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A New Solvable Procedure For Fuzzy Linear Complementarity Problem With Symmetric Trapezoidal Intuitionistic Fuzzy Numbers Approach

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Abstract

A new method namely, algebraic elimination method is proposed for finding a complementarity feasible solution to the Symmetric Trapezoidal Intuitionistic fuzzy linear complementarity problem which has applications in non-linear programming. Then, the algebraic elimination method is extended to Symmetric Trapezoidal
Intuitionistic fuzzy quadratic programming problems. The solution procedure is illustrated by means of numerical examples.

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1 Introduction

The linear complementarity (LC) problem is one of the most widely studied problems of mathematical programming since it arise in a variety of applications [3, 4, 6] in engineering, economics and applied sciences. Several methods have been proposed for solving LC problems. Iterative methods for the solution of LC problem were considered in [2]. Yassine discussed a comparative study between Lemke's method and the Interior point method for the monotone linear complementarity problem.

A Fuzzyquadratic programming (QP) problem is a special case of non- linear programming problem. The two important methods for solving QP problem are Wolfe's method and Beale's method [1]. Wolfe used Kuhn Tucker conditions to transform the QP problem into linear inequalities and complementarity slackness. Also, FQPP can be transformed into FLCP using the KKT conditions and then, it can be solved by Lemke's algorithm.

In this paper, we propose a new method namely, algebraic elimination method for finding a complementarity feasible solution to the FLCP. In the algebraic elimination method, we first reduce the FLCP into an inequality system by using the relation $W \geq 0$ and then, the resulting reduced inequality system is solved by matrix/algebraic method. The proposed method for solving FLCP is very simple and easy to understand and also, to apply. Further, we extend the algebraic elimination method for solving Fuzzy quadratic programming problems with linear constraints after converting to FLCP. Numerical examples are given for better understanding of the solution procedures of the proposed method.

2 Preliminaries

Definition 1 (Fuzzy set)**.** A Fuzzy set Ã is defined by $\tilde{A} = \{x, \mu_A(x)\}; x \in A, \mu_A(x) \in [0,1]$. In the pair $(x,\ \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval $[0,1]$ called membership function.

Definition 2 (Symmetric Trapezoidal intuitionistic fuzzy number (STIFN))**.** A Symmetric Trapezoidal intuitionistic fuzzy number (STIFN) is an intuitionistic fuzzy set in R with the following function $\mu_{\tilde{A}^I(X)}$ and non-membership function $v_{\tilde{A}^I(X)}$.

$$\mu_{\tilde{A}^I} = \begin{cases} 0, & x < a_1 - h \\ \dfrac{x-(a_1-h)}{h}, & a_1 - h \leq x \leq a_1 \\ 0, & a_1 \leq x \leq a_2 \\ \dfrac{(a_2+h)-x}{h}, & a_2 \leq x \leq a_2 + h \\ 1, & a_2 + h \leq x \end{cases} \qquad \nu_{\tilde{A}^I} = \begin{cases} 1, & x < a_1 - h' \\ \dfrac{x-(a_1-h')}{h'}, & a_1 - h' \leq x \leq a_1 \\ 0, & a_1 \leq x \leq a_2 \\ \dfrac{(a_2+h')-x}{h'}, & a_2 \leq x \leq a_2 + h' \\ 1, & a_2 + h' \leq x \end{cases}$$

Where $a_1 \leq a_2$ and $h, h' \geq 0$. This STIFN is denoted by $\tilde{A}^I = \{(a_1,\ a_2,\ h,\ h); (a_1, a_2, h', h')\}$ for our Convenience. STIFN is denoted by $\tilde{A}^I = (a_1, a_2, h, h')$ throughout this paper.

Definition 3. Modified arithmetic Operations on Symmetric trapezoidal intuitionistic fuzzy numbers (STIFNS). Let $\tilde{A}^I = (a_1, a_2, h, h')$ and $\tilde{B}^I = (b_1, b_2, k, k')$ be two Symmetric trapezoidal intuitionistic fuzzy numbers. Then

(i) Addition: $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, h + k, h' + k')$

(ii) Subtraction: $\tilde{A}^I - \tilde{B}^I = (a_1 - b_2, a_2 - b_1, h + k, h' + k')$

(iii) Multiplication:

$$\tilde{A}^I \times \tilde{B}^I = \left(\frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - W\right), \left(\frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - W.|w - w'|.|w - w_1'|\right)$$

Where

$$w = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(m) \cdot \max(m) - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right)$$
$$w' = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(n) \cdot \max(n) - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right)$$
$$w_1' = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(n') \cdot \max(n') - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right)$$
$$m = (a_1 b_1, a_1 b_2, a_2 b_1, a_2, b_2),$$
$$n = (a_1 - h)(b_1 - k).(a_1 - h)(b_1 + k).(a_2 + h)(b_1 - k).(a_2 + h)(b_2 + k),$$
$$n' = (a_1 - h')(b_1 - k').(a_1 - h')(b_1 + k').(a_2 + h')(b_1 - k').(a_2 + h')(b_2 + k')$$

(iv) Division:

$$\tilde{A}^I \times \tilde{B}^I = \left(\frac{a_1 + a_2}{b_1 + b_2} - w, \frac{a_1 + a_2}{b_1 + b_2} + |w - w'||w - w_1\right)$$

Where

$$w = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(m), \max(m) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\}$$
$$w' = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(n), \max(n) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\}$$
$$w_1^1 = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(n^1), \max(n^1) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\},$$
$$m = \left(\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right), n = \left(\left[\frac{a_1 - h}{b_1 - k}\right], \left[\frac{a_1 - h}{b_2 + k}\right], \left[\frac{a_2 + h}{b_1 - k}\right], \left[\frac{a_2 + h}{b_2 + k}\right]\right),$$
$$n' = \left(\left[\frac{a_1 - h'}{b_1 - k'}\right], \left[\frac{a_1 - h'}{b_2 + k'}\right], \left[\frac{a_2 + h'}{b_1 - k'}\right], \left[\frac{a_2 + h'}{b_2 + k'}\right]\right).$$

Definition 4. Let $F(S)$ be the set of all Symmetric trapezoidal intuitionistic fuzzy numbers. For $\tilde{A}^I = \{(a_1, a_2, h, h); (a_1, a_2, h', h')\} \in \mathfrak{R}(s)$ we define a ranking function $F : F(s) \to \mathfrak{R}$ by $F(\tilde{A}^I) = \dfrac{(a_1 + a_2)}{2} + (h - h')$.

3 Intuitionistic Fuzzy Linear Complementarity Problem (IFLCP)

3.1 Fuzzy Linear Complementarity Problem (FLCP)

Assume that all parameters in (1)-(3) are fuzzy and are described by fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \quad (1)$$

$$\tilde{W}_j \geq 0, Z_j \geq 0, \qquad j = 1, 2, 3, \ldots, n \quad (2)$$

$$\tilde{W}_j\tilde{Z}_j = 0, \qquad j = 1, 2, 3, \ldots\ldots\ldots\ldots n \quad (3)$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy complementary variables.

3.2 Intuitionistic Fuzzy Quadratic Programming Problem (QPP) as a Intuitionistic fuzzy Linear Complimentarity Problem (LCP)

Consider the following Quadratic Programming Problem
Minimize $\tilde{f}^I(\tilde{x})^I = \tilde{c}^{t^I}\tilde{x}^I + \frac{1}{2}\tilde{x}^{t^I}\tilde{H}^I\tilde{x}^I$ Subject to $\tilde{A}^I\tilde{x}^I \leq \tilde{b}^I$, and $\tilde{x}^I \geq 0$
Where $\tilde{c}$ an n-vector of fuzzy numbers is, $\tilde{b}$ is an m-vector, $\tilde{\text{A}}$ is an $m \times n$ fuzzy matrix and $\tilde{H}$ is an $n \times n$ fuzzy symmetric matrix. Let $\tilde{y}$ denotes the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian multiplier vectors of the constraints $\tilde{\text{A}}\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ respectively, then the Kuhn-Tucker conditions can be written as $\tilde{\text{A}}^I\tilde{x}^I + \tilde{y}^I = \tilde{b}^I - \tilde{H}^I\tilde{x}^I - \tilde{A}^{t^I}\tilde{u}^I + \tilde{v}^I = \tilde{c}^I\tilde{x}^{t^I}\tilde{v}^I = \tilde{0}^I, \tilde{u}^{t^I}\tilde{y}^I = \tilde{0}^I$ And $\tilde{x}^I, \tilde{y}^I\tilde{u}^I, \tilde{v}^I \geq \tilde{0}^I$. Now Letting $\tilde{M}^I = \begin{bmatrix} \tilde{0}^I & -\tilde{A}^I \\ \tilde{A}^{t^I} & \tilde{H}^I \end{bmatrix}, \tilde{q}^I = \begin{bmatrix} \tilde{b}^I \\ \tilde{c}^I \end{bmatrix}, \tilde{w}^I = \begin{bmatrix} \tilde{y}^I \\ \tilde{v}^I \end{bmatrix}$ and $\tilde{z}^I = \begin{bmatrix} \tilde{u}^I \\ \tilde{v}^I \end{bmatrix}$ the Kuhn-Tucker conditions can be expressed as the LCP. $\tilde{W}^I - \tilde{M}^I\tilde{Z}^I = \tilde{q}^I, \tilde{W}^{t^I}\tilde{Z}^I = \tilde{0}^I, (\tilde{W}^I, \tilde{Z}^I) \geq \tilde{0}^I$.

4 Proposed Algorithm

Now, we propose a new method namely, algebraic elimination method for finding a complementarity feasible solution to the LC problem. The proposed method proceeds as follows:

Step 1. If $M \geq 0$ and $q \leq 0$, then reduce the given system $-MZ = q, Z \geq 0$ by taking $W = 0$ and solve the reduced system by matrix/algebraic method. Say $Z = Z^o$. Then, $(w = 0,\ Z = Z^o)$ is a complementarity feasible solution to LC problem.

Step 2. If there is no negative term in q, then $(W = q, Z = 0)$ is a complementarity feasible solution to LC problem.

Step 3. If $q < 0$ for atleast one term, move on to Step 4.

Step 4. Construct the inequality system $-MZ \leq q; -Z \leq 0$ from the given system using $W \geq 0$.

Step 5. Eliminate the Z-variables one by one upto only one of the Z-variable obtained. Say Z_k.

Step 6. Find the greatest lower bound of all maximum possible values of Z_k. Say, Z_{k^o} The values of all possible Z_j's are computed using backward substitution method and basic algebraic method by making the involved inequality into equality. Say $Z_j = Z_{j^o}, j = 1, 2, \ldots, n$.

Step 7. If $m = n$, go to Step 9. Otherwise, move on to Step 8.

Step 8. (a) Take $W_j = 0$ if $Z_{j^o} \neq 0$, $j = 1, 2, \ldots, n$ (b) Substituting the values of w_j, $j = 1, 2, \ldots, n$ and the values of $Z_j, j = 1, 2, \ldots, n$ in $W - MZ = q$ obtained in 8(a), compute the rest of W and Z-values. Thus, W- value and Z-value are obtained. Say, W^o and $Z^o(c)(W^o,\ Z^o)$ is a complementarity feasible solution to the LC problem.

Now, the proposed method is illustrated with the help of the following examples.

Example 5. Consider the following FLC problem

$$\begin{bmatrix} \tilde{2}^I & \tilde{1}^I \\ \tilde{1}^I & \tilde{2}^I \end{bmatrix} \begin{bmatrix} -\tilde{5}^I \\ -\tilde{6}^I \end{bmatrix} (\tilde{0}^I, \tilde{0}^I)\tilde{W}^I - \tilde{M}^I\tilde{Z}^I = \tilde{q}^I; \tilde{W}^I, \tilde{Z}^I \geq 0; \tilde{W}^I\tilde{Z}^I = 0.$$

Where $\tilde{M}^1 = \begin{bmatrix} \tilde{1}^I & \tilde{0}^I & \tilde{2}^I \\ \tilde{3}^I & \tilde{2}^I & -\tilde{1}^I \\ -\tilde{2}^1 & \tilde{1}^I & \tilde{0}^I \end{bmatrix}$ and $\tilde{q}^I = \begin{bmatrix} -\tilde{1}^I \\ \tilde{2}^I \\ -\tilde{3}^I \end{bmatrix}$

Now, by the Step 3. and the Step 4. We have the following inequality system $-\tilde{M}^I\tilde{Z}^I \leq \tilde{q}^I, \tilde{Z}^I \geq 0$. The Table form of the above problem can be given in Table 1.

Table 1:

Z_1	Z_2	Z_3	Inequality	Q	$Eqn.$ No	Operation
$(0,0,0,0)$	$(0,0,0.0)$	$-(1,1,0,0)$	$\leq$	$-(4,8,2,2)$	(A)	
$(0,0,0,0)$	$-(1,3,2,2)$	$-(1,1,0,0)$	$\leq$	$(0,0,0,0)$	(B)	
$(1,1,0,0)$	$(1,1,0,0)$	$(0,0,0,0)$	$\leq$	$-(3,5,2,2)$	(C)	

Z_1	Z_2	Z_3	Inequality	Q	$Eqn.$ No	Operation
$-(1,1,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$\leq$	$(0,0,0,0)$	(D)	
$(0,0,0,0)$	$-(1,1,0,0)$	$(0,0,0,0)$	$\leq$	$(0,0,0,0)$	(E)	
$(0,0,0,0)$	$(0,0,0,0)$	$-(1,1,0,0)$	$\leq$	$(0,0,0,0)$	(F)	

Now, we proceed to the Step 5, eliminating Z_1 from the Table 1, we obtain the following Table.
We proceed to the Step 6. To the Step 9, we conclude that $-(1,1,0,0)\tilde{Z}_2^I \geq -(2,4,2.3,2.3)$. Now since that $-(1,1,0,0)\tilde{Z}_2^I \geq (2,4,2.3,2.3)$ corresponds to (H) and since(H) corresponds to (D) and (C), we have $(1,1,0,0)$ $\tilde{Z}_1^I = (0,0,0,0)$. Now, since $(1,1,0,0)\tilde{Z}_2^I \geq -(2,4,2.3,2.3)$, we have $(1,1,0,0)$ $\tilde{w}_2^I = (0,0,0,0)$. Now substituting the values of $(1,1,0,0)\tilde{Z}_2^I \geq -(2,4,2.3,2.3)$, $((1,1,0,0)$ $\tilde{w}_2^I = (0,0,0,0)$ and $(1,1,0,0)\tilde{Z}_1^I = (0,0,0,0)$, in the given system we have $(1,1,0,0)$ $\tilde{w}_2^I - (1,3,2,2)\tilde{Z}_3^I = -(1,1,0,0)$; $(1,1,0,0)\tilde{Z}_3^I = (1,3,2,2) + (1,3,2,2)\tilde{Z}_2^I$; $(1,1,0,0)$

$\tilde{w}_3^I = -(2,4,2.3,2.3)$. $+ (1,1,0,0)\tilde{Z}_{2;}^I\tilde{w}_1^I \geq (0,0,0,0)$; $\tilde{w}_3^I \geq (0,0,0,0)$; $\tilde{Z}_3^I \geq (2,4,2.3,2.3)$; $\tilde{Z}_3^I\tilde{w}_3^I = (0,0,0,0)$.This implies that $\tilde{Z}_3^I = (7,9,3.7,3.7)$, $\tilde{w}_1^I = (12.5,17.5,10.5,10.5)$, $\tilde{Z}_2^I = (1,3,2,2)$ and $\tilde{w}_3^I = (0,0,0,0)$. Thus the complimentarity feasible solution to the FLC problem is $\tilde{w}^I = ((12.5,17.5,10.5,10.5),\ \ 0,0,0,0),(0,0,0,0))$ and $\tilde{Z}^I = ((0,0,0,0),(1,3,2,2),(7,9,3.7,3.7))$.

Example 6. Consider the following QP problem
Minimize $Z = -6x1^{+}X2^2$ Subject to $x+ \leq 4; xl, x2 \geq 0$.
Here $W = \begin{bmatrix} V \\ Y \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}; Z = \begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}; C = (-6,0)^T$; $Q = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}; A = [1\ 1]$ and $b = [4]$ Now $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ and
$q = \begin{bmatrix} -6 \\ 0 \\ 4 \end{bmatrix}$ thus STIFN Problem for the proposed algorithm becomes
Now consider the following SITFLC problem which corresponds to the given data that
Now by step 3 and the step 4, we have the following inequality system -$\tilde{M}^I\tilde{Z}^I \leq \tilde{q}^I, \tilde{Z}^I \geq 0$. The table form of the above problem can be given as follows:

Table 2:

Z_1	Z_2	Z3	inequality	q	Eqn. No	Operation
$-(1,1,0,0)$	$(0,0,0,0)$	$-(1,3,2,2)$	$\leq$	$-(1,1,0,0)$	(A)	
$-(2,4,2,2)$	$-(1,3,2,2)$	$(1,1,0,0)$	$\leq$	$(1,3,2,2)$	(B)	
$(1,3,2,2)$	$-(1,1,0,0)$	$(0,0,0,0)$	$\leq$	$-(2,4,2,2)$	(C)	
$-(1,1,0,0)$	$(0,0,0,0)$	$(0,0,0,0)$	$\leq$	$(0,0,0,0)$	(D)	
$(0,0,0,0)$	$-(1,1,0,0)$	$(0,0,0,0)$	$\leq$	$(0,0,0.0)$	(E)	
$(0,0,0,0)$	$(0,0,0,0)$	$-(1,1,0,0)$	$\leq$	$(0,0,0,0)$	(F)	

Now, we proceed to the Step 5, eliminating Z_1 from the Table 2, we obtain the following table
we conclude that $(1,1,0,0)\tilde{Z}_3^I \geq (5.5,10.5,2.67,2.67)$
Now, since $(1,1,0,0)\tilde{Z}_3^I \geq (5.5,10.5,2.67,2.67)$, Corresponds to (A)
Now, since $(1,1,0,0)\tilde{Z}_3^I \geq$ (5.5,10.5,2.67,2.67), we have $(1,1,0,0)\tilde{w}_3^I = (0,0,0,0)$.
Now, substituting the values of $(1,1,0,0)\tilde{Z}_3^I \geq (5.5,10.5,2.67,2.67)$ and $(1,1,0,0)$ $\tilde{w}_3^I = (0,0,0,0)$ the given system we have $(1,1,0,0)\tilde{w}^I = -(4,8,2,2) + (1,1,0,0)\tilde{Z}_3^I$; $(1,1,0,0)\tilde{w}_2^I = (1,3,2,2)\tilde{Z}_2^I + (1,1,0,0)Z_3$; $(1,1,0,0)\tilde{Z}_1^I + (1,1,0,0)\tilde{Z}_2^I = (3,5,2,2)$; $\tilde{Z}_1^I \geq (0,0,0,0)\tilde{Z}_2^I \geq (0,0,0,0)$, $\tilde{Z}_3^I \geq (5.5,10.5,2.67,2.67)$, $\tilde{w}_1^I \geq (0,0,0,0)\tilde{w}_2^I \geq (0,0,0)$, $\tilde{Z}_2^I\tilde{w}_2^I = (0,0,0,0)$, $\tilde{Z}_1^I\tilde{w}_1^I = (0,0,0,0)$. This implies that $\tilde{w}_1^I = (0,0,0,0)$, $\tilde{Z}_2^I = (0,0,0,0)$, $\tilde{Z}_1^I = (3,5,2,2)$; , $\tilde{Z}_3^I = (5.5,10.5,2.67,2.67)$ and $\tilde{w}_2^I = (5,7,0.67,0.67)$.
Therefore, the complimentarity feasible solution to the STIFLC Problem is
$\tilde{w}^I = ((0,0,0,0),\ (5,7,0.67,0.67),(0,0,0,0))$, $\tilde{Z}^I = ((3,5,2,2),(0,0,0,0),\ (5.5,10.5,2.67,2.67))$.
Thus the optimal solution to the given QP problem is $X1 = (3,5,2,2)$, $X2 = (0,0,0,0)$ and $\min K = -(12,36,9.5,9.5)$.

5 Conclusion

In this paper, we propose a new method namely, algebraic elimination method for finding a complementarity feasible solution to the Intuitionistic fuzzy linear complementarity problem. The proposed method is easy to understand and apply. Then, we extend algebraic elimination method to quadratic programming problems. With the help of numerical examples, the proposed method is illustrated for better understanding of the solution procedures. Since complementarity is central to all constrained optimization problems, the algebraic elimination method can serve decision makers by providing an appropriate best solution to a variety of linear programming complementarity models in a simple and effective manner.

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