject Classification: 54H25; 47H10; 47S40; 54A40

Key Words and Phrases: fuzzy metric, strong fuzzy metric space, continuous t-norm, control function

1 Introduction

Zadeh invented the theory of fuzzy sets in 1965[9].The theory of fuzzy set has been developed extensively by many authors in different fields such as control theory, engineering sciences neural networks, etc. The concept of fuzzy metric space was introduced initially by Kramosil and Michalek [6]. Later on, George and Veeramani [1] gives the modified notion of fuzzy metric spaces due to Kramosil and Michalek and analysed a Hausdorff topology of fuzzy metric spaces. Recently, Gregori et al. [4] gave many interesting examples of fuzzy metrics in the sense of George and Veeramani and have also applied these fuzzy metrics to color image processing. In 1988, Grabiec [3] proved an analog of the Banach contraction theorem in fuzzy metric spaces. In his proof, he used a fuzzy version of Cauchy sequence. M.S.Khan et al. propounded a new notion of Banach fixed point theorem in metric spaces by introducing a control function called an altering distance function in 1984[5]. Recently, Shen et al.[9] introduced the notion of control function in fuzzy metric space $(X, M, *)$ is given by

$$\varphi(M(Tx, Ty, t)) \geq k(t).\varphi(M(x, y, t)), \forall x, y \in X, t > 0, \quad (1)$$

obtained fixed point result for self-mappingT.

In this paper, we prove some fixed point theorems in the sense of strong fuzzy metric spaces in the contraction of (1) by using the control function, which are the generalization of some existing results in the literature.

2 Preliminaries

The basic definitions are recalled here.

Definition 1. A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = (x, \mu A(x)) : x \in A, \mu A(x) \in [0, 1]$. In the pair $(x, \mu A(x))$, the first element x belongs to the classical set A, the second element $\mu A(x)$ belongs to the interval $[0, 1]$, is called the membership function.

Definition 2. A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0, 1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$

Example 3. *1. Lukasievicz t-norm:* $a * b = max\{a + b - 1, 0\}$

2. Product t-norm: $a * b = a.b$

3. Minimum t-norm: $a * b = min(a, b)$

Definition 4. A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty) \to [0, 1]$ satisfies the following conditions: $\forall x, y, z \in X$ and $s, t > 0$

(KM1) $M(x, y, 0) = 0$,$t > 0$

(KM2) $M(x, y, t) = 1$ if and only if $x = y, t > 0$

(KM3) $M(x, y, t) = M(y, x, t)$

(KM4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$

(KM5) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left-continuous.

Then M is called a fuzzy metric on X.

Definition 5. A fuzzy metric space is an ordered triple such that X is a nonempty set,$*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty) \to [0, 1]$ satisfies the following conditions: $\forall x, y, z \in X$ and $s, t > 0$

(GV1) $M(x, y, t) > 0$,$t > 0$

(GV2) $M(x, y, t) = 1$ if and only if $x = y, t > 0$

(GV3) $M(x,y,t) = M(y,x,t)$

(GV4) $M(x,z,t+s)M(x,y,t) * M(y,z,s)$

(GV5) $M(x,y,.) : (0,\infty) \to [0,1]$ is continuous.

Then M is called a fuzzy metric on X.

Definition 6. Let $(X, M, *)$ be a fuzzy metric space. M is said to be strong if it satisfies the following additional axiom:

(GV4') $M(x,z,t) \geq M(x,y,t) * M(y,z,t)$, $\forall x,y,z \in X, t > 0$,

then $(X, M, *)$ is called a strong fuzzy metric space.

Definition 7. Let $(X, M, *)$ be a fuzzy metric space, for $t > 0$ the open ball $B(x,r,t)$ with a centre $x \in X$ and a radius $0 < r < 1$ is defined by
$B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}$.
A subset $A \subset X$is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x,r,t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is topology on$X$, called the topology induced by the fuzzy metric M.

Definition 8. Let $(X, M, *)$ be a fuzzy metric space

1. A sequence $\{x_n\}$ in X is said to be convergent to a point x in $(X, M, *)$ if $\lim_{t\to\infty}(M(x,y,t) = 1$ for all $t > 0$.
2. sequence $\{x_n\}$ in X is called a Cauchy sequence in $(X, M, *)$, if for each $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$
3. A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.
4. A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact

Lemma 9. *Let $(X, M, *)$ be a fuzzy metric space. For all $u, v \in X, M(u,v,.)$ is non-decreasing function.*

Proof. If $M(u,v,t) > M(u,v,s)$ for some $0 < t < s$.
Then $M(u,v,t) * M(v,v,s-t) \leq M(u,v,s) < M(u,v,t)$,
Thus $M(u,v,t) < M(u,v,t) < M(u,v,t)$, (since $M(v,v,s-t) = 1$)
which is a contradiction □

3 Main Results

Definition 10. A function $\varphi : [0,1] \to [0,1]$ is called control function or an altering distance function if it satisfies the following properties:

(CF1) φ is strictly decreasing and continuous;

(CF2) $\varphi(\lambda) \geq 0, \forall\lambda \neq 1$ if $\varphi(\lambda) = 0$ if and only if $\lambda = 1$. It is obvious that $\lim_{\lambda \to 1^-} \varphi(\lambda) = \varphi(1) = 0$.

(CF3) $\varphi(\lambda * \mu) \leq \varphi(\lambda) + \varphi(\mu)$, $\lambda, \mu \in \{M(u, Tu, t) : u \in X, t > 0\}$

Theorem 11. *Let $(X,\ M,\ *)$ be a complete strong fuzzy metric space with continuous t-norm $*$ and let T is a self-mapping in X. If there exists a control function φ and $\lambda_i = \lambda_i(t)$,*
$i = 1,\ 2,\ 3,\ .,6,\ \lambda_i \geq 0$ and $\lambda_1 + \lambda_2 + \lambda_3 + 2\lambda_4 + \lambda_5 + \lambda_6 < 1$ such that

$$\varphi(M(Tu,\ Tv,\ t)) \leq \lambda_1 \varphi(M(u,\ Tu,\ t)) + \lambda_2 \varphi(M(v,\ Tv,\ t)) + \lambda_3 \varphi(M(Tu,\ v,\ t)) + \lambda_4 \varphi(M(u,\ Tv,\ t)) + \lambda_5(M(u,\ v,\ t)) + \lambda_6 \varphi\{\max(M(u,\ Tu,\ t),\ M(v,\ Tv,\ t)) \tag{2}$$

Then T has a unique fixed point in X.

Proof. Let u be any arbitrary point in X and define a sequence $\{u_n\} \in X$ such that $u_{n+1} = Tu_n$.
Assume that $u_{n+1} = Tu_n = u_n$ for some $n \in \mathbb{N}$, then $u_n$is a fixed point of T.
Suppose $u_{n+1} \neq u_n$, put $u = u_{n-1}$ and $v = u_n$ in equation (2) we get

$$\begin{aligned}
\varphi(M(Tu_{n-1},\ Tu_n,\ t)) \leq{}& \lambda_1 \varphi(M(u_{n-1},\ Tu_{n-1},\ t)) + \lambda_2 \varphi(M(u_n,\ Tu_n,\ t)) + \\
& \lambda_3 \varphi(M(Tu_{n-1},\ u_n,\ t)) + \lambda_4 \varphi(M(u_{n-1},\ Tu_n,\ t)) + \\
& \lambda_5 \varphi(M(u_{n-1},\ u_n,\ t)) + \\
& \lambda_6 \varphi\{\max(M(u_{n-1},\ Tu_{n-1},\ t),\ M(u_n,\ Tu_n,\ t))\} \\
\varphi(M(u_n,\ u_{n+1},\ t)) \leq{}& \lambda_1 \varphi(M(u_{n-1},\ u_n,\ t)) + \lambda_2 \varphi(M(u_n,\ u_{n+1},\ t)) + \lambda_3 \varphi(M(u_n,\ u_n,\ t)) + \\
& \lambda_4 \varphi(M(u_{n-1},\ u_{n+1},\ t)) + \lambda_5 \varphi(M(u_{n-1},\ u_n,\ t)) + \\
& \lambda_6 \varphi\{\max(M(u_{n-1},\ u_n,\ t),\ M(u_n,\ u_{n+1},\ t))\} \\
\varphi(M(u_n,\ u_{n+1},\ t)) \leq{}& \lambda_1 \varphi(M(u_{n-1},\ u_n,\ t)) + \lambda_2 \varphi(M(u_n,\ u_{n+1},\ t)) + \lambda_3 \varphi(M(1)) + \\
& \lambda_4 \varphi(M(u_{n-1},\ u_{n+1},\ t)) + \lambda_5 \varphi(M(u_{n-1},\ u_n,\ t)) + \\
& \lambda_6 \varphi\{\max(M(u_{n-1},\ u_n,\ t),\ M(u_n,\ u_{n+1},\ t))\}
\end{aligned}$$

$$\begin{aligned}
\varphi(M(u_n,\ u_{n+1},\ t)) \leq{}& \lambda_1 \varphi(M(u_{n-1},\ u_n,\ t)) + \lambda_2 \varphi(M(u_n,\ u_{n+1},\ t)) + \\
& \lambda_4 \varphi(M(u_{n-1},\ u_{n+1},\ t)) + \lambda_5 \varphi(M(u_{n-1},\ u_n,\ t)) + \\
& \lambda_6 \varphi\{\max(M(u_{n-1},\ u_n,\ t),\ M(u_n,\ u_{n+1},\ t))\}
\end{aligned} \tag{3}$$

Here $(X,\ M,\ *)$ is a strong fuzzy metric space then we have

$M(u_{n-1},\ u_{n+1},\ t) \geq M(u_{n-1},\ u_n,\ t) * M(u_n,\ u_{n+1},\ t)$ (by using (GV4'))

$$\varphi(M(u_{n-1},\ u_{n+1},\ t)) \geq \varphi((M(u_{n-1},\ u_n,\ t)) * (M(u_n,\ u_{n+1},\ t)))(by using(CF3))$$

$$\varphi(M(u_{n-1},\ u_{n+1},\ t)) \geq \varphi((M(u_{n-1},\ u_n,\ t)) + (M(u_n,\ u_{n+1},\ t))) \tag{4}$$

Using above inequalities in (3) we get

$$\varphi(M(u_n, u_{n+1}, t)) \le \lambda_1\varphi(M(u_{n-1}, u_n, t)) + \lambda_2\varphi(M(u_n, u_{n+1}, t)) + \lambda_4[\varphi(M(u_{n-1}, u_n, t)) + \varphi(M(u_n, u_{n+1}, t))] + \lambda_5\varphi(M(u_{n-1}, u_n, t)) + \lambda_6\varphi\{\max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))\}$$

$$\varphi(M(u_n, u_{n+1}, t)) \le \lambda_1\varphi(M(u_{n-1}, u_n, t)) + \lambda_2\varphi(M(u_n, u_{n+1}, t)) + \lambda_4\varphi(M(u_{n-1}, u_n, t)) + \lambda_4\varphi(M(u_n, u_{n+1}, t)) + \lambda_5\varphi(M(u_{n-1}, u_n, t)) + \lambda_6\varphi\{\max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t))\} \quad (5)$$

$$\text{If } \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) = M(u_{n-1}, u_n, t) \quad (6)$$

Then the above inequality (4) becomes

$$\varphi(M(u_n, u_{n+1}, t)) \le \lambda_1\varphi(M(u_{n-1}, u_n, t)) + \lambda_2\varphi(M(u_n, u_{n+1}, t)) + \lambda_4\varphi(M(u_{n-1}, u_n, t)) + \lambda_4\varphi(M(u_n, u_{n+1}, t)) + \lambda_5\varphi(M(u_{n-1}, u_n, t)) + \lambda_6\varphi(M(u_{n-1}, u_n, t))$$

We obtained

$$\varphi(M(u_n, u_{n+1}, t)) \le \frac{\lambda_1 + \lambda_4 + \lambda_5 + \lambda_6}{(1 - (\lambda_2 + \lambda_4))}\varphi(M(u_{n-1}, u_n, t)) < \varphi(M(u_{n-1}, u_n, t)) \quad (7)$$

Similarly,

$$\text{If } \max(M(u_{n-1}, u_n, t), M(u_n, u_{n+1}, t)) = M(u_n, u_{n+1}, t) \quad (8)$$

Then the inequality (5) becomes

$$\varphi(M(u_n, u_{n+1}, t)) \le \frac{\lambda_1 + \lambda_4 + \lambda_5}{(1 - (\lambda_2 + \lambda_4 + \lambda_6))}\varphi(M(u_{n-1}, u_n, t)) < \varphi(M(u_{n-1}, u_n, t)) \quad (9)$$

Hence $\varphi(M(u_n, u_{n+1}, t)) < \varphi(M(u_{n-1}, u_n, t))$

This gives $(M(u_n, u_{n+1}, t)) > (M(u_{n-1}, u_n, t))$

Since the sequence $\{M(u_n, u_{n+1}, t)\}$ is non decreasing

Taking limit $n \to \infty$, we get

$$\lim_{n\to\infty} M(u_n, u_{n+1}, t) = q(r), \text{for } q : (0, \infty) \to [0, 1] \quad (10)$$

Suppose that $q(r) \ne 1$ for some $r > 0$ as $n \to \infty$,

Now (7) becomes,

$$\varphi(q(r)) \le \frac{\lambda_1 + \lambda_4 + \lambda_5 + \lambda_6}{(1 - (\lambda_2 + \lambda_4))}\varphi(q(r)) < \varphi(q(r)) \quad (11)$$

which is a contradiction.

Hence $\lim_{n\to\infty} M(u_n,\ u_{n+1},\ t) = 1,\ t > 0$

Next we prove that the sequence $\{u_n\}$ is a Cauchy's sequence.

Assume that $\{u_n\}$ is not a Cauchy's sequence then for any $0 < \epsilon < 1,\ t > 0$ then there exists sequence $\{u_{n_k}\}$ and $\{u_{m_k}\}$ where $n_k,\ m_k \geq n$ and $n_k,\ m_k \in \mathbb{N}\ \ (n_k > m_k)$

$$\text{such that } M(u_{n_k},\ u_{m_k},\ t) \leq 1 - \epsilon \tag{12}$$

Let n_k be least integer exceeding m_k satisfying the above property

$$\text{That is } M(u_{n_k-1},\ u_{m_k},\ t) > 1 - \epsilon,\ n_k,\ m_k \in \mathbb{N} \quad \text{and } t > 0 \tag{13}$$

Put $u = u_{n_k-1}$ and $v = u_{m_k-1}$

$$\begin{aligned}\varphi(M(Tu_{n_k-1},\ Tu_{m_k-1},\ t)) \leq\ & \lambda_1\varphi(M(u_{n_k-1},\ Tu_{n_k-1},\ t)) + \lambda_2\varphi(M(u_{m_k-1},\ Tu_{m_k-1},\ t)) + \\ & \lambda_3\varphi(M(Tu_{n_k-1},\ u_{m_k-1},\ t)) + \lambda_4\varphi(M(u_{n_k-1},\ Tu_{m_k-1},\ t)) + \\ & \lambda_5\varphi(M(u_{n_k-1},\ u_{m_k-1},\ t)) + \\ & \lambda_6\varphi\{\max(M(u_{n_k-1},\ Tu_{n_k-1},\ t),\ M(u_{m_k-1},\ Tu_{m_k-1},\ t))\}\end{aligned}$$

$$\begin{aligned}\varphi(M(u_{n_k},\ u_{m_k},\ t)) \leq\ & \lambda_1\varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \lambda_2\varphi(M(u_{m_k-1},\ u_{m_k},\ t)) + \\ & \lambda_3\varphi(M(u_{n_k},\ u_{m_k-1},\ t)) + \lambda_4\varphi(M(u_{n_k-1},\ u_{m_k},\ t)) + \\ & \lambda_5\varphi(M(u_{n_k-1},\ u_{m_k-1},\ t)) + \\ & \lambda_6\varphi\{\max(M(u_{n_k-1},\ u_{n_k},\ t),\ M(u_{m_k-1},\ u_{m_k},\ t))\}\end{aligned} \tag{14}$$

If $\max(M(u_{n_k-1},\ u_{n_k},\ t),\ M(u_{m_k-1},\ u_{m_k},\ t)) = M(u_{n_k-1},\ u_{n_k},\ t)$

$$\begin{aligned}\varphi(M(u_{n_k},\ u_{m_k},\ t)) \leq\ & \lambda_1\varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \lambda_2\varphi(M(u_{m_k-1},\ u_{m_k},\ t)) + \\ & \lambda_3\varphi(M(u_{n_k},\ u_{m_k-1},\ t)) + \lambda_4\varphi(M(u_{n_k-1},\ u_{m_k},\ t)) + \\ & \lambda_5\varphi(M(u_{n_k-1},\ u_{m_k-1},\ t)) + \lambda_6\varphi(M(u_{n_k-1},\ u_{n_k},\ t))\end{aligned} \tag{15}$$

By (GV4'), (CF3) and (CF1) it follows that

$$\varphi(M(u_{n_k},\ u_{m_k-1},\ t)) \leq \varphi(M(u_{n_k},\ u_{m_k},\ t)) + \varphi(M(u_{m_k},\ u_{m_k-1},\ t)) \tag{16}$$

and $\varphi(M(u_{n_k-1},\ u_{m_k-1},\ t)) \leq \varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \varphi(M(u_{n_k},\ u_{m_k-1},\ t))$

Applying the previous inequalities we get

$$\begin{aligned}\varphi(M(u_{n_k-1},\ u_{m_k-1},\ t)) \leq\ & \varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \varphi(M(u_{n_k},\ u_{m_k},\ t)) + \\ & \varphi(M(u_{m_k},\ u_{m_k-1},\ t))\end{aligned} \tag{17}$$

Also (13) and (CF1) we get

$$\varphi(M(u_{n_k-1},\ u_{m_k},\ t)) \leq \varphi(1 - \epsilon). \tag{18}$$

Substituting (16), (17), and (18) in (15) we have

$$\begin{aligned}\varphi(M(u_{n_k},\ u_{m_k},\ t)) \leq\ & \lambda_1\varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \lambda_2\varphi(M(u_{m_k-1},\ u_{m_k},\ t)) + \\ & \lambda_3\varphi(M(u_{n_k},\ u_{m_k},\ t)) + \lambda_3\varphi(M(u_{m_k},\ u_{m_k-1},\ t)) + \lambda_4\varphi(1 - \epsilon) + \\ & \lambda_5\varphi(M(u_{n_k-1},\ u_{n_k},\ t)) + \lambda_5\varphi(M(u_{n_k},\ u_{m_k},\ t)) + \\ & \lambda_5\varphi(M(u_{m_k},\ u_{m_k-1},\ t)) + \lambda_6\varphi(M(u_{n_k-1},\ u_{n_k},\ t))\end{aligned}$$

$$(1-\lambda_3-\lambda_5)\,\varphi\,(M\,(u_{n_k},\ u_{m_k},\ t)) \le (\lambda_1+\lambda_5+\lambda_6)\,\varphi\,(M\,(u_{n_k-1},\ u_{n_k},\ t)) + (\lambda_2+\lambda_3+\lambda_5)\,\varphi\,(M\,(u_{m_k},\ u_{m_k-1},\ t)) + \lambda_4\varphi\,(1-\epsilon) \tag{19}$$

Using (12) we obtain,

$$\varphi\,(M\,(u_{n_k},\ u_{m_k},\ t)) > \varphi(1-\epsilon) \tag{20}$$

$$(1-\lambda_3-\lambda_5)\,\varphi(1-\epsilon) \le (\lambda_1+\lambda_5+\lambda_6)\,\varphi\,(M\,(u_{n_k-1},\ u_{n_k},\ t)) + (\lambda_2+\lambda_3+\lambda_5)\,\varphi\,(M\,(u_{m_k},\ u_{m_k-1},\ t)) + \lambda_4\varphi\,(1-\epsilon) \tag{21}$$

Taking $k \to \infty$ in above inequality we obtain

$$(1-\lambda_3-\lambda_5)\,\varphi(1-\epsilon) \le \ \lambda_4\varphi\,(1-\epsilon) \tag{22}$$

$$Thatis\,(1-\lambda_3-\ \lambda_4-\lambda_5)\,\varphi(1-\epsilon) \le 0,$$

which implies that $\epsilon = 0$ and we get a contradiction.
Hence $\{u_n\}$ is a Cauchy's sequence.
Since X is complete and there exist $z \in X$ such that $\lim_{n\to\infty} u_n = z$
That is $M\,(u_n,\ z,\ t) = 1$ as $n \to \infty$
Put $u = u_{n-1}$ and $v = z$ in equation (2) we get

$$\varphi\,(M\,(u_n,\ Tz,\ t)) \le \lambda_1\varphi\,(M\,(u_{n-1},\ u_n,\ t)) + \lambda_2\varphi\,(M\,(z,\ Tz,\ t)) + \lambda_3\varphi\,(M\,(u_n,\ z,\ t)) + \lambda_4\varphi\,(M\,(u_{n-1},\ Tz,\ t)) + \lambda_5\,(M\,(u_{n-1},\ z,\ t)) + \ \lambda_6\varphi\{\max\,(M\,(u_{n-1},\ u_n,\ t)\,,\ M\,(z,\ Tz,\ t))\ \} \tag{23}$$

Taking $n \to \infty$ in (23)

$$(1-\lambda_2-\lambda_4-\ \lambda_6)\,\varphi\,(\ M\,(z,\ Tz,\ t)) \le 0,\ t > 0 \tag{24}$$

Therefore $M\,(z,\ Tz,\ t) = 1$, and $z = Tz$.
To prove Uniqueness,
Suppose that w is another fixed point of T, that is $Tw = w$ where $q \ne z$

$$(1-\lambda_3-\lambda_4-\lambda_5)\varphi(\ M\,(z,\ w,\ t)) \le 0,\ t > 0 \tag{25}$$

Hence $z = w$ is the unique fixed point of T. □

Corollary 12. *Let $(X,\ M,\ *)$ be a complete strong fuzzy metric space with continuous t-norm $*$ and let T is a self-mapping in X. If there exists a control function φ and $\lambda_i = \lambda_i\,(t)$,*
$i = 1,\ 2,\ 3,\ .,6,\ \lambda_i \ge 0$ and $\lambda_1+\lambda_2+\lambda_3+2\lambda_4+\lambda_5 < 1$ such that

$$\varphi\,(M\,(Tu,\ Tv,\ t)) \le \lambda_1\varphi\,(M\,(u,\ Tu,\ t)) + \lambda_2\varphi\,(M\,(v,\ Tv,\ t)) + \lambda_3\varphi\,(M\,(Tu,\ v,\ t)) + \lambda_4\varphi\,(M\,(u,\ Tv,\ t)) + \lambda_5\,(M\,(u,\ v,\ t)) \tag{26}$$

Then T has a unique fixed point in X.

Proof. The proof of the above theorem 3.1 considering the fuzzy contraction on the fuzzy metric space

$$(X,\ M,\ *), \varphi\left(M\left(Tu,\ Tv,\ t\right)\right) \leq \lambda_1\varphi\left(M\left(u,\ Tu,\ t\right)\right) + \lambda_2\varphi\left(M\left(v,\ Tv,\ t\right)\right) + \lambda_3\varphi\left(M\left(Tu,\ v,\ t\right)\right) + \lambda_4\varphi\left(M\left(u,\ Tv,\ t\right)\right) + \lambda_5\left(M\left(u,\ v,\ t\right)\right).$$

□

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