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3-difference cordiality of some special trees

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Abstract

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denotes the number of edges labelled with 1 and not labeled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate the 3-difference cordial labeling behavior of x-tree and y-tree.

AMS Subject Classification: 05C78
Key Words and Phrases: X-tree, Y-tree, difference cordial.

1 Introduction

Graphs considered here are finite, undirected and simple. The graph labeling was first introduced by Rosa [1] in the name of graceful labeling. Recently k-difference cordial labeling [3] has been introduced and studied their behavior in [3, 4, 5, 6]. A connected acyclic graph is called a tree. In this paper we investigate the 3-difference cordial labeling behavior of X-tree, Y-tree, coconut tree and some caterpillar and lob star. Terms not defined here are used in the sense of Harary [2].

2 k-Difference cordial labeling

Definition 1. Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, \ldots, k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denotes the number of edges labelled with 1 and not labeled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

Let X_n be the tree with $V(X_n) = \{u, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(X_n) = \{uu_1, uv_1, ux_i, uy_1, u_iu_{i+1}, v_iv_{i+1}, x_ix_{i+1}, y_iy_{i+1} : 1 \leq i \leq n\}$. Then X_n is called the X-tree.

Theorem 2. The tree X_n is 3-difference cordial

Proof. Let $G = X_n$. let $V(G) = \{u, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(G) = \{uu_1, uv_1, ux_i, uy_1, u_iu_{i+1}, v_iv_{i+1}, x_ix_{i+1}, y_iy_{i+1} : 1 \leq i \leq n\}$. Then G has 4n+1 vertices and 4n edges.

Case 1. $n \equiv 0 \pmod 4$

Fix the label 2 to the vertex u. Consider the vertices u_i. Assign the label 1,3,1,2 to the vertices u_1, u_2, u_3, u_4 respectively. Then we assign the label 1,3,1,2 to the next four vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 2. Next we move to the vertices v_i. Assign the label 1 to the vertices v_{12i+1}, v_{12i+3}, v_{12i+6} and v_{12i+9} for all the values of i=0,1,2,... and we assign the label 3 to the vertices v_{12i+2}, v_{12i+5}, v_{12i+7} and v_{12i+10} for all i=0,1,2,... For all the values of i=0,1,2,..., assign the label 2 to the vertices v_{12i+4}, v_{12i+8} and v_{12i+11}. Now we assign the label 2 to the vertices v_{12i} for i=1,2,3,... Next we move to the vertices x_i. Assign the label 3,1,3,2 to the vertices x_1, x_2, x_3, x_4 respectively. Then we assign the label 3,1,3,2 to the next four vertices x_5, x_6, x_7, x_8 respectively. Continuing this process, we assign the label to the next four vertices and so on. Therfore the last vertex x_n received the label 2.Now our attention move to the vertices y_i. Assign the label 1,3,2,2 to the vertices y_1, y_2, y_3, y_4 respectively. Then we assign the label 1,3,2,2 to the next four vertices y_5, y_6, y_7, y_8 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex y_n received the label 2.

Case 2. $n \equiv 1 \pmod 4$

Fix the label 1 to the vertex u. Now we consider the vertices u_i. Assign the label 3 to the vertex u_1. Then we assign the label 1,1,2,3 to the vertices u_2, u_3, u_4, u_5 respectively. Now we assign the label 1,1,2,3 to the next four vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 3. Next our attention move to the vertices v_i. Fix the label 3 to the first vertex v_1. For all the values of i=0,1,2,... , assign the label 1 to the vertices v_{12i+2}, v_{12i+3}, v_{12i+8} and v_{12i+10}. Assign the label 2 to the vertices v_{12i+4}, v_{12i+6} and v_{12i+7} for all i=0,1,2,... and we assign the label 2 to the vertices v_{12i} for i=1,2,... Then we assign the label 3 to the vertices v_{12i+5}, v_{12i+9} and v_{12i+11} for all i=0,1,2,... and we assign the label 3 to the vertices v_{12i+1}

for i=1,2,3,... Now we move to the vertices x_i. Fix the label 2 to the vertex x_1. Assign the label 3,1,3,2 to the vertices x_2, x_3, x_4, x_5 respectively. Then we assign the label 3,1,3,2 to the next four vertices x_6, x_7, x_8, x_9 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Therefore the last vertex x_n received the label 2. Now we move to the vertices y_i. Fix the label 2 to the vertex y_1. Assign the label 1,3,2,2 to the next four vertices y_2, y_3, y_4, y_5 respectively. Then we assign the label 1,3,2,2 to the next four vertices y_6, y_7, y_8, y_9 respectively. Continuing like this, we assign the label to the next four vertices and so on. Clearly the last vertex y_n received the label 2.

Case 3. $n \equiv 2 \pmod 4$

First we fix the label 2 to the vertex u. Now we consider the vertices u_i. Fix the label 1,3 to the vertices u_1, u_2 respectively. Assign the label 1,1,2,3 to the vertices u_3, u_4, u_5, u_6 respectively. Then we assign the label 1,1,2,3 to the next four vertices u_7, u_8, u_9, u_10 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 3. Next we move to the vertices v_i. Fix the label 1,3 to the vertices v_1, v_2 respectively. Assign the label 2 to the vertices v_{12i+3}, v_{12i+4} and v_{12i+9} for all i=0,1,2,... and we assign the label 2 to the vertices v_{12i+1} for i=1,2,3,... For all the values i=0,1,2,..., assign the label 1 to the vertices v_{12i+5}, v_{12i+7} and v_{12i+11}. Assign the label 1 to the vertices v_{12i} for all i=1,2,3,... Then we assign the label 3 to the vertices v_{12i+6}, v_{12i+8} and v_{12i+10} for i=0,1,2,3,... and we assign the label 3 to the vertices v_{12i+2} for i=1,2,3,... Now our attention move to the vertices x_i. Fix the label 2,1 to the vertices x_1, x_2 respectively. Assign the label 3,1,2,3 to the vertices x_3, x_4, x_5, x_6 respectively. Then we assign the label 3,1,2,3 to the next four vertices x_7, x_8, x_9, x_{10} respectively. Continuing like this, we assign the label to the next four vertices and so on. Therefore the last vertex x_n received the label 3. Next we move to the vertices y_i. Fix the label 2,3 to the vertex y_1, y_2 respectively. Assign the label 2,2,1,3 to the next four vertices y_3, y_4, y_5, y_6 respectively. Then we assign the label 2,2,1,3 to the next four vertices y_7, y_8, y_9, y_{10} respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex y_n received the label 3.

Case 4. $n \equiv 3 \pmod 4$

Fix the label 1 to the vertex u. First we consider the vertices u_i. Fix the label 1,3,2 to the vertices u_1, u_2, u_3 respectively. Assign the label 1,3,1,2 to the vertices u_4, u_5, u_6, u_7 respectively. Then we assign the label 1,3,1,2 to the next four vertices u_8, u_9, u_10, u_11 respectively. Continuing like this, we assign the label to the next four vertices and so on. therefore the last vertex u_n received the label 2. Next our attention move to the vertices v_i. Fix the label 3,2,1 to the vertices v_1, v_2, v_3 respectively. Assign the label 2 to the vertices v_{12i+4}, v_{12i+5} and v_{12i+10} for all the values of i=0,1,2,... and we assign the label 2 to the vertices v_{12i+2} for i=1,2,3,... Then we assign the label 1 to the vertices v_{12i+6} and v_{12i+11} for i=0,1,2,3,... and we assign the label 1 to the vertices v_{12i} and v_{12i+1} for all i=1,2,3,... For all the values of i=0,1,2,3,..., assign the label 3 to the vertices v_{12i+7}, v_{12i+8} and v_{12i+9}. Then we assign the label 3 to the vertices v_{12i+3} for all i=1,2,3,... Now we move to the vertices x_i. Fix the label 1,2,3 to the vertices x_1, x_2, x_3 respectively. Assign the label 3,3,2,1 to the vertices x_4, x_5, x_6, x_7 respectively. Then we assign the label 3,3,2,1 to the next four vertices x_8, x_9, x_10, , x_11 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex x_n received the label by the integer 1. Now our attention move to the vertices y_i. Fix the label 1,3,2 to the vertex y_1, y_2, y_3 respectively. Assign the label 1,3,2,2 to the vertices y_4, y_5, y_6, y_7 respectively. Then we assign the label 1,3,2,2 to the next four vertices y_8, y_9, y_10, y_11 respectively. Continuing like this, we assign the label to the next four vertices and so on. Therefore the last vertex y_n received the label by the integer 2. Hence the edge condition is $e_f(0) = e_f(1) = 2n$. Also the vertex condition is given in table 2.1

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 4, 7 \pmod{12}$	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$	$\frac{4n-1}{3}$
$n \equiv 2, 5, 8, 11 \pmod{12}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$
$n \equiv 0, 6, 9 \pmod{12}$	$\frac{4n}{3}$	$\frac{4n+3}{3}$	$\frac{4n}{3}$
$n \equiv 0, 6, 9 \pmod{12}$	$\frac{4n-1}{3}$	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$

Table 1:

□

Let Y_n be the tree with $V(Y_n) = \{u, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(Y_n) = \{uu_1, uv_1, uw_1, u_iu_{i+1}, v_iv_{i+1}, w_iw_{i+1}, 1 \leq i \leq n-1\}$. Then Y_n is called the Y- tree.

Theorem 3. The tree Y_n is 3-difference cordial

Proof. Let $G = Y_n$.let $V(G) = \{u, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G) = \{uu_1, uv_1, uw_1, u_iu_{i+1}, v_iv_{i+1}, w_iw_{i+1}, 1 \leq i \leq n-1\}$. Clearly the order and size of G are 3n+1 and 3n.

Case 1. $n \equiv 0 \pmod 4$

Fix the label 2 to the vertex u. First we consider the vertices u_i. Assign the label 3,3,1,,2 to the first four vertices u_1, u_2, u_3, u_4 respectively. Then we assign the label 3,3,1,,2 to the next four vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label by the integer 2. Next we move to the vertices v_i. Assign the label 1,1,3,2 to the vertices v_1, v_2, v_3, v_4 respectively. Then we assign the label 1,1,3,2 to the next four vertices v_5, v_6, v_7, v_8 respectively. Continuing like this, we assign the label to the next four vertices and so on. Therefore the last vertex v_n received the label by the integer 2. Now our attention move to the vertices w_i. Assign the label 1,3,2,2 to the first four vertices w_1, w_2, w_3, w_4 respectively. Then we assign the label 1,3,2,2 to the next four vertices w_5, w_6, w_7, w_8 respectively. Continuing this way, we assign the label to the next four vertices and so on. Clearly the last vertex w_n received the label by the integer 2.

Case 2. $n \equiv 1 \pmod 4$

Fix the label 1 to the vertex u. Now we consider the vertices u_i. Fix the label 1 to the vertex u_1. Assign the label 3,3,2,1 to the vertices u_2, u_3, u_4, u_5 respectively. Then we assign the label 3,3,2,1 to the next four vertices u_6, u_7, u_8, u_9 respectively. Continuing this way, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label by the integer 1. Next we move to the vertices v_i. Fix the label 2 to the vertex v_1. Assign the label 1,1,3,2 to the vertices v_2, v_3,

v_4, v_5 respectively. Then we assign the label 1,1,3,2 to the next four vertices v_6, v_7, v_8, v_9 respectively. Proceeding like this, we assign the label to the next four vertices and so on. Therefore the last vertex v_n received the label 2. Now our attention move to the vertices w_i. Fix the label 3 to the vertex w_1. Assign the label 2,2,1,3 to the vertices w_2, w_3, w_4, w_5 respectively. Then we assign the label 2,2,1,3 to the next four vertices w_6, w_7, w_8, w_9 respectively. Continuing like this, we assign the label to the next four vertices and so on. Clearly the last vertex w_n received the label by the integer 3.

Case 3. $n \equiv 2 \pmod 4$

Fix the label 2 to the vertex u. First we consider the vertices u_i. Fix the label 1,3 to the vertices u_1 and u_2 respectively. Assign the label 3,3,2,1 to the vertices u_3, u_4, u_5, u_6 respectively. Then we assign the label 3,3,2,1 to the next four vertices u_7, u_8, u_9, u_{10} respectively. Continuing this way, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 1. Next we move to the vertices v_i. Fix the label 1,3 to the vertex v_1 and v_2 respectively. Assign the label 1,1,2,3 to the vertices v_3, v_4, v_5, v_6 respectively. Then we assign the label 1,1,2,3 to the next four vertices v_7, v_8, v_9, v_{10} respectively. Proceeding like this, we assign the label to the next four vertices and so on. Therefore the last vertex v_n received the label by the integer 3. Now our attention move to the vertices w_i. Fix the label 2,1 to the vertex w_1 and w_2 respectively. Assign the label 2,2,1,3 to the vertices w_3, w_4, w_5, w_6 respectively. Then we assign the label 2,2,1,3 to the next four vertices w_7, w_8 , w_9, w_{10} respectively. Continuing this way, we assign the label to the next four vertices and so on. Clearly the last vertex w_n received the label by the integer 3.

Case 4. $n \equiv 3 \pmod 4$

Fix the label 1 to the vertex u. Now we consider the vertices u_i. Fix the label 1,2,3 to the vertices u_1, u_2 , u_3 respectively. Assign the label 2,2,1,3 to the vertices u_4, u_5, u_6 , u_7 respectively. Then we assign the label 2,2,1,3 to the vertices u_8, u_9, u_{10}, u_{11} respectively. Proceeding like this, we assign the label to the next four vertices and so on. Therefore the last vertex u_n received the label 3. Next our attention move to the vertices v_i. Fix the label 1,3,2 to the vertices v_1 ,v_2 , v_3 respectively. Assign the label 1,1,3,2 to the vertices v_4, v_5, v_6, v_7 respectively. Then we assign the label 1,1,3,2 to the next four vertices v_8, v_9, v_{10}, v_{11} respectively. Continuing like this, we assign the label to the next four vertices and so on. Clearly the last vertex v_n received the label 2. Now we move to the vertices w_i. Fix the label 1,3,2 to the vertices w_1, w_2, w_3 respectively. Assign the label 3,3,1,2 to the vertices w_4, w_5, w_6, w_7 respectively. Then we assign the label 3,3,1,2 to the next four vertices w_8 , w_9, w_{10}, w_{11} respectively. Proceeding like this, we assign the label to the next four vertices and so on. Therefore the last vertex w_n received the label by the integer 2. Clearly the vertex and edge conditions are given by the following tables

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 4$	n	$n+1$	n
$n \equiv 1, 2, 3 \pmod 4$	$n+1$	n	n

Table 2:

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0, 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1, 3 \pmod 4$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 3:

□

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