International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 361-370
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

The Arithmetic Geometric Mean (AGM) inequality approach to compute EOQ/EPQ under Fuzzy Environment

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Abstract

In this article, a new method by using the Arithmetic Geometric Mean (AGM) inequality is proposed to compute the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) without taking differential calculus or algebraic manipulations under fuzzy environment. In this paper EOQ/EPQ with shortage and without shortage are also discussed. Here the demand rate and shortage cost are taken as triangular fuzzy number. The proposed method finds both the optimal ordering quantity and the total relevant cost. The effectiveness of the proposed method illustrated by means of numerical examples.

AMS Subject Classification: 03E72, 90C05, 90C33

Key Words and Phrases: Inventory, Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), Arithmetic Geometric Mean (AGM) inequality, Cost comparison optimization, triangular fuzzy number, Shortage level

1 Introduction

The lot size model has been studied extensively since the economic order quantity (EOQ) model was first introduced in 1913 by Harris [7]. The economic production quantity (EPQ) model was then developed by Taft [6] later; the EOQ/EPQ models were extended to consider shortages. The EOQ is one of the most popular and successful optimization models in supply chain management due to its simplicity of use and simplicity of concepts. The most popular approach to derive the EOQ formula in operations management text books is to apply differential calculus by taking first and second order derivatives of the average cost per unit. Grubbstrom [8] presented an alternate method to obtain the EOQ without taking derivative. Grubbsrom and Erdem [9] then extended the approach to solve the EOQ model allowing for shortages. Cardenas-Barron [2] further used the same approach to obtain the economic production quantity (EPQ). Later Minner [10] Wee et al. [12] proposed another different approach to obtain the EOQ without differential calculus and algebraic manipulations. He compared the cost co-efficient when a local cost minimum is obtained, took the time horizon to infinity to get the EOQ and then proved its the global minimum. Later the cost comparisons and the arithmetic geometric mean (AGM) inequality. These methods were developed by minner [10] and Teng [11] respectively. Minners method has been modified by Wee et al. [12] . The modified cost comparison method is simpler than Minners method. Tengs AGM method is also very simple but he is not the first researcher to apply the well known AGM inequality in to optimization functions. It can be argued that Garver (1935) and niven (1981). The arithmetic geometric mean is simple unfortunately the AGM method does not derive the mathematical expression for the shortage level for EOQ/EPQ models. So Cardenas-Barron [3] explained how to find the shortage level.

1.1 Notations:

The following notation is used throughout the entire paper.
D - the demand rate per unit of time, q - lot size per order (Production run), C_1 - inventory carrying (holding) cost, C_2 - shortage cost per item, C_3 - setup cost per order, r - the fill rate, k - the production rate, d - the decreasing (demand) rate in the production model, T - cycle time, TC - the total relevant cost per unit of time, S - shortage level, Q^* - the optimal order quantity

1.2 Assumptions:

Apart from the traditional EOQ/EPQ model some changes are made here as follows
(i) We may assume that the demand rate (D) is uncertain, take it as a triangular fuzzy number, (ii) Replenishment is instantaneous, (iii) Quantity discount is not allowed, (iv) Both the initial and the ending inventory levels are zero, (v) In case, shortages are allowed and it is uncertain so consider it as a triangular fuzzy number

This paper is organized as follows: Section 2 provides some basic idea about the triangular fuzzy number and its arithmetic operations, Section 3 gives methodology to find EOQ and EPQ under fuzzy environment using AGM inequality. In section 4, the effectiveness of the proposed method is illustrated by means of examples. Finally we conclude the paper.

2 Preliminieries

2.1 Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = (x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]$ in the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$ belong to the closed interval [0,1] called membership function

2.2 Fuzzy Number

The notation of fuzzy numbers was introduced by Dubois D. and Prade H [5]. A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}} : R \to [0, 1]$ is called a fuzzy number if,

1. A fuzzy set $\tilde{A}$ is normal
2. $\tilde{A}$ is convex
3. $\mu_{\tilde{A}}$ is continuous and
4. Supp $\tilde{A}$ is bounded, where supp $\tilde{A} = x \in R : \mu_{\tilde{A}}(x) > 0$

2.3 Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where a_2 is the central value$\mu_{\tilde{A}}(a_2) = 1$, a_1 is the left spread and a_3 is the right spread, and $a_1 < a_2 < a_3$ are defined in R. The membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{(a_3 - x)}{(a_3 - a_2)} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

2.4 Arithmetic Operations on Triangular Fuzzy Number Using Function Principle

The arithmetic operations on triangular fuzzy number using function principle were given by Chen, S. H. [4]. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then,

i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ Where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers

ii) The product of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$ where $T = [a_1b_1, a_2b_2, a_3b_3]$

$c_1 = minT, c_2 = a_2b_2, c_3 = maxT$

If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\tilde{A}\times\tilde{B} = [a_1b_1, a_2b_2, a_3b_3]$

iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$

then the subtraction of $\tilde{B}$ from $\tilde{A}$ is

$\tilde{A} + \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$, Where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv) $\dfrac{1}{B} = B^{-1} = (\dfrac{1}{b_3}, \dfrac{1}{b_2}, \dfrac{1}{b_1})$ Where b_1, b_2, b_3 are non zero real numbers,

then $\dfrac{\tilde{A}}{\tilde{B}} = (\dfrac{a_1}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_1})$
v) Let $\alpha \in R$, then $\alpha\tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3)$ if $\alpha \succeq 0$
$\alpha\tilde{A} = (\alpha a_3, \alpha a_2, \alpha a_1)$ if $\alpha < 0$

3 Methodology

3.1 Proposed method and Analysis

All previous researchers for deriving EOQ/EPQ with or without using derivatives, a simplest method by using arithmetic geometric mean (AGM) inequality theorem is proposed to obtain the optimal order quantity that minimizes the total relevant cost. We know that, the arithmetic mean is always greater than or equal to geometric mean.

In short, for any positive real numbers $a_1, a_2, a_3, ...a_n$ such that $\dfrac{a_1 + a_2 + a_3... + a_n}{n} \geq (a_1.a_2.a_3...a_n)^{\frac{1}{n}}$ the equation holds only if $a_1 = a_2 = a_3 = ... = a_n$ We will use this theorem to compute EOQ/EPQ with shortages and without shortages.

3.2 Case (1): EOQ without shortages

It is clear from the ordinary EOQ model without shortage that the total relevant cost per unit time as follows

$TC = \dfrac{qC_1}{2} + \dfrac{\tilde{D}C_3}{q}$ Where $\tilde{D} = (d_1, d_2, d_3)$

By using Arithmetic Geometric Mean (AGM) inequality,We can get that,

$$TC = \frac{qC_1}{2} + \frac{\tilde{D}C_3}{q} = \frac{qC_1 + 2(\frac{\tilde{D}C_3}{q})}{2} \geq \sqrt{2C_1C_3\tilde{D}}$$

when the inequality, $\dfrac{qC_1}{2} = \dfrac{\tilde{D}C_3}{q}$ ......(3.1)

Holds TC has a minimum, since minimum total cost occurs at the point where annual holding costs and annual ordering costs are equal. Therefore we obtain from equation (3.1) that the optimal order quantity $Q^* = \sqrt{\dfrac{2C_3\tilde{D}}{C_1}}$

Hence the minimum total relevant cost per unit time is $TC = \sqrt{2C_1C_3\tilde{D}}$

3.3 Case (2):EOQ with Shortages

According to Wee et al. [12] and Cardenas-Barron [3], the fill rate r is proportion of immediate filling demand in the inventory cycle. Then $1 - r$ is the proportion of the demand that is shortages and the optimal fill rate is given by $r = \dfrac{\tilde{C_2}}{C_1 + \tilde{C_2}}$

The Shortage level S is given by $S = q - rq = q(1 - r) = q(1 - \dfrac{\tilde{C_2}}{C_1 + \tilde{C_2}})$

On simplifying we get, $S = (\dfrac{C_1}{C_1 + \tilde{C_2}})q$

In the EOQ model with shortages, we know that the total relevant cost is

$TC = \dfrac{C_1qr^2}{2} + \dfrac{\tilde{C_2}q(1-r)^2}{2} + \dfrac{\tilde{D}C_3}{q}$ where, $\tilde{D} = (d_1, d_2, d_3), \tilde{C_2} = (C_{21}, C_{22}, C_{23})$

By using AGM inequality, we can get that

$$TC = \frac{C_1 q r^2}{2} + \frac{\tilde{C}_2 q(1-r)^2}{2} + \frac{\tilde{D}C_3}{q} = \frac{q[C_1 r^2 + \tilde{C}_2(1-r)^2] + 2(\frac{\tilde{D}C_3}{q})}{2} \geq \sqrt{2C_3\tilde{D}[C_1 r^2 + \tilde{C}_2(1-r)^2]}$$

when the inequality, $\frac{C_1 q r^2}{2} + \frac{\tilde{C}_2 q(1-r)^2}{2} = \frac{\tilde{D}C_3}{q}$ ...... (3.2)

Holds TC has a minimum, since minimum total cost occurs at the point where annual holding costs and annual ordering costs are equal. Therefore we obtain from equation (3.2) that the optimal order quantity $Q^* = \sqrt{\frac{2C_3\tilde{D}}{C_1 r^2 + \tilde{C}_2(1-r)^2}}$

Substituting the optimal fill rate $r = \frac{\tilde{C}_2}{C_1 + \tilde{C}_2}$ we get,

$$Q^* = \sqrt{\frac{2C_3\tilde{D}}{C_1}(\frac{C_1 + \tilde{C}_2}{\tilde{C}_2})}$$

Hence, the minimum total relevant cost per unit time is $TC = \sqrt{2C_3\tilde{D}[C_1 r^2 + \tilde{C}_2(1-r)^2]}$

Substituting the optimal fill rate $r = \frac{\tilde{C}_2}{C_1 + \tilde{C}_2}$ we get,

$$TC = \sqrt{2C_1C_3\tilde{D}(\frac{\tilde{C}_2}{C_1 + \tilde{C}_2})}$$

3.4 Case (3): EPQ without Shortages

It is clear from the traditional EPQ model without shortage that the total relevant cost per unit time as follows,

$TC = \frac{qC_1}{2}(\frac{k-d}{k}) + \frac{\tilde{D}C_3}{q}$ Where $\tilde{D} = (d_1, d_2, d_3)$

By using Arithmetic Geometric Mean (AGM) inequality,We can get that,

$$TC = \frac{qC_1}{2}(\frac{k-d}{k}) + \frac{\tilde{D}C_3}{q} = \frac{qC_1(\frac{k-d}{k}) + 2(\frac{\tilde{D}C_3}{q})}{2} \geq \sqrt{2C_1C_3\tilde{D}(\frac{k-d}{k})}$$

when the inequality, $\frac{qC_1}{2}(\frac{k-d}{k}) = \frac{\tilde{D}C_3}{q}$ ......(3.3)

Holds TC has a minimum, since minimum total cost occurs at the point where annual holding costs and annual ordering costs are equal. Therefore we obtain from equation (3.3) that the optimal order quantity $Q^* = \sqrt{\frac{2C_3\tilde{D}}{C_1}(\frac{k-d}{k})}$

Hence the minimum total relevant cost per unit time is $TC = \sqrt{2C_1C_3\tilde{D}(\frac{k-d}{k})}$

3.5 Case (4): EPQ with Shortages

According to Cardenas-Barron [3], the equation $S = q - sq$ cannot be used to calculate the Shortage level for the EPQ with Shortage because it is not valid in this case. However, in the EPQ model with Shortage, the indifference between storage and shortage level gives the relation $C_1 rT = \tilde{C}_2(1-r)T$

Therefore in every cycle one must have

$C_1 * (Positive\ inventory\ interval) = \tilde{C}_2 * (Shortages\ interval)$

Also, $rT = t_3 + t_4$ and $(1-r)T = t_1 + t_2$
Therefore, $C_1 rT = \tilde{C}_2(t_1 + t_2)$
Also, it is easy to understand the following simple geometric relation
$t_1 = \frac{S}{\tilde{d}}$ and $t_2 = \frac{S}{k-\tilde{d}}$
And We know that, $q = \tilde{d}T \Rightarrow T = \frac{q}{\tilde{d}}$
Substitute T, t_1, t_2 in $C_1 rT = \tilde{C}_2(t_1 + t_2)$ we get,
$C_1(\frac{\tilde{C}_2}{C_1 + C_2})(\frac{q}{\tilde{d}}) = \tilde{C}_2(\frac{S}{\tilde{d}} + \frac{S}{k-\tilde{d}})$ Simplifying we get the Shortage level is,
$S = \frac{C_1 q}{C_1 + \tilde{C}_2}(\frac{k-\tilde{d}}{k})$
In the EPQ model with shortages, we know that the total relevant cost is
$TC = \frac{C_1 q r^2}{2}(\frac{k-\tilde{d}}{k}) + \frac{\tilde{C}_2 q(1-r)^2}{2}(\frac{k-\tilde{d}}{k}) + \frac{\tilde{d}C_3}{q}$ where, $\tilde{d} = (d_1, d_2, d_3), \tilde{C}_2 = (C_{21}, C_{22}, C_{23})$
By using AGM inequality, we can get that

$$TC = \frac{C_1 q r^2}{2}(\frac{k-\tilde{d}}{k}) + \frac{\tilde{C}_2 q(1-r)^2}{2}(\frac{k-\tilde{d}}{k}) + \frac{\tilde{d}C_3}{q} = \frac{q[C_1 r^2 + \tilde{C}_2(1-r)^2] + 2(\frac{\tilde{d}C_3}{q})}{2}$$

$$\geq \sqrt{2C_3\tilde{d}[C_1 r^2 + \tilde{C}_2(1-r)^2]}$$

when the inequality, $\frac{C_1 q r^2}{2}(\frac{k-\tilde{d}}{k}) + \frac{\tilde{C}_2 q(1-r)^2}{2}(\frac{k-\tilde{d}}{k}) = \frac{\tilde{d}C_3}{q}$ ...... (3.4)
Holds TC has a minimum, since minimum total cost occurs at the point where annual holding costs and annual ordering costs are equal. Therefore we obtain from equation (3.4) that the optimal order quantity $Q^* = \sqrt{\frac{2C_3\tilde{d}k}{(k-\tilde{d})(C_1 r^2 + \tilde{C}_2(1-r)^2)}}$

Substituting the optimal fill rate $r = \frac{\tilde{C}_2}{C_1 + \tilde{C}_2}$ we get, $Q^* = \sqrt{\frac{2C_3\tilde{d}}{C_1}(\frac{k}{k-\tilde{d}})(\frac{C_1 + \tilde{C}_2}{\tilde{C}_2})}$

Hence, the minimum total relevant cost per unit time is $TC = \sqrt{2C_3\tilde{d}(\frac{k-\tilde{d}}{k})[C_1 r^2 + \tilde{C}_2(1-r)^2]}$

Substituting the optimal fill rate $r = \frac{\tilde{C}_2}{C_1 + \tilde{C}_2}$ we get, $TC = \sqrt{2C_1 C_3\tilde{d}(\frac{k-\tilde{d}}{k})(\frac{\tilde{C}_2}{C_1 + \tilde{C}_2})}$

4 Illustrated Examples

4.1 EOQ without Shortages

A soap retailer purchase around 9000 pieces of soaps for his annual requirement. Ordering one months usage at a time. Each piece costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15 percentage of the average inventory per year.
You have been asked to suggest a more economic purchasing policy for the retailer. What advice would you offer and how much would it save the retailer per year under uncertainty demand?

Solution: Given that,

$\tilde{D} = (d_1, d_2, d_3) = (8500, 9000, 9500)$ Soaps/year, $q = \dfrac{(8500, 9000, 9500)}{12} = (708.33, 750, 791.67)$,

$C = Rs.20$/soap, $C_3 = Rs.15$, $C_1 = \dfrac{15}{100} \times 20 = Rs.3$/soap/year

Total relevant Cost $= \dfrac{qC_1}{2} + \dfrac{\tilde{D}C_3}{q} = \dfrac{(708.33, 750, 791.67) \times 3}{2} + \dfrac{(708.33, 750, 791.67) \times 15}{(708.33, 750, 791.67)}$

$= (1223.55, 1305, 1388.68)$

Optimal ordering quantity $Q^* = \sqrt{\dfrac{2C_3\tilde{D}}{C_1}} = \sqrt{\dfrac{2 \times 15 \times (8500, 9000, 9500)}{3}} = (291.55, 300, 308.22)$

That is, the retailer need to purchase around 300 soaps

Minimum total relevant cost per unit time $TC = \sqrt{2C_1C_3\tilde{D}} = \sqrt{2 \times 3 \times 15 \times (8500, 9000, 9500)}$

$= (874.64, 900, 924.66)$

Hence if the retailer purchases around 300 soaps each time, the net saving to the retailer will be $Rs.(1223.55, 1305, 1388.68) - (874.64, 900, 924.66) = Rs.(298.89, 405, 514.04)$

4.2 EPQ with Shortages

The demand for an item in a company is more or less 18,000 units per year, and the company can produce the item at a rate of 3,000 per month. The cost of one set-up is Rs. 500 and the holding cost of 1 unit per month is 15 paise. The shortage cost of one unit is around 20 rupees per month. Determine (i) Optimum production quantity and the number of shortages, (ii) Optimum cycle time and optimum production time (iii) Maximum inventory level in the cycle, and (iv) Total associated cost per year if the cost of the item is Rs. 20 per unit.

Solution:

Given that,

$C_1 = Re.0.15$ per month, $\tilde{C_2} = (C_{21}, C_{22}, C_{23}) = Rs.(18, 20, 22), C_3 = Rs.500$

$k = 3000$ units per month

$\tilde{d} = (d_1, d_2, d_3) = (17500, 18000, 18500)$ units per year

$= (1458.33, 1500, 1541.67)$ units per month

(i) Optimum production quantity is given by $Q^* = \sqrt{\dfrac{2C_3\tilde{d}}{C_1}(\dfrac{k}{k - \tilde{d}})(\dfrac{C_1 + \tilde{C_2}}{\tilde{C_2}})}$

$= \sqrt{\dfrac{2 \times 500 \times (1458.33, 1500, 1541.67)}{0.15}(\dfrac{3000}{3000 - (1458.33, 1500, 1541.67)})(\dfrac{0.15 + (18, 20, 22)}{(18, 20, 22)})}$

$= (3950.6587, 4488.8751, 5100.7758)$ units

Number of Shortage is given by $S = \dfrac{C_1Q^*}{C_1 + \tilde{C_2}}(\dfrac{k - \tilde{d}}{k})$

$= \dfrac{0.15 \times (3950.6587, 4488.8751, 5100.7758)}{0.15 + (18, 20, 22)}(\dfrac{3000 - (1458.33, 1500, 1541.67)}{3000})$

$= (13.7479, 16.7080, 20.4916)$

(ii) Optimum cycle time between set-ups $= \dfrac{Q^*}{k} = \dfrac{(3950.6587, 4488.8751, 5100.7758)}{3000}$

$= (1.3169, 1.4963, 1.7003)$

Optimum production time $= \dfrac{Q^*}{\tilde{d}} = \dfrac{(3950.6587, 4488.8751, 5100.7758)}{(1458.33, 1500, 1541.67)}$

$= (2.5626, 2.9926, 3.4977)$ months

(iii) Maximum inventory level $= \frac{k-\tilde{d}}{k}Q^* - S$
$= (\frac{3000-(1458.33, 1500, 1541.67)}{3000})(3950.6587, 4488.8751, 5100.7758) - (13.7479, 16.7080, 20.4916)$
$= (2009.7519, 2227.7296, 2465.7392)$units
(iv) Total associated (relevant) cost $TC = \sqrt{2C_1C_3\tilde{d}(\frac{k-\tilde{d}}{k})(\frac{\tilde{C_2}}{C_1+\tilde{C_2}})}$
$= \sqrt{2 \times 0.15 \times 500 \times (1458.33, 1500, 1541.67)(\frac{3000-(1458.33, 1500, 1541.67)}{3000})(\frac{(18, 20, 22)}{0.15+(18, 20, 22)})}$
$= Rs.(302.2395, 334.1669, 369.1249)$

5 Conclusion

In this note, a earliest way to complete the EOQ/EPQ with or without shortage by using well known Arithmetic Geometric Mean (AGM) inequality neither using complex differential calculus nor the tedious algebraic manipulations. In real life situation, the demand rate and the shortage costs are not predictable. So here we use the fuzzy concepts to compute the both. The main contribution of this paper is to present an alternate method for deriving EOQ/EPQ and it should be ease compare to the ordinary method. Finally, this simple approach should be considered as a more accessible approach to ease the learning of inventory theory in fuzzy environment for students who lack knowledge of calculus.

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