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Solving Fuzzy Sequential Linear Programming Problem Using Move Limits Algorithm

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Abstract

In literature, variety of methods have been proposed using numerical analysis to solve the complexity of optimization problems having non-linearity in objective function and also in constraints. In this paper a new technique is proposed to solve Fuzzy Sequential linear programming problem. An algorithmic procedure is given by using move limits methodology to solve Fuzzy Sequential linear programming problem. The algorithm is successfully tested in Fuzzy Sequential linear programming problem having nonlinearity in both objective function and constraints. This procedure is illustrated with numerical examples.

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Key Words and Phrases: Fuzzy Numbers, Hexagonal Fuzzy Numbers, Sequential Linear Programming problem, Move Limits, Fuzzy Ranking function.

1 Introduction

Fuzzy sets have been introduced by Lotfi.A.Zadeh (1965) [18] as a mathematical way of representing impreciseness or vagueness in everyday life. Fuzzy set theory has been practically applied to many disciplines such as control theory and operational research, mathematical modelling and industrial applications. The concept of fuzzy optimization in wide-ranging was first proposed by Tanaka et al, 1974 [14]. (Zimmerman, 1978), Proposed the first formatting of fuzzy linear programming. A most advantageous solution of nonlinear fuzzy programming problems introduced by (Kumar and Kaur, 2010; Kheirfam, 2011) [[7],[8]].

Optimization problems usually have very complex and highly non-linear implicit formulations. Therefore, the sequential linear programming (SLP) method is a popular approach to deal with the complexity and non-linearity of optimization problems [[4],[9],[10]]. The method performs very well in convex programming problems with nearly linear objective and constraint inequality functions [11]. The inherent conceptual simplicity, the SLP techniques are not globally convergent [1]. For under-constrained problems, where there are fewer active constraints than there are design variables, the method often performs poorly because the linearized domain is unbounded [16].

Pope introduced move limits in the early 70s, researchers proposed a variety of techniques to define the move limits. Haftka and Gurdal [4] suggests to choose as move limit the 10–30% of the value that a design variable has at the beginning of the iteration cycle. Vanderplaats and Kodyalam [17] proved the move limits are very large in the first design cycles and they are then reduced by 50% if the design violates the constraints more than at the previous iteration; anyhow the move limits are never reduced to less than 25%. John et al. [6] solved the linearized sub-problems with the Simplex algorithm. Move limits are reduced by means of the parameter each time the design does not improve in the current iterate. Move limits [[3],[13],[15]] created based on the difference that exists between the non-linear functions and the linearized functions.

The paper is organized as follows: Basic concepts of fuzzy sets and hexagonal fuzzy numbers are given in Section 2, and Section 3 deals with the concepts of move limits. In section 4 proposed algorithms is given to solve FSLPP. Finally in Section 5, the efficiency of the proposed method is explained by means of an example.

2 Preliminaries

Definition 1 (Fuzzy set [18]). A fuzzy set $\tilde{A}$ is defined by

$$\tilde{A} = \{(\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x})) : \mathrm{x} \in \mathrm{A}, \mu_{\mathrm{A}}(\mathrm{x}) \in [0, 1]\}.$$

In the pair $(\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}))$, the first element x belongs to the classical set A and the second element $\mu_{\mathrm{A}}(\mathrm{x})$ belongs to the interval [0, 1] called membership function.

Definition 2 (Fuzzy Number [18]). A fuzzy set $\tilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number:

1. $\tilde{A}$ must be a normal fuzzy set;
2. $\alpha_{\tilde{A}}$ must be a closed interval for every $\alpha \in [0, 1]$; and
3. the support of $\tilde{A}$ must be bounded.

Definition 3 (Hexagonal Fuzzy Number). A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy numbers [2] denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0, & x < a_1; \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_2-a_1}\right), & a_1 \le x \le a_2; \\ 1 + \frac{1}{2}\left(\frac{x-a_3}{a_3-a_2}\right), & a_2 \le x \le a_3; \\ 1, & a_3 \le x \le a_4; \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5; \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_5}{a_6-a_5}\right), & a_5 \le x \le a_6; \\ 0, & x > a_6. \end{cases}$$

Definition 4 (Operations of Hexagonal Fuzzy numbers [12]). Following are the three operations that can be performed on hexagonal fuzzy numbers, suppose $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy Numbers then

Addition: $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
Subtraction: $\tilde{A}_H - \tilde{B}_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$
Multiplication: $\tilde{A}_H * \tilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

Definition 5 (Ranking Function).

$$R(\tilde{A}_H) = \left(\frac{2a_1 + 10a_2 + 15a_3 + 15a_4 + 10a_5 + 2a_6}{54}\right)\left(\frac{19w}{54}\right)$$

where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are hexagonal fuzzy numbers.

Definition 6 (Normal Fuzzy Number). When $w = 1$ the hexagonal fuzzy number is a normal fuzzy number.

3 Move Limits Methodology in FSLPP

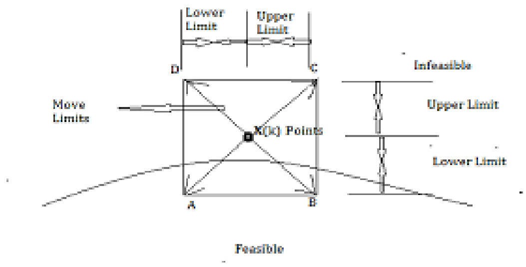
Fuzzy Sequential Linear Programming Problem defined as

$$\begin{aligned} &\text{Minimize } f(\tilde{x}) = f(\tilde{x}^{(k)}) + \nabla f(\tilde{x}^{(k)}).\tilde{d} \\ &\text{Subject to} \\ &\qquad h(\tilde{x}) = h(\tilde{x}^{(k)}) + \nabla h_j^T(\tilde{x}^{(k)}).\tilde{d} = 0 \\ &\qquad g(\tilde{x}) = g(\tilde{x}^{(k)}) + \nabla g_j^T(\tilde{x}^{(k)}).\tilde{d} \le 0 \\ &\qquad -\nabla_{ij}{}^{(k)} \le \tilde{d}^{(k)} \le \nabla_{iu}{}^{(k)} \\ &\qquad \text{and } \tilde{x}_j \ge 0 \text{ for all } i, j = 1, 2, 3 \ldots p \end{aligned}$$

may not have a bounded solution, or the changes in design may become too large, thus invalidating the linear approximations. Here we introducing change in limits of points, then the constraints are called move limits. It can be expressed as:

$$-\nabla_{ij}^{(k)} \le \tilde{d}^{(k)} \le \nabla_{iu}^{(k)} \quad \textit{for all } i = 1, 2, 3 \ldots p$$

where $\nabla_{ij}^{(k)}$ and $\nabla_{iu}^{(k)}$ are the maximum allowed decrease and increase in the i^{th} design variable, respectively, at the k^{th} iteration. The problem is still linear in terms of di, so FLP methods can be used to solve it. Note that the iteration counter k is used to specify $\nabla_{ij}^{(k)}$ and $\nabla_{iu}^{(k)}$. The move limits may change at each and every iteration. The below diagram shows the concept of move limits.



It shows the effect of imposing the move limits on changes in the design $x^{(k)}$; the new point estimate is required to stay in the rectangular area ABCD [5].

3.1 Selection Procedure of Move Limits

Many times lower and upper bounds are specified on the real design variables x_i. Therefore, move limits must also be selected to remain within these specified bounds. In linear approximations of the functions, the design changes should not be very large and the move limits should not be extremely large. Usually $\nabla_{ij}^{(k)}$ and $\nabla_{iu}^{(k)}$ are selected as some fraction of the current design variable values this may vary from 1% to 100 % [5]. If the resulting LP problem turns out to be infeasible, the move limits will need to be relaxed, larger changes in the design must be allowed and the sub problem solved again. We have to select proper move limits and adjust them at every iteration to solve the problem successfully.

4 Proposed Algorithms to Solve FLSPP Using Move Limits

Step 1: Select Fuzzy Sequential Linear Programming Problem (FSLPP) to which the optimization in needed

$$\begin{aligned}
&Max\ or\ Min\ \tilde{z} = \textstyle\sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \\
&Subject\ to \\
&\qquad \textstyle\sum_{j=1}^{n} a_{ij}\tilde{x}_j \le \tilde{b}_i\ ;\ i = 1,2,3..... \\
&\qquad \textstyle\sum_{j=1}^{n} a_{ij}\tilde{x}_j \ge \tilde{b}_i\ ; i = m_0+1, m_1+1, ...m \\
&and\ \tilde{x}_j \ge 0\ \ for\ all\, j = 1,2,3.....
\end{aligned}$$

Step 2: Covert Fuzzy Sequential Linear Programming Problem (FSLPP) into Hexagonal Fuzzy Sequential Linear Programming Problem (HFSLPP).

$$\begin{aligned}
&Max\ or\ Min\ \tilde{z} = \textstyle\sum_{j=1}^{n} (c_1, c_2, c_3, c_4, c_5, c_6)\,\tilde{x}_j \\
&Subject\ to \\
&\qquad \textstyle\sum_{j=1}^{n} (a_1, a_2, a_3, a_4, a_5, a_6)\tilde{x}_j \le\ or \ge\ or\ = (b_1, b_2, b_3, b_4, b_5, b_6) \\
&\quad and\ \tilde{x}_j \ge 0\ \ for\ all\, j = 1,2,3.....
\end{aligned}$$

Step 3: Reformulate the HFSLPP, using the Ranking function

$$\begin{aligned}&Max\ or\ Min\ \tilde{z} = \textstyle\sum_{j=1}^{n} R(c_1, c_2, c_3, c_4, c_5, c_6)\,\tilde{x}_j\\ &Subject\ to\\ &\qquad \textstyle\sum_{j=1}^{n} R(a_1, a_2, a_3, a_4, a_5, a_6)\tilde{x}_j \leq\ or \geq\ or\ =\ R(b_1, b_2, b_3, b_4, b_5, b_6)\\ &\qquad and\ \ \tilde{x}_j \geq 0\ \ for\ all\, j = 1, 2, 3.....\end{aligned}$$

Step 4: Estimate the point as $\tilde{x}^{(k)}$ and K. (Initially, let k = 0 and declare two small positive numbers $\in_1 = 0.01$ and $\in_2 = 0.01$).
Step 5: Compute the fuzzy cost and fuzzy constraint functions at the point $\tilde{x}^{(k)}$.

$$\begin{aligned}&Minimize\, f(\tilde{x})\ = f(\tilde{x}^{(k)}) + \nabla f(\tilde{x}^{(k)}).\tilde{d}\\ &Subject\ to\\ &\qquad h\,(\tilde{x})\ = h(\tilde{x}^{(k)}) + \nabla h_j^T(\tilde{x}^{(k)}).\tilde{d} = 0\\ &\qquad g(\tilde{x})\ \ = g(\tilde{x}^{(k)}) + \nabla g_j^T(\tilde{x}^{(k)}).\tilde{d} \leq 0\\ &\qquad and\ \ \tilde{x}_j \geq 0\ \ for\ all\, j = 1, 2, 3.....\,p\end{aligned}$$

Step 6: Select move limits at $\nabla_{ij}^{(k)}$ and $\nabla_{iu}^{(k)}$. (To optimize the solution, the moving limits are considered as a fraction indicates the deviation of the current position of the solution to where the optimization is needed).
Step 7: Create the modified FLPP containing moving limits and fuzzy cost and fuzzy constraints function gradients.

$$\begin{aligned}&Minimize\, f(\tilde{x})\ = f(\tilde{x}^{(k)}) + \nabla f(\tilde{x}^{(k)}).\tilde{d}\\ &Subject\ to\\ &\qquad h\,(\tilde{x})\ = h(\tilde{x}^{(k)}) + \nabla h_j^T(\tilde{x}^{(k)}).\tilde{d} = 0\\ &\qquad g\,(\tilde{x})\ = g(\tilde{x}^{(k)}) + \nabla g_j^T(\tilde{x}^{(k)}).\tilde{d} \leq 0\\ &\qquad -{\nabla_{ij}}^{(k)}\ \leq \tilde{d}^{(k)}\ \leq\ {\nabla_{iu}}^{(k)}\\ &\qquad and\ \ \tilde{x}_j \geq 0\ \ for\ all\ i, j = 1, 2, 3.....\,p\end{aligned}$$

Step 8: Convert the modified FLPP into standard Fuzzy LPP. Solve the standard FLPP by using fuzzy Simplex method, to get $\tilde{d}^{(k)}$.
Step 9: Check if $\tilde{g}_i \leq \in_1$ and $\left|\tilde{h}_i\right| \leq \in_1$; $i = 1, 2...$ and $\left\|\tilde{d}_i^k\right\| \leq \in_2$; $i = 1, 2...$ then stop. Otherwise, go to step 4, by taking $\tilde{x}^{(k+1)} = \tilde{x}^{(k)} + \tilde{d}^{(k)}$. Set $k = k + 1$.

5 Numerical Example

$$\begin{aligned}&\text{Minimize } f(\widetilde{x}) = \widetilde{x}_1^2 + \widetilde{x}_2^2 - 3\widetilde{x}_1\widetilde{x}_2\\ &\text{Subject to Constraints}\\ &\qquad \tfrac{\widetilde{x}_1^2}{6} + \tfrac{\widetilde{x}_2^2}{6} \leq 1\\ &\qquad\quad -\widetilde{x}_1 \leq 0\\ &\qquad\quad -\widetilde{x}_2 \leq 0\\ &\text{Non} - \text{negative restrictions}\,\widetilde{x}_1, \widetilde{x}_2 \geq 0\end{aligned}$$

Solution: The Formulated Fuzzy Sequential Linear Programming Problem is converted in to Hexagonal Fuzzy Sequential Linear Programming Problem as

$$\text{Minimize}\, f(\widetilde{x}) = (0, 0.5, 1, 1, 1.5, 2; 1)\widetilde{x}_1^2 + (0, 0.5, 1, 1, 1.5, 2; 1)\widetilde{x}_2^2 - (1, 2, 3, 3, 4, 5; 1)\widetilde{x}_1\widetilde{x}_2$$

Subject to

$$\frac{(0,0.1,0.15,0.15,0.3,0.35;1)}{6}\widetilde{x}_1^2+\frac{(0,0.1,0.15,0.15,0.3,0.35;1)}{6}\widetilde{x}_2^2 \leq (0,0,0,0,0,0;1)$$
$$-(0,0.5,1,1,1.5,2;1)\widetilde{x}_1 \leq (0,0,0,0,0,0;1)$$
$$-(0,0.5,1,1,1.5,2;1)\widetilde{x}_2 \leq (0,0,0,0,0,0;1)$$

Non negative restriction $\widetilde{x}_1, \widetilde{x}_2 \geq (0,0,0,0,0,0)$. By using Ranking function, we obtained

$$R(0,0.5,1,1,1.5,2;1) = 0.35$$
$$R(1,2,3,3,4,5;1) = 1.05$$
$$R(0,0.1,0.15,0.15,0.3,0.35;1) = 0.05$$

Therefore

$$\text{Minimize } f(\widetilde{x}) = 0.35\widetilde{x}_1^2 + 0.35\widetilde{x}_2^2 - 1.05\widetilde{x}_1\widetilde{x}_2$$

Subject to

$$0.05\widetilde{x}_1^2 + 0.05\widetilde{x}_2^2 \leq 0$$
$$-0.35\widetilde{x}_1 \leq 0$$
$$-0.35\widetilde{x}_2 \leq 0$$

Non negative restriction $\widetilde{x}_1, \widetilde{x}_2 \geq 0$.

Let us calculate fuzzy cost and fuzzy constraints gradients functions at the point $k = 0$ and $\widetilde{x}^{(k)}$. Choose $\in_1 = \in_2 = 0.01$. $\widetilde{x}^{(0)} = (1, 1)$.

Iter	$\widetilde{x}^{(k)}$	Linearization	M.Lts	Solutions	Con's
1	$\widetilde{x}^{(0)} = (1.00, 1.00)$	Min $f(\widetilde{x}) = -0.35\tilde{d}_1 - 0.35\tilde{d}_2 - 0.35$ Subject to $0.1\tilde{d}_1 + 0.1\tilde{d}_2 \leq 0.25$ $-0.35\tilde{d}_1 \leq 0.35$ $-0.35\tilde{d}_2 \leq 0.35$ $-0.15 \leq \tilde{d}_1 \leq 0.15$ $-0.15 \leq \tilde{d}_2 \leq 0.15$ and $\tilde{d}_1, \tilde{d}_2 \geq 0$.	15%	$\tilde{x}_1 = 1.15$ $\tilde{x}_2 = 1.15$	No
2	$\widetilde{x}^{(1)} = (1.15, 1.15)$	Min $f(\widetilde{x}) = -0.40\tilde{d}_1 - 0.40\tilde{d}_2 - 1.3225$ Subject to $0.115\tilde{d}_1 + 0.115\tilde{d}_2 \leq 0.218$ $-0.35\tilde{d}_1 \leq 1.15$ $-0.35\tilde{d}_2 \leq 1.15$ $-0.11 \leq \tilde{d}_1 \leq 0.11$ $-0.11 \leq \tilde{d}_2 \leq 0.11$ and $\tilde{d}_1, \tilde{d}_2 \geq 0$	11%	$\tilde{x}_1 = 1.26$ $\tilde{x}_2 = 1.26$	No
3	$\widetilde{x}^{(2)} = (1.26, 1.26)$	Min $f(\widetilde{x}) = -0.441\tilde{d}_1 - 0.441\tilde{d}_2 - 0.556$ Subject to $0.126\tilde{d}_1 + 0.126\tilde{d}_2 \leq 0.19$ $-0.35\tilde{d}_1 \leq 0.441$ $-0.35\tilde{d}_2 \leq 0.441$ $-0.06 \leq \tilde{d}_1 \leq 0.06$ $-0.06 \leq \tilde{d}_2 \leq 0.06$ and $\tilde{d}_1, \tilde{d}_2 \geq 0$.	6%	$\tilde{x}_1 = 1.32$ $\tilde{x}_2 = 1.32$	No
4	$\widetilde{x}^{(3)} = (1.32, 1.32)$	Min $f(\widetilde{x}) = -0.462\tilde{d}_1 - 0.462\tilde{d}_2 - 0.609$ Subject to $0.132\tilde{d}_1 + 0.132\tilde{d}_2 \leq 0.176$ $-0.35\tilde{d}_1 \leq 0.462$ $-0.35\tilde{d}_2 \leq 0.462$ $-0.01 \leq \tilde{d}_1 \leq 0.01$ $-0.01 \leq \tilde{d}_2 \leq 0.01$ and $\tilde{d}_1, \tilde{d}_2 \geq 0$.	1%	$\tilde{x}_1 = 1.33$ $\tilde{x}_2 = 1.33$	Yes

Therefore the criteria $\tilde{g}_i \leq \in_1$ and $\left|\tilde{h}_i\right| \leq \in_1$; $i = 1, 2...$ and $\left\|\tilde{d}_i^k\right\| \leq \in_2$; $i = 1, 2...$ is satisfied. The optimal solution is $\tilde{x}_1 = 1.33$ and $\tilde{x}_2 = 1.33$.

6 Conclusion

In this paper, a new algorithm is proposed to solve fuzzy sequential linear programming problem by using move limits methodology. The algorithm is successfully tested in Fuzzy Sequential linear programming problem having nonlinearity in both objective function and also in constraints. An illustration is given for the solvation of fuzzy sequential linear programming problem using the newly proposed algorithm. The optimal solutions obtained from the proposed algorithm are more approximate than many other solutions obtained from the existing methods.

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