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A Methodology to Combine Intuitionistic Pentagonal Fuzzy Interval Focal Elements and Their Corresponding Basic Probability Assignments of Two Variables in Evidence Theory

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Abstract

In this paper a method is proposed to combine intuitionistic fuzzy focal elements and their corresponding Basic Probability Assignments of two variables using Dempster Shafer Theory (DST) of evidence. Further, we discussed about interval focal elements by combining non-membership interval focal elements and their corresponding Basic Probability Assignments, under interval valued intuitionistic pentagonal fuzzy arithmetic operations. A numerical example is provided to illustrate the efficiency of the proposed model.

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Key Words and Phrases: Fuzzy sets, Dempster-Shafer Theory (DST), Basic Probability Assignments, Intuitionistic Pentagonal Fuzzy Number, Interval valued pentagon intuitionistic fuzzy arithmetic operations.

1 Introduction

Probability theory is a very strong and well established mathematical tool to deal with objective uncertainty which uncertainty arises from heterogeneity or the random character of natural processes. However, all uncertainties arising in different situations are not of objective type. Such problems cannot be handled by traditional probability theory. Uncertainty may arise due to scarce or incomplete information or data, measurement error or data obtain from expert judgement or subjective interpretation of available data or information. These are subjective nature. Traditional probability theory is inappropriate to represent subjective uncertainty. To overcome the limitations of probabilistic method, Dempster put forward a theory and now it is known as evidence theory (or) Dempster-Shafer theory in 1976. This theory is nowadays widely used for the objective and subjective uncertainty analysis. There are three basic functions that are important to understanding and applying D-S theory, the basic belief mass function which specifies the belief mass distribution overall possible sub-sets of a frame of discernment, the belief function, and the plausibility function. Atanassov [3] introduced the concept of intuitionistic fuzzy set (IFS), which is generalization of concept of fuzzy sets [4]. Gau and Buehrer [6] introduced the concept of vague set. But Bustince and Burillo [7] showed that vague sets are intuitionistic fuzzy sets (IFSS). Herrere et al [2] developed an aggregation process for combining numerical interval valued and linguistic information. The interval valued intuitionistic fuzzy sets(IVIFSS), introduced by Atanassov and Gargo [7], each of which is characterized by a membership and a non-membership function whose values are rather than exact numbers, are very useful to describe the decision information in the process of decision making.

In this paper, interval focal elements and the corresponding BPAs, under interval valued intuitionistic pentagonal fuzzy operations rather than interval valued fuzzy arithmetic operations and highly acquired better results.

The paper is organized as follows: The Dempster - Shafer theory is given in Section 2 and Section 3 deals with basic definitions and intuitionistic pentagonal fuzzy operations the concepts of algebraic combination of focal elements is provided in Section 4. Finally, in Section 5 the effectiveness of the proposed method is illustrated by means of example and some concluding remarks are given in Section 6.

2 Preliminaries

2.1 Dempster-Shafer Theory: (D-S Theory)

The mathematical theory of evidence, also known as the D-S theory, is a generalization of the Bayesian theory of probability (l). A belief structure is a mass function $m : 2^{\Theta} \to [0,1]$ such that

(i) $m(\emptyset) = 0$

(ii) $\sum_{A \subseteq \Theta} m(A) = 1$

where $\emptyset$ is an empty set and A is any subset of Θ. The mass $m(A)$ is expressed as the amount of belief that is allocated to A.

Given two independent mass functions m_1 and m_2 Dempster's combination rule can be used to combine the functions into single mass function (2). The combined mass function is

$$m(C) = \frac{\sum\limits_{A\cap B=C} m_1(A)m_2(B)}{1 - \sum\limits_{A\cap B=\emptyset} m_1(A)m_2(B)} \quad \text{where } C \neq \emptyset, m_{12}(\emptyset) = 0.$$

3 Intutuinistic Fuzzy Sets:[11]

Definition 1. Given a fixed set $X = \{x_1, x_2, \ldots, x_n\}$, an intuitionistic fuzzy set (IFS) is defined as $\tilde{A}^I = (\langle x_i, \mu_{\tilde{A}^I}(x_i), \theta_{\tilde{A}^I}(x_i)\rangle / x_i \in X)$ which assigns to each element x_i, a membership degree $\mu_{\tilde{A}^I}(x_i) and$ a non-membership degree $\theta_{\tilde{A}^I}(x_i)$ under the condition $0 \leq \mu_{\tilde{A}^I}(x_i) + \theta_{\tilde{A}^I}(x_i) \leq 1$, for all $x_i \in X$

Definition 2. Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$ and $X(\neq \emptyset)$ be a given set. An IFS A in X is defined as

$$\tilde{A}^I = (\langle x_i, \mu_{\tilde{A}^I}(x_i),\ \theta_{\tilde{A}^I}(x_i)\rangle / x_i \epsilon X),$$

where $\mu_{\tilde{A}^I} : X \to D[0,1], \theta_{\tilde{A}^I} : X \to D[0,1]$ with the condition $0 \leq \sup(\mu_{\tilde{A}^I}(x_i)) + \sup(\theta_{\tilde{A}^I}(x_i)) \leq 1$ for any $x \in X$.

Definition 3. A $(\alpha,\ \beta)-$ cut set of a intuitionistic fuzzy number is defined as $A^I_{\alpha,\beta} = \{(x/\mu_{\tilde{A}^I}(x) \geq \alpha, 4r_{\tilde{A}^I}(x) \leq \beta\}$, where $0 \leq \alpha \leq 1; 0 \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$.

Definition 4 (Yun and Lee, 2013)**.** A pentoganal intuitionistic fuzzy number (PIFN) $\tilde{A}^{Ip}$ is defined as an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^{Ip}}(x)$ and non membership function $\nu_{\tilde{A}^{Ip}}(x)$:

$$\mu_{\tilde{A}^{Ip}} = \begin{cases} \dfrac{x-a_1}{2a_2-2a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{1}{2} + \dfrac{x-a_2}{2a_3-2a_2}, & a_2 \leq x \leq a_3 \\ 1, & x = a_3 \\ 1 - \dfrac{x-a_3}{2a_4-2a_3}, & a_3 \leq x \leq a_4 \\ \dfrac{a_3-x}{2a_3-2a_4}, & a_4 \leq x \leq 3 \\ 0, & \text{Otherwise} \end{cases}$$

and

$$\nu_{\tilde{A}^{Ip}} = \begin{cases} \dfrac{2a_2 - a_1 - x}{2a_2 - 2a_1}, & a_1 \le x \le a_2 \\ \dfrac{1}{2} - \dfrac{x - a_2}{2a_3 - 2a_2}, & a_2 \le x \le a_3 \\ 0, & x = a_3 \\ \dfrac{x - a_3}{2a_4 - 2a_3}, & a_3 \le x \le a_4 \\ \dfrac{x - 2a_4 + a_3}{2a_3 - 2_4}, & a_4 \le x \le 3 \\ 1, & \text{Otherwise} \end{cases}$$

Where $a_1 \le x \le a_2 \le a_3 \le x \le a_4$ and $0 \le \mu_{\tilde{A}^{Ip}}(x) + 4\nu_{\tilde{A}^{Ip}}(x) \le 1$ or $\mu_{\tilde{A}^{Ip}}(x) = 4\nu_{\tilde{A}^{Ip}}(x)$, for all $x \in R$

Definition 5. A positive interval valued intuitionistic fuzzy number is denoted as $\{(a_1,\ a_2\ (a'_1,\ a'_2)\}$ where all a'_is and $a'_is > 0$ for all $i = 1, 2$.

A negative interval valued intuitionistic fuzzy number is denoted as $\{a_2(a'_1,\ a'_2)\}$ where all a_i's and $a'_is < 0$ for all $i = 1, 2$.

Definition 6. A positive pentagon fuzzy number $\tilde{A}_p$ is denoted as $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ where $a'_is > 0$ for all $i = 1, 2, 3, 4, 5$ (e.g) $\tilde{A}_p = (3, 5, 7, 9, 11)$.

A negative pentagon fuzzy number $\tilde{A}_p$ is denoted as $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ where $a'_is < 0$ for all $i = 1, 2, 3, 4, 5$ (e.g) $\tilde{A}_p = (-5, -4, -3, -2, -1)$.

Definition 7 (Arithmetic operations of interval valued intuitionistic pentagonal fuzzy numbers using Function Principle)**.** The following are the arirthmetic operations that can be performed on interval intuitionistic pentagonal fuzzy numbers:
Let

$$\tilde{A}^{Ip} = \{(a_1,\ b_1,\ c_1,\ d_1,\ e_1\ (a_2,\ b_2,\ c_2,\ d_2,\ e_2)\}$$

and

$$\tilde{B}^{Ip} = \{(a_3,\ b_3,\ c_3,\ d_3,\ e_3\ (a_4,\ b_4,\ c_4,\ d_4,\ e_4)\}.$$

Then

(i) Addition:

$$\tilde{A}^{Ip} + \tilde{B}^{Ip} = \{(a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3, e_1 + e_3); \\ (a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4, e_2 + e_4)\}$$

(ii) Subtraction:

$$\tilde{A}^{Ip} - \tilde{B}^{Ip} = \{(a_1 - e_3, b_1 - d_3, c_1 - c_3, d_1 - b_3, e_1 - a_3); \\ (a_2 - e_4, b_2 - d_4, c_2 - c_4, d_2 - b_4, e_2 - a_4)\}$$

4 Algebraic Combination of Focal Elements: [2]

Let X_1 and X_2 be two variables whose values are represented by Dempster- Shafer structure with focal elements $A_1, A_2, A_3, \ldots, A_n$ and $B_1, B_2, B_3, \ldots, B_m$ which are considered as intervals and their corresponding Basic Probability Assignments (BPA) are as follows:

$m(A_i) = a_i$ and $m(B_i) = b_j, i = 1, 2, 3, \ldots, n \& j = 1, 2, 3, \ldots, m$ respectively.

where $\sum_{i=1}^{n} a_i = 1$ and $\sum_{j=1}^{m} b_j = 1$

Initially we combine all the fuzzy focal elements using fuzzy arithmetic which will produce $\dagger nm^1$ number of fuzzy focal elements and there after the corresponding basic probability assignments of resulting fuzzy focal elements will be calculated as follows.

4.1 Addition of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i + B_j) = \frac{m(A_i) + m(B_j)}{\sum_i \sum_j (m(A_i) + m(B_j))} \tag{1}$$

4.2 Subtraction of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i - B_j) = \frac{m(A_i)\,(1 - m(B_j))}{\sum_i \sum_j (m(A_i)\,(1 - m(B_j)))} \tag{2}$$

4.3 Multiplication of Fuzzy Focal Elements

$$m(C_{ij}) = m(A_i B_j) = \frac{m(A_i) m(B_j)}{\sum_i \sum_j (m(A_i) m(B_j))} \tag{3}$$

$$m(C_{ij}) = m(A_i / B_j) = \frac{m(A_i)/m(B_j)}{\sum_i \sum_j (m(A_i)/m(B_j))} \tag{4}$$

Finally, we arrange all the focal elements in increasing order of the left end point.

5 Numerical Example:[6]

Suppose Basic Probability Assignments (BPA) of two parameters is assigned by an expert and which are given in the following tables:

5.1 Addition of Focal Elements

Number of focal elements of the first parameter is four and the second parameter is five respectively. After adding the focal elements using interval valued intuitionistic pentagonal fuzzy arithmetic we get twenty numbers of focal elements. Now the

Interval valued Focal Elements		BPA
Membership value	Non Membership value	
$[5, 7]$	$[4, 8]$	0.15
$[7, 15]$	$[6, 16]$	0.20
$[9, 18]$	$[16, 20]$	0.30
$[11, 21]$	$[18, 22]$	0.35

Table 1: BPA of the first parameter

Interval valued Focal Elements		BPA
Membership value	Non Membership value	
$[25, 28]$	$[24, 29]$	0.55
$[29, 32]$	$[28, 33]$	0.25
$[30, 35]$	$[29, 34]$	0.15
$[34, 43]$	$[33, 43]$	0.05

Table 2: BPA of the second parameter

corresponding basic probability assignments of resulting focal elements are calculated using (1) and arranging all the focal elements in increasing order of the left end point are given in Table 3.

Interval valued Focal Elements		BPA	Interval valued Focal Elements		BPA
Membership value	Non Membership value		Membership value	Non Membership value	
$[30, 37]$	$[28, 39]$	0.087	$[36, 47]$	$[34, 49]$	0.126
$[32, 43]$	$[30, 45]$	0.099	$[37, 50]$	$[35, 52]$	0.130
$[34, 41]$	$[32, 43]$	0.100	$[39, 51]$	$[37, 53]$	0.141
$[35, 44]$	$[33, 46]$	0.109	$[41, 48]$	$[39, 56]$	0.208

Table 3: Basic Probability Assignments of resulting focal elements using algebraic addition

5.2 Substitution of Focal Elements

Subtracting all the focal elements using interval valued intuitionistic pentagonal fuzzy arithmetic we get twenty members of focal elements. Now, the corresponding Basic Probability Assignments of resulting focal elements are calculated 2 and arranging all the focal elements in increasing order of the left end point are given in the following Table 4.

Interval valued Focal Elements		BPA	Interval valued Focal Elements		BPA
Membership value	Non Membership value		Membership value	Non Membership value	
$[-42, -33]$	$[-44, -31]$	0.085	$[-31, -22]$	$[-33, -20]$	0.121
$[-40, -27]$	$[-42, -25]$	0.098	$[-30, -21]$	$[-32, -19]$	0.129
$[-37, -25]$	$[-39, -23]$	0.103	$[-29, -20]$	$[-31, -18]$	0.132
$[-35, -19]$	$[-37, -17]$	0.112	$[-28, -15]$	$[-30, -13]$	0.220

Table 4: Basic Probability Assignments of resulting focal elements using algebraic subtraction

6 Conclusion

D-S theory is more fruitful in situation when cost of technical difficulties impossible to make enough observation to quantify the models with real data. The experts provide opinion in terms of basic probability assignments of focal elements in evidence theory. However, due to presence of uncertainty focal elements can sometimes be treated as pentagonal fuzzy numbers. In this paper, we have considered the interval valued Intuitionistic pentagonal Fuzzy Numbers which is proposed to combine intuitionistic pentagonal fuzzy focal elements and their corresponding Basic Probability Assignments (BPA) of two variables under interval valued intuitionistic pentagonal fuzzy arithmetic operations. From Tables 3 and 4, it is observed that arithmetic operations have the significance of simple calculation and high accuracy in addition.

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