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Algorithm for finding Location Domination Number of a Graph connected by a Bridge

G.Rajasekar[1] and K.Nagarajan[2]

[1]*Department of Mathematics, Jawahar Science College, Neyveli, India*
grsmaths@gmail.com
[2]*Department of Mathematics, Jawahar Science College, Neyveli, India*
n2a4g1r1j1@gmail.com

Abstract

Locating-Dominating set (LD- set) of a graph G, is a dominating set S with the property that every vertices v in $V(G) - S$ is uniquely identified by the set of neighbours of v which are in S. This paper presents the precise algorithm for finding the locating-domination number of graphs which are connected by bridge.

AMS Subject Classification: 05C69
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1 Introduction

Fault diagnosis in multiprocessor systems and safeguard related problem are the major applications of Locating-domination sets and it is done by modelling as a graph where vertices are processors, rooms and edges are links between processors, adjacent rooms.

For any vertex v in $V(G) - S$, $S(v)$ is the set of vertices in S which are adjacent to v. Locating-dominating set was introduced by Slater [9, 10] which is defined as follows. A dominating set S is defined to be a locating-dominating set if $S(v) \neq S(w)$, for any $v,\ w \in V(G) - S$. A locating dominating set is denoted by LD-set. The minimum cardinalities of an LD-set in G is called the location-domination number of G and it is denoted by $RD(G)$. An LD-set with $RD(G)$ elements is called as a referencing-dominating set or an RD-set.

Locating-dominating set is NP-complete was proved by Colboum et al. [3]. Slater formulated linear-time algorithm for solving Locating-Dominating-Set in tree [10]. A LD-set will be both locating and dominating set but the inverse is not true always. Slater [7, 10] determined that the graph can have atmost $n + 2^n - 1$ vertices if the RD-set has n vertices and shown that for any trees T_n with n vertices should have $RD(T_n) > \frac{n}{3}$.

2 Location Domination Number of a Graph connected by a Bridge

Let G_1, G_2 be any two graphs and G be a graph attained by connecting G_1 and G_2 by a bridge $e = v_1v_2$ where $v_1 \in V(G_1)$, $v_2 \in V(G_2)$ and it is shown in Figure. Therefore the graph G has $V(G) = V(G_1) \cup V(G_2)$ with $V(G_1) \cap V(G_2) = \phi$ and $E(G) = E(G_1) \cup E(G_2) \cup \{e\}$
For any RD-set S of the graph G,

$$S = S \cap V(G) = S \cap (V(G_1) \cup V(G_2)) = (S \cap V(G_1)) \cup (S \cap V(G_2))$$

and as $(S \cap V(G_1)) \cap (S \cap V(G_2)) = \phi$,

$$\begin{aligned} |S| &= |S \cap V(G_1)| + |S \cap V(G_2)| - |(S \cap V(G_1)) \cap (S \cap V(G_2))| \\ &= |S \cap V(G_1)| + |S \cap V(G_2)|. \end{aligned} \tag{1}$$

Theorem 1. *Let G_1 and G_2 be any two graphs. Let G be a graph obtained by connecting G_1 and G_2 by a bridge. Then $RD(G) = RD(G_1) + RD(G_2) - 1$ or $RD(G) = RD(G_1) + RD(G_2)$*

Proof. Let $e = v_1v_2$ be the bridge connecting G_1 and G_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$.
Claim: $RD(G) \leq RD(G_1) + RD(G_2)$.

Let S_1 and S_2 be any RD-set of G_1 and G_2. Consider $S = S_1 \bigcup S_2$, then S dominates G as S_i dominates G_i, for $i = 1$ and 2. i.e. $S(u) \neq \phi$ for all $u \in V(G) - S$. Now let us show that $S(u) \neq S(v)$ for any $u, v \in V(G) - S$.

Suppose $S(u) = S(v)$ then $S_1(u) \cup S_2(u) = S_1(v) \cup S_2(v)$. As $V(G_1) \bigcap V(G_2) = \phi$, we get $S_1 \bigcap S_2 = \phi$. Thus $S_1(u) \bigcap S_2(u) = \phi$, $S_1(u) \bigcap S_2(v) = \phi$, $S_1(v) \bigcap S_2(u) = \phi$ and $S_1(v) \bigcap S_2(v) = \phi$. Therefore

$$S_1(u) = S_1(v) \tag{2}$$

and

$$S_2(u) = S_2(v) \tag{3}$$

If $u,\ v$ belongs to $V(G_1) - S_1$ then by Equation (2), S_1 cannot be the RD-set of G_1. This is a contradiction to the assumption that S_1 is the RD-set of G_1. Therefore $S(u) \neq S(v)$ for all $u, v \in V(G_1) - S_1$.

Similarly if $u, v \in V(G_2) - S_2$ then by Equation (3), S_2 cannot be RD-set of G_2. Therefore $S(u) \neq S(v)$ for all $u, v \in V(G_2) - S_2$

If $u \in V(G_1) - S_1$ and $v \in V(G_2) - S_2$ then $S_1(u) \neq \phi$ and $S_2(v) \neq \phi$. Now by Equation (2) and (3) we get $S_1(v) \neq \phi$ and $S_2(u) \neq \phi$. Since $S_1(v) \neq \phi$ and $v \in V(G_2) - S_2$ as well as v_1v_2 is the bridge connecting G_1 and G_2 we have $v = v_2$ and $S_1(v) = S_1(v_2) = \{v_1\}$.

Similarly, since $S_2(u) \neq \phi$ and $u \in V(G_1) - S_1$ as well as v_1v_2 is the bridge between G_1 and G_2 we get $u = v_1$ and $S_2(u) = S_2(v_1) = \{v_2\}$. i.e. $v_2 \in S_2$, this contradict the assumption that $v = v_2 \in V(G_2) - S_2$. Hence $S(u) \neq S(v)$ for all $u, v \in V(G) - S$. Therefore $S = S_1 \cup S_2$ is an LD-set of G. Hence $RD(G) \leq RD(G_1) + RD(G_2)$.
Claim: $RD(G_1) + RD(G_2) - 1 \leq RD(G)$

Let S be an RD-set of G such that $RD(G) = RD(G_1) + RD(G_2) - k$ where $k \geq 2$. Then by Equation (1), $|S| = |S \cap V(G_1)| + |S \cap V(G_2)| = RD(G_1) + RD(G_2) - k$

where $k \geq 2$. Therefore $|S \cap V(G_1)| = RD(G_1) - m$ and $|S \cap V(G_2)| = RD(G_2) - n$ with $m + n = k$.

Suppose if neither $m = 0$ nor $n = 0$, then $S \cap V(G_1)$ will not be the LD-set of G_1. But as S is the RD-set of G, some of the vertices of $S \cap V(G_2)$ must locate and dominate the vertices of G_1 which are not located and dominated by the vertices of $S \cap V(G_1)$. But as v_1v_2 is the only edge which connect G_1 with G_2, we must have that $v_2 \in S \cap V(G_2)$ and $v_1 \notin S \cap V(G_1)$. Similarly if we consider the set $S \cap V(G_2)$, then it is not the LD-set of G_2, and hence by above argument we have $v_1 \in S \cap V(G_1)$ and $v_2 \notin S \cap V(G_2)$. This contradiction shows that both m and n cannot be non-zero simultaneously. Hence either $m = 0$ or $n = 0$.

Without loss of generality let us take $m = 0$, then $|S \cap V(G_1)| = RD(G_1)$ and $|S \cap V(G_2)| = RD(G_2) - k$ where $k \geq 2$. This implies that $G_2 - \{v_2\}$ has a LD-set $S \cap V(G_2)$ with cardinality less than or equal to $RD(G_2) - k$. Hence $(S \cap V(G_2)) \cup \{v_2\}$ is an LD-set of G_2 with cardinality less than or equal to $RD(G_2) - (k-1)$ where $k > 1$. This contradicts the hypothesis of the RD-set. Hence $k \leq 1$, thus $RD(G_1) + RD(G_2) - 1 \leq RD(G)$.

Both the claims infer us that

$$RD(G_1) + RD(G_2) - 1 \leq RD(G) \leq RD(G_1) + RD(G_2)$$ □

Theorem 2. *Let G_1 and G_2 be any two graphs with $\mathscr{S}_1$, $\mathscr{S}_2$ the set of all RD-set of G_1, G_2 respectively. Let G be a graph obtained by connecting G_1 and G_2 by a bridge v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$. If either $\mathscr{S}_1$ does not have any RD-set which contains v_1 or $\mathscr{S}_2$ does not have any RD-set which contains v_2 then $RD(G) = RD(G_1) + RD(G_2)$.*

Proof. Let v_1 is not contained in any of the RD-set of G_1. Now let us show that $RD(G) = RD(G_1) + RD(G_2)$. Suppose we assume that G has a RD-set S such that $|S| = RD(G) < RD(G_1) + RD(G_2)$ then by Theorem 1, $|S| = RD(G) = RD(G_1) + RD(G_2) - 1$. Now by Equation (1) we get

$$|S| = |S \cap V(G_1)| + |S \cap V(G_2)| = RD(G_1) + RD(G_2) - 1 \tag{4}$$

Therefore either $|S \cap V(G_1)| = RD(G_1) - 1$ or $|S \cap V(G_2)| = RD(G_2) - 1$.
Case 1: $|S \cap V(G_1)| = RD(G_1) - 1$

Then $S \cap V(G_1)$ will not be an LD-set of the graph G_1. Therefore some vertices of G_1 are located and dominated by the vertices of $S \cap V(G_2)$. But as G_1 and G_2 are connected only by a single edge, v_1 is the only vertex in G_1 which can be located and dominated by the vertex v_2 of G_2. Hence $G_1 - \{v_1\}$ has a LD-set $S \cap V(G_1)$ with cardinality $RD(G_1) - 1$. This implies that G_1 has a LD-set $(S \cap V(G_1)) \cup \{v_1\}$ with cardinality $RD(G_1)$. This contradicts the hypothesis that $\{v_1\}$ is not contained in any RD-set of G_1 and thereby $|S \cap V(G_1)| \neq RD(G_1) - 1$.
Case 2: $|S \cap V(G_2)| = RD(G_2) - 1$

As $|S \cap V(G_2)| = RD(G_2) - 1$, the set $S \cap V(G_2)$ cannot be the LD-set of G_2. Hence some vertices of $V(G_2)$ must be located and dominated by the vertices of G_1. Since there is only one edge connecting G_1 with G_2, the vertex v_1 of G_1 must locate and dominate the vertex v_2 of G_2 and thereby $v_1 \in S \cap V(G_1)$. But by Equation (4), $|S \cap V(G_1)| = RD(G_1)$, so $S \cap V(G_1)$ is an RD-set of G_1 which contains v_1. This contradiction indicates that $|S \cap V(G_2)| \neq RD(G_2) - 1$.

Therefore Case 1 and Case 2 infer that $RD(G) \neq RD(G_1) + RD(G_2) - 1$. Hence RD-set of G must have a cardinality of $RD(G_1) + RD(G_2)$.

Similarly we can show that $RD(G) = RD(G_1) + RD(G_2)$ if none of the RD-set in S_2 contain the vertex v_2. □

Remark 3. *If both graphs G_1 and G_2 does not have any RD-set in $\mathscr{S}_1$ and $\mathscr{S}_2$ which contains $\{v_1\}$ and $\{v_2\}$ respectively, then the graph formed by connecting G_1 and G_2 by the bridge v_1v_2 will have the RD-set with $RD(G_1) + RD(G_2)$ number of vertices only.*

Theorem 4. *Let G_1 and G_2 be any two graphs and $\mathscr{S}_1$, $\mathscr{S}_2$ be the set of all RD-set of G_1, G_2 respectively. Let G be a graph obtained by connecting G_1 and G_2 by a bridge v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$. If both $\mathscr{S}_1$ and $\mathscr{S}_2$ does not have any RD-set S_1 and S_2 respectively which satisfy the condition that for $i = 1$ and 2, $v_i \in S_i$ and $S_i - \{v_i\}$ is the RD-set of $G_i - \{v_i\}$ then RD-set of G is nothing but the RD-set of G_1 union RD-set of G_2.*

Proof. By Theorem 1 and Remark 3, if $\mathscr{S}_1$, $\mathscr{S}_2$ or both does not contain v_1, v_2 or v_1 and v_2 respectively then $RD(G) = RD(G_1) + RD(G_2)$.

Now let us assume that $\mathscr{S}_1$ and $\mathscr{S}_2$ contain atleast one RD-set S_1 and S_2 such that $v_1 \in S_1$, $v_2 \in S_2$ as well as satisfy the hypothesis of the theorem. Now we have to prove that $S_1 \cup S_2$ is the minimal LD-set of G, i.e. $RD(G) = RD(G_1) + RD(G_2)$.

Let S be the RD-set of G such that $|S| = RD(G) = RD(G_1) + RD(G_2) - 1$. Now we will show that such the RD-set does not exist. By Equation (1),

$$RD(G) = |S| = |S \cap V(G_1)| + |S \cap V(G_2)| = RD(G_1) + RD(G_2) - 1 \quad (5)$$

Therefore either $|S \cap V(G_1)| \leq RD(G_1) - 1$ or $|S \cap V(G_2)| \leq RD(G_2) - 1$.

Without loss of generality let us assume that $|S \cap V(G_1)| \leq RD(G_1) - 1$. Therefore $S_3 = S \cap V(G_1)$ will not be the LD-set of G_1. Hence there exist some vertices $u, v \in V(G_1) - S_3$ such that $S_3(u) = S_3(v)$.

Claim: Either $u = v_1$ or $v = v_1$

Suppose $u \neq v_1$ and $v \neq v_1$ then u, v are not adjacent to any vertices of G_2. Therefore $S(u) \cap V(G_2) = \phi = S(v) \cap V(G_2)$. Now

$S(u) = S(u) \cap (V(G_1) \cup V(G_2)) = S(u) \cap V(G_1) = S_3(u)$

Similarly $S(v) = S_3(v)$, hence $S(u) = S(v)$. Thus S could not be the RD-set of G. Therefore either $u = v_1$ or $v = v_1$, so let us assume that $u = v_1$.

Therefore for all $v, w \in V(G_1) - (S_3 \cup \{u\})$, $S_3(v) \neq S_3(w)$, hence S_3 is the LD-set of $G_1 - \{v_1\}$. This implies that $S_3 \cup \{v_1\}$ is the LD-set of G_1 with cardinality

$$|S_3 \cup \{v_1\}| = |S \cap V(G_1)| + |\{v_1\}| \leq RD(G_1) - 1 + 1 = RD(G) \quad (6)$$

But by the definition of RD-set,

$$RD(G_1) \leq |S_3 \cup \{v_1\}| \quad (7)$$

From Equation (6) and (7), $RD(G_1) = |S_3 \cup \{v_1\}|$. Hence G_1 has an RD-set $S_3 \cup \{v_1\}$ such that $S_3 = (S_3 \cup \{v_1\}) - \{v_1\}$ is the RD-set of $G_1 - \{v_1\}$. This contradicts the hypothesis of the theorem. Hence $|S \cap V(G_1)| > RD(G_1) - 1$

and similarly one can show that $|S \cap V(G_2)| > RD(G_2) - 1$. Therefore by Equation (1), $RD(G) = |S| \geq RD(G_1) + RD(G_2)$. With the light of Theorem 1, $RD(G) = RD(G_1) + RD(G_2)$. □

Theorem 5. *Let G_1, G_2 be any two graphs with the set of all RD-set $\mathscr{S}_1$ and $\mathscr{S}_2$ respectively. Let us connect G_1 and G_2 by a single edge v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$ to form the graph G.*

Let $\mathscr{S}_1$ has an RD-set S_1 as such $v_1 \in S_1$ but none of them fulfil the condition that $S_1 - \{v_1\}$ is the RD-set of $G_1 - \{v_1\}$. While $\mathscr{S}_2$ has a RD-set S_2 such that $v_2 \in S_2$ which satisfy the conditions that $S_2 - \{v_2\}$ is the RD-set of $G_2 - \{v_2\}$ and there exist some vertex $w \in S_2$ which is adjacent to v_2. If $\mathscr{S}_1$ and $\mathscr{S}_2$ satisfies the above condition then $RD(G) = RD(G_1) + RD(G_2) - 1$.

Proof. Let us consider $S = (S_1 \cup S_2) - \{v_2\}$, then

$$S(v_2) \supseteq \{v_1, w\} \text{ and } S(u) = \begin{cases} S_1(u), & u \in V(G_1) - S_1 \\ S_2(u) - \{v_2\}, & u \in V(G_2) - S_2 \end{cases}$$

As S_1, $S_2 - \{v_2\}$ are RD-set of G_1, $G_2 - \{v_2\}$ respectively and $S(v_2) \supseteq \{v_1, w\}$, it obviously that $S(u) \neq S(v)$ for all $u, v \in V(G) - S$. Thus S is a LD-set of G with $|S| = |S_1| + |S_2| - 1 = RD(G_1) + RD(G_2) - 1$. The Theorem 1 ensures that S as a minimal LD-set of G and its cardinality is $RD(G_1) + RD(G_2) - 1$. □

Theorem 6. *Let G be a graph constructed by connecting the graphs G_1 and G_2 by a bridge v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$. Let $\mathscr{S}_1$, $\mathscr{S}_2$ be the set of all RD-set of G_1 and G_2 with the properties*

(i) $\mathscr{S}_1$ has some RD-set S_1 such that $v_1 \in S_1$ but none of them satisfies the condition that $S_1 - \{v_1\}$ is RD-set of $G_1 - \{v_1\}$.

(ii) $\mathscr{S}_2$ has a RD-set S_2 as such $v_2 \in S_2$ and $S_2 - \{v_2\}$ is the RD-set of $G_2 - \{v_2\}$ as well as v_2 is not adjacent to any vertices of S_2.

If $S_1(u) \neq \{v_1\}$ for all $u \in V(G_1) - S_1$ then $RD(G) = RD(G_1) + RD(G_2) - 1$ otherwise $RD(G) = RD(G_1) + RD(G_2)$.

Proof. If v_2 is adjacent to some vertices of S_2, then by Theorem 5, $RD(G)$ would be $RD(G_1) + RD(G_2) - 1$. Now let us take that v_2 is not adjacent to any vertices of S_2.

First let us prove that $RD(G) = RD(G_1) + RD(G_2) - 1$ if $S_1(u) \neq \{v_1\}$ for all $u \in V(G_1) - S_1$. Consider $S = (S_1 \cup S_2) - \{v_2\} = S_1 \cup (S_2 - \{v_2\})$, then

$$S(v_2) = \{v_1\} \text{ and } S(u) = \begin{cases} S_1(u), & u \in V(G_1) - S_1 \\ S_2(u) - \{v_2\}, & u \in V(G_2) - S_2 \end{cases}$$

As $S_1, S_2 - \{v_2\}$ are the RD-set of G_1, $G_2 - \{v_2\}$ and $S(v_2) = \{v_1\} \neq S(u)$ for all $u \in V(G)$, we have that $S(u) \neq S(v)$ for all $u, v \in V(G)$. Thus S is an LD-set of G with $RD(G_1) + RD(G_2) - 1$ number of vertices. But by Theorem 1, S must be a minimal LD-set of G.

Now let us prove for the case $S_1(u) = \{v_1\}$ for some unique $u \in V(G_1) - S_1$, the vertex u is unique because S_1 is the RD-set of G_1. We prove

$RD(G) = RD(G_1) + RD(G_2)$ by the method of contradiction by assuming that G has an RD-set S such that $RD(G) = RD(G_1) + RD(G_2) - 1$. By Equation (1),

$$|S| = |S \cap V(G_1)| + |S \cap V(G_2)| = RD(G_1) + RD(G_2) - 1 \quad (8)$$

Therefore either $|S \cap V(G_1)| \leq RD(G_1) - 1$ or $|S \cap V(G_2)| \leq RD(G_2) - 1$.
Case 1: Suppose $|S \cap V(G_1)| = RD(G_1) - 1$ then $S \cap V(G_1)$ cannot be the LD-set of G_1. As S is the RD-set of G and v_1 is the only vertex in G_1 which is adjacent to the vertex v_2 of G_2, one must have that $S \cap V(G_1)$ as the LD-set of $G_1 - \{v_1\}$. Thus G_1 has a RD-set $(S \cap V(G_1)) \cup \{v_1\}$ such that $S \cap V(G_1)$ is the RD-set of $G_1 - \{v_1\}$. This contradict the hypothesis of the theorem. Thus $|S \cap V(G_1)| \neq RD(G_1) - 1$.
Case 2: Suppose $|S \cap V(G_2)| = RD(G_2) - 1$ then $S \cap V(G_2)$ will not be the LD-set of G_2. As S is the RD-set of G and v_2 is the only vertex in G_2 which is adjacent to the vertex v_1 of G_1, we should have that $v_2 \notin S$ and $S(v_2) \supseteq \{v_1\}$.

By Equation (8), $|S \cap V(G_1)| = RD(G_1)$, therefore $S \cap V(G_1)$ is an RD-set of G_1. Hence by our assumption there exists some vertex $u \in V(G_1) - (S \cap V(G_1))$ such that it is dominated and uniquely identified by the vertex v_1. Hence $S(v_2)$ cannot be equal to $\{v_1\}$, therefore $S(v_2) \supset \{v_1\}$. i.e. $S \cap V(G_2)$ has a vertex which is adjacent to v_2.

Now we have shown that G_2 has a RD-set $(S \cap V(G_2)) \cup \{v_2\}$ such that some vertices of the RD-set is adjacent to v_2. This contradicts the hypothesis of the theorem, hence $|S \cap V(G_2)| \neq RD(G_2) - 1$.

From both Case 1 and Case 2 one can conclude that S cannot be LD-set of G if $|S| = RD(G_1) + RD(G_2) - 1$. Thus by Theorem 1, $RD(G)$ must be the sum of the locating domination number of the graphs G_1 and G_2. □

Theorem 7. *Let G_1 and G_2 be any two graphs and G be a graph obtained by connecting G_1 and G_2 by a bridge v_1v_2 where $v_1 \in V(G_1)$ and $v_2 \in V(G_2)$. Suppose for $i = 1$ and 2, G_i has the RD-set S_i such that $v_i \in S_i$ and $S_i - \{v_i\}$ is the RD-set of $G_i - \{v_i\}$ then $RD(G) = RD(G_1) + RD(G_2) - 1$.*

Proof. By considering $S = (S_1 \cup S_2) - \{v_1\}$, it is obvious that the set S is the RD-set of G with cardinality $RD(G_1) + RD(G_2) - 1$. □

3 Algorithm to calculate Location Domination Number of a Graph connected by a Bridge

From the theorems let us frame the algorithm for determining the RD-set of graph obtained by connecting by a bridge.
Input: Graph G_1, G_2 and the bridge v_1v_2
Output: RD-set of the graph G formed by connecting G_1 and G_2 by the edge v_1v_2

Step 1: Check whether both $\mathscr{S}_1$ and $\mathscr{S}_2$ (the set of all RD-set of the graphs G_1 and G_2) have a RD-set that contains v_1 and v_2 respectively. If so, proceed to Step 2 otherwise $RD(G_1) + RD(G_2)$ is the location domination number of the graph G.

Step 2: Formulate the set $\mathscr{S}_{11}$ and $\mathscr{S}_{21}$ as follows:
$\mathscr{S}_{11} = \{S \in \mathscr{S}_1 \mid S - \{v_1\}$ is the RD-set of the graphs $G_1 - \{v_1\}\}$ and

$\mathscr{S}_{21} = \{S \in \mathscr{S}_2 \mid S - \{v_2\}$ is the RD-set of the graphs $G_2 - \{v_2\}\}$
if $\mathscr{S}_{11} = \phi$ and $\mathscr{S}_{21} = \phi$
$RD(G) = RD(G_1) + RD(G_2)$
else if $\mathscr{S}_{11} \neq \phi$ and $\mathscr{S}_{21} \neq \phi$
$RD(G) = RD(G_1) + RD(G_2) - 1$
else
if $\mathscr{S}_{11} \neq \phi$
Proceed to Step 3
else
Proceed to Step 4
end
end

Step 3: Interchange all labels related to the two graphs. i.e. graph G_1 is labelled as G_2 and initially considered G_2 is relabelled as G_1 and so on. Proceed to Step 4.

Step 4: Formulate the set $\mathscr{S}_{22}$ as follows:
$\mathscr{S}_{22} = \{S \in \mathscr{S}_{21} \mid$ there exist some vertex $w \in S$ which is adjacent to $v_2\}$
if $S_{22} \neq \phi$
$RD(G) = RD(G_1) + RD(G_2) - 1$
else
Proceed to Step 5
end

Step 5: Form the set $\mathscr{S}_{12}$ with the property that
$\mathscr{S}_{12} = \{S \in \mathscr{S}_{11} \mid S(u) \neq \{v_1\}$ for all $u \in V(G_1) - S\}$
if $S_{12} \neq \phi$
$RD(G) = RD(G_1) + RD(G_2) - 1$
else
$RD(G) = RD(G_1) + RD(G_2)$
end

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