International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 279-286
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Accurate 2-Domination in Fuzzy Graphs using Strong Arcs

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Abstract

In this paper, the concept of accurate 2-dominating set and accurate 2-domination number of a fuzzy graph was introduced. In a fuzzy graph G, a subset D of V is said to be an accurate 2-dominating set of G, if $V-D$ has no 2-dominating set of same fuzzy cardinality $|D|$. The minimum cardinality taken over all accurate 2-dominating sets of G is called the accurate 2-domination number of G and it is denoted as $\gamma_{fa2}(G)$. Here, some properties of accurate 2-dominating sets and also some bounds of accurate 2-domination numbers are found.

AMS Subject Classification: 03E72, 05C69, 05C72

Key Words and phrases: 2-dominating sets, accurate dominating sets, accurate 2-dominating sets, accurate 2-dominatiion number and strong arcs.

1 Introduction

Ore [9] and Berge [1] initiated the study on dominating sets in graphs. Then, the domination number and the independent domination number of graphs are introduced by Cockayne and Hedetniemi [2]. The n-domination in graphs was introduced by Fink and Jacobson [3] in the year 1985. Accurate domination in graphs was introduced by Kulli and Kattimani [4].

The concept of fuzzy relation was introduced by Zadeh [12] in his classical paper in 1965. Somansundaram and Somasundaram [11] discussed domination in fuzzy graphs using effective edges. Rosenfeld [10] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Nagoor Gani and Chandrasekaran [5] discussed domination in fuzzy graph using strong arcs. Nagoor gani and Vadivel [8] discussed the concepts of domination, independent domination and irredundance in fuzzy graphs using strong arcs. Nagoor Gani and Prasanna Devi [6] introduced the concept of 2- domination in fuzzy graphs. The concepts of accurate 2- domination number using strong arcs are discussed in this paper.

(See [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12])

2 Preliminaries

Definition 1. A fuzzy graph $G =< \sigma, \mu >$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where for all $x, y \in V$ we have $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

Definition 2. A fuzzy graph $H =< \tau, \rho >$ is called a fuzzy subgraph of G if $\tau(v_i) \leq \sigma(v_i)$ for all $v_i \in V$ and $\rho(v_i, v_j) \leq \mu(v_i, v_j)$ for all $v_i, v_j \in V$.

Definition 3. An arc (u, v) in a fuzzy graph G is said to be strong, if $\mu^\infty(u, v) = \mu(u, v)$ and then the nodes u and v are said to be strong neighbours. The strong neighbourhood of the node u is defined as $N_S(u) = \{v \in V : (u, v) \text{ is a strong arc}\}$.

Definition 4. The strong neighbourhood degree of a node u is defined as $dN_S(u) = \sum\limits_{v \in N_S(u)} \sigma(v)$. The minimum strong neighbourhood degree of a fuzzy graph G is defined as $\delta_{N_S}(G) = \min\{dN_S(u)/u \in V\}$. The maximum strong neighbourhood degree of a fuzzy graph G is defined as $\Delta_{N_S}(G) = \max\{dN_S(u)/u \in V\}$.

Definition 5. The underlying crisp graph of a fuzzy graph $G =< \sigma, \mu >$ is denoted by $G^* =< \sigma^*, \mu^* >$, where $\sigma^* = \{v_i \in V/\sigma(v_i) > 0\}$ and $\mu^* = \{(v_i, v_j) \in V/\mu(v_i, v_j) \text{ is a strong arc}\}$.

Definition 6. In a fuzzy graph G. A subset D of V is said to be dominating set of G, if every node in $V - D$ has atleast one strong neighbour in D. The minimum cardinality taken over all dominating sets of fuzzy graph G is called the domination number of G and it is denoted by $\gamma_f(G)$.

Definition 7. In a fuzzy graph G. A subset D of V is said to be 2- dominating set of G, if every node in $V - D$ has atleast two strong neighbours in D. The minimum cardinality taken over all 2- dominating sets of fuzzy graph G is called the 2- domination number of G and it is denoted by $\gamma_{f2}(G)$.

Definition 8. In a fuzzy graph G. A subset D of V is said to be an accurate dominating set of G, if $V - D$ has no dominating set of same fuzzy cardinality $|D|$. The minimum cardinality taken over all accurate dominating sets of fuzzy graph G is called the accurate domination number of G and it is denoted by $\gamma_{fa}(G)$.

3 Accurate 2-domination in fuzzy graphs

In this section, we define accurate 2-dominating set and accurate 2-domination number of a fuzzy graph with suitable example. We also discuss some properties on accurate 2-domination number of fuzzy graphs using strong arcs.

Definition 9. A 2-dominating set D, where $D \subseteq V$, of fuzzy graph G is said to be an accurate 2-dominating set of G if $V-D$ has no 2-dominating set of same fuzzy cardinality $|D|$.The minimum cardinality taken over all accurate 2-dominating sets of G is called the accurate 2-domination number of G and it is denoted by $\gamma_{fa2}(G)$.

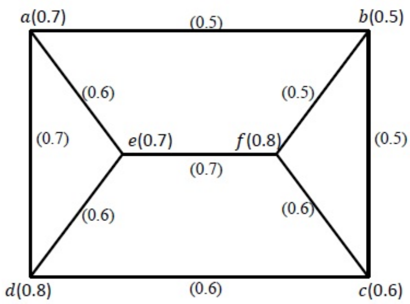

Figure 1: Fuzzy Graph *G*

Example 10. *Here* $D_1 = \{a,d,f\}$, $D_2 = \{b,c,e\}$, $D_3 = \{e,f\}$, $D_4 = \{a,c\}$, $D_5 = \{a,b\}$, $D_6 = \{a,b,c,d\}$, $D_7 = \{a,b,e,f\}$, $D_8 = \{c,d,e,f\}$, $D_9 = \{d,f\}$, $D_{10} = \{c,e\}$ *are some of the dominating set of fuzzy graph G.*

D_1, D_2, D_6, D_7, D_8 *are the accurate 2-dominating sets.*

The fuzzy cardinality of $|D_1|$, $|D_2|$, $|D_6|$, $|D_7|$ *and* $|D_8|$ *are 2.3, 1.8, 2.6, 2.7 and 2.9 respectively.*

Then, the accurate 2-domination number is

$\gamma_{fa2}(G) = \min\{|D_1|,|D_2|,|D_6|,|D_7|,|D_8|\}$

$\gamma_{fa2}(G) = \min\{2.3, 1.8, 2.6, 2.7, 2.9\}$

$\gamma_{fa2}(G) = 1.8$

Therefore, the accurate 2-domination number is 1.8 and the minimum accurate 2-dominating set is D_2.

Theorem 11. *In a fuzzy graph G, is there exists a node* $v \in V$*, whose strong neighbours* $|N_S(v)| = 0$ *or* 1*, then* v *belongs to every accurate 2-dominating set of G.*

Proof. Let G be a fuzzy graph.

Let $v \in V(G)$, be a node which has strong neighbour atmost one. ie., $|N_S(v)| = 0$ or 1.

Case (i): Assume that, v doesnot have any strong neighbours in G. ie., $|N_S(v)| = 0$.

If $v \in V-D$, then either any node in accurate 2-dominating set D or any nodes in $V(G)$ cannot dominate v. So, v must be dominated by itself, ie., $v \in D$.

Hence, if $|N_S(v)| = 0$, then v belongs to every accurate 2-dominating set of G.

Case (ii): Assume that, v has only one strong neighbour in G, $|N_S(v)| = 1$

Let u be the strong neighbours of v in G.

Suppose, u dominates v, then $u \in D$ and $v \in V - D$.

Since, v has only one strong neighbours in D then D cannot be an accurate 2-dominating set of G. Therefore, v should be dominated by itself.

Hence, $|N_S(v)| = 1$, then v belongs to every accurate 2-dominating set of G. □

Corollary 12. *For any fuzzy graph G. If $|N_S(v)| = 0$ or 1, then the accurate 2-dominating number and 2-domination number of G must be same, ie., $\gamma_{fa2}(G) = \gamma_{f2}(G)$.*

Proof. Let G be any fuzzy graph and a subset $D \subseteq V$ be a 2-dominating set of G, then every nodes in $V - D$ must have two strong neighbours in D.

By theorem 11, If $|N_S(v)| = 0$ or 1, then v belongs to every accurate 2-dominating set of G. So, v must belongs to D.

Since, $v \in D$ and v has atmost one strong neighbours in $V(G)$, then $V - D$ does not have any 2-dominating set of same fuzzy cardinality $|D|$. Therefore, the 2-dominating set D itself forms an accurate 2-dominating set of G.

Hence the proof. □

Theorem 13. *Every accurate 2-dominating set of a fuzzy graph G is also a 2-dominating set of G.*

Proof. Let G be a fuzzy graph and D a subset V, be an accurate 2-dominating set of G. Since, D is an accurate 2-dominating set, then every node $v \in V - D$ has atleast two strong neighbours in D. That is, each node in $V - D$ is dominated by atleast two nodes in D.

Therefore, every accurate 2-dominating set D itself forms 2- dominating set of fuzzy graph G. □

Theorem 14. *Every accurate 2-dominating set of a fuzzy graph G is also an accurate dominating set of G.*

Proof. Let G be a fuzzy graph and a subset $D \subseteq V$, be an accurate 2-dominating set of G.

Since, D is an accurate 2-dominating set, then every nodes in $V - D$ has atleast two strong neighbours in D, i.e., every nodes in $V - D$ will be dominated by atleast two nodes in D. Therefore, D is also a dominating set of G and if $V - D$ does not contains any dominating set of same fuzzy cardinality $|D|$. Then D itself forms an accurate dominations set of G. □

Theorem 15. *For any fuzzy graph G, $\gamma_{f2}(G) \leq \gamma_{fa2}(G)$.*

Proof. Assume that, $D \subseteq V$ be the minimum accurate 2-dominating set of fuzzy graph G.

By theorem 13, every accurate 2-dominating set of a fuzzy graph G is also a 2-dominating set of G. Then, every nodes in $V - D$ has atleast two strong neighbours in D, therefore, every node $u \in V - D$ is dominated by atleast two nodes in D. Therefore, D is a 2-dominating set of G. Hence, $\gamma_{f2}(G) \leq \gamma_{fa2}(G)$. □

Theorem 16. *For any fuzzy graph G, $\gamma_{fa}(G) \leq \gamma_{fa2}(G)$.*

Proof. Assume that, $D \subseteq V$ be the minimum accurate 2-dominating set of fuzzy graph G.

By theorem 14, every accurate 2-dominating set of a fuzzy graph G is also an accurate dominating set of G. Then, every nodes in $V - D$ has atleast two strong neighbours in D, therefore, every node $u \in V - D$ is dominated by atleast two nodes in D. Therefore, D is also a dominating set of G and if $V - D$ does not have any dominating set of same fuzzy cardinality $|D|$. Then, D itself forms an accurate dominating set of G. Hence, $\gamma_{fa}(G) \leq \gamma_{fa2}(G)$. □

Theorem 17. *For any fuzzy graph G. If D be an accurate 2-dominating set of fuzzy graph G, then $V - D$ need not to be an accurate 2-dominating set of G.*

Proof. Let G be a fuzzy graph and D, a subset of V, be an accurate 2-dominating set of G.

Case (i): By theorem 11, if $|N_S(v)| = 0$ or 1, then v belongs to every accurate 2-dominating sets of G. That is, $v \in D$, has atmost one strong neighbour in $V - D$. Therefore, $V - D$ has atmost one strong neighbour which dominates v. Hence, $V - D$ cannot be a 2-dominating set of G and also it cannot be an accurate 2-dominating set of G.

Case (ii): Assume that, D be an accurate 2-dominating set of G. If every node $v \in D$ has atleast two strong neighbours in $V - D$ and the fuzzy cardinality of $|V - D| \neq |D|$ then $V - D$ is an accurate 2-dominating set of G.

Therefore by case(i) and (ii), if D be any accurate 2-dominating set of G, then $V - D$ need not be an accurate 2-dominating set of fuzzy graph G. □

Theorem 18. *For any fuzzy graph G, $\gamma_{fa2}(G) \leq |V| - \gamma_{fa}(G) + 1$.*

Proof. Assume that, a subset D of V, be the minimum dominating set of fuzzy graph G. Therefore, for any node $v \in D$, $(V - D) \cup \{v\}$ forms an accurate dominating set of G. And let us denote it as $A = (V - D) \cup \{v\}$.

Hence, if $V - A$ has atleast two strong neighbours in A, then the accurate dominating set A itself forms an accurate 2-dominating set of G.

That is, $|A| \leq |(V - D) \cup \{v\}|$
$|A| \leq |V| - |D| + 1$
Thus, $\gamma_{fa2}(G) \leq |V| - \gamma_{fa}(G) + 1$. □

Theorem 19. *For any fuzzy graph G, bounds of accurate 2-domination number is*

$$\frac{|V|}{\Delta_{N_S}(G) + 1} \leq \gamma_{fa2}(G) \leq \frac{|V|\Delta_{N_S(G)}}{\Delta_{N_S(G)} + 1} + 1$$

Proof. Let G be a fuzzy graph and a subset $D \subseteq V$, be an accurate 2-dominating set of G. Since, D is an accurate 2-dominating set of G, then each nodes in $V - D$ must have atleast two strong neighbours in D.

Case (i): Let us prove that $\gamma_{fa2}(G) \geq \frac{|V|}{\Delta_{N_S}(G)+1}$

If D be an accurate dominating set of G and there exists a node $v \in V$, which has maximum strong neighbours in $V(G)$, then v can dominate itself and number of nodes in Δ_{N_S}. Therefore, accurate domination number of fuzzy graph cannot be less than $\frac{|V|}{\Delta_{N_S}(G)+1}$. That is

$$\gamma_{fa}(G) \geq \frac{|V|}{\Delta_{N_S}(G)+1} \tag{1}$$

Since $\gamma_{fa}(G) \leq \gamma_{fa2}(G)$. Then,

$$\gamma_{fa2}(G) \geq \frac{|V|}{\Delta_{N_S}(G)+1} \tag{2}$$

Case (ii): Now, let us prove, $\gamma_{fa2}(G) \leq \frac{|V|\Delta_{N_S(G)}}{\Delta_{N_S(G)}+1} + 1$.

By theorem 18, $\gamma_{fa2}(G) \leq |V| - \gamma_{fa}(G) + 1$

By equation 1, $\gamma_{fa2}(G) \leq |V| - \frac{|V|}{\Delta_{N_S}(G)+1} + 1$

$$\gamma_{fa2}(G) \leq \frac{|V|(\Delta_{N_S(G)}+1) - |V|}{\Delta_{N_S(G)}+1} + 1$$

$$\gamma_{fa2}(G) \leq \frac{|V|\Delta_{N_S(G)}}{\Delta_{N_S(G)}+1} + 1 \tag{3}$$

Therefore, by equation 2 and 3, we get $\frac{|V|}{\Delta_{N_S}(G)+1} \leq \gamma_{fa2}(G) \leq \frac{|V|\Delta_{N_S(G)}}{\Delta_{N_S(G)}+1} + 1$. □

Corollary 20. *For any fuzzy graph G,* $\gamma_{fa2}(G) \geq \frac{|V|}{\Delta_{N_S}(G)+1}$.

Proof. Let G be a fuzzy graph and D be a minimum accurate 2-dominating set of fuzzy graph G, where $D \subseteq V$. Let S be the set of all strong arcs between the sets D and $V - D$.

If there exist a node $u \in V$, whose $|dN_S(u)| = \Delta_{N_S}(G)$, then the strong neighbourhood degree of every node $v \in D$ does not exceed $\Delta_{N_S}(G)$. Therefore,

$$S \leq \Delta_{N_S}(G).\gamma_{fa2}(G) \tag{4}$$

And also since every node $v \in V - D$ has atleast two strong neighbours in D. Then,

$$S \geq 2(|V| - \gamma_{fa2}(G)) \tag{5}$$

From equations 4 and 5,

$2(|V| - \gamma_{fa2}(G)) \leq \Delta_{N_S}(G).\gamma_{fa2}(G)$

$2|V| - 2\gamma_{fa2}(G) \leq \Delta_{N_S}(G).\gamma_{fa2}(G)$

$2|V| \leq \Delta_{N_S}(G).\gamma_{fa2}(G) + 2\gamma_{fa2}(G)$

$2|V| \leq (\Delta_{N_S}(G)+2).\gamma_{fa2}(G)$

$\therefore \gamma_{fa2}(G) \geq \frac{|V|}{\Delta_{N_S}(G)+1}$. □

4 Conclusion

In this paper, the accurate 2-dominating set and accurate 2-domination number are defined. Some results on accurate 2-dominating sets and accurate 2-domination number are discussed. Also, bounds of accurate 2-domination number of fuzzy graphs are found.

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