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Degree of Vertices in Operations of Intuitionistic Fuzzy Graphs

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Abstract

An intuitionistic fuzzy graph can be obtained from two given intuitionistic fuzzy graphs using Union, Join, Cartesian product and Composition. In this paper we find the degree of a vertex in intuitionistic fuzzy graphs formed by these operations in terms of the degree of vertices in the given intuitionistic fuzzy graphs in some particular cases.

AMS Subject Classification: 03E72, 03F55

Key Words and Phrases: Degree of a vertex, Union, Join, Cartesian product, Composition.

1 INTRODUCTION

Intuitionistic Fuzzy Graph theory was introduced by Atanassov in [1]. In [6] A.Nagoor Gani and Radha introduced the degree of a vertex in some fuzzy graphs. In [7] A.Nagoor Gani and Shajitha Begum introduced degree, order and size in intuitionistic fuzzy graph. A.Nagoor Gani and H.Sheik Mujibur Rahman introduced the union, join, Cartesian product and Composition of some intuitionistic fuzzy graphs in [8] and [9]. In this paper we introduce the degree of a vertex in intuitionistic fuzzy graphs for some particular cases.

2 DEGREE OF A VERTEX IN UNION

For any $u_1 \in V_1 \cup V_2$, we have three cases to consider
Case i: Either $u \in V_1$or $u \in V_2$ but not both. Then no edge incident at u lies in $E_1 \cap E_2$. So,

$$(\mu_2 \cup \mu_2^{'})(uv) = \begin{cases} \mu_2(uv), \ if \ u \in V_1(i.e.) if \ uv \in E_1 \\ \mu_2^{'}(uv), \ if \ u \in V_2(i.e.) if \ uv \in E_2 \end{cases} .$$

Hence if $u \in V_1$, then the membership degree of u is
$d_{\mu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1} \mu_2(uv) = d_{\mu(G_1)}(u)$, Similarly if $u \in V_2$, then the membership degree of u is $d_{\mu(G_1 \cup G_2)}(u) = \sum_{uv \in E_2} \mu_2^{'}(uv) = d_{\mu(G_2)}(u)$

$$(\nu_2 \cup \nu_2^{'})(uv) = \begin{cases} \nu_2(uv), \ if \ u \in V_1(i.e.) if \ uv \in E_1 \\ \nu_2^{'}(uv), \ if \ u \in V_2(i.e.) if \ uv \in E_2 \end{cases} .$$

Hence if $u \in V_1$, then the non membership degree of u is
$d_{\nu(G_1 \cup G_2)}(u) = \sum_{uv \in E_1} \nu_2(uv) = d_{\nu(G_1)(u)}$, Similarly if $u \in V_2$, then the non membership degree of u is $d_{\nu(G_1 \cup G_2)}(u) = \sum_{uv \in E_2} \nu_2^{'}(uv) = d_{\nu(G_2)}(u)$
Case ii : $u \in V_1 \cap V_2$ but no edge incident at u lies in$E_1 \cap E_2$. Then any edge incident at u is either E_1 or inE_2 but not both. Also all these edges will be included in$G_1 \cup G_2$. Hence the membership degree of u is

$$d_{\mu(G_1 \cup G_2)}(u) = \sum_{uv \in E} (\mu_2 \cup \mu_2^{'})(uv) = \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E_2} \mu_2^{'}(uv) = d_{\mu(G_1)}(u) + d_{\mu(G_2)}(u)$$

Similarly the non membership degree of u is

$$d_{\nu(G_1 \cup G_2)}(u) = \sum_{uv \in E} (\nu_2 \cup \nu_2^{'})(uv) = \sum_{uv \in E_1} \nu_2(uv) + \sum_{uv \in E_2} \nu_2^{'}(uv) = d_{\nu(G_1)}(u) + d_{\nu(G_2)}(u)$$

Case iii: $u \in V_1 \cap V_2$ and some edges incident at u are in$E_1 \cap E_2$. Any edge uv which is in $E_1 \cap E_2$ appear only once in $G_1 \cup G_2$ and for this uv, $(\mu_2 \cup \mu_2^{'})(uv) = \mu_2(uv) \vee \mu_2^{'}(uv)$ and $(\nu_2 \cup \nu_2^{'})(uv) = \nu_2(uv) \wedge \nu_2^{'}(uv)$, By definition,

$$\begin{aligned} d_{\mu(G_1 \cup G_2)}(u) &= \sum_{uv \in E} (\mu_2 \cup \mu_2^{'})(uv) \\ &= \sum_{uv \in E_1 - E_2} \mu_2(uv) + \sum_{uv \in E_2 - E_1} \mu_2^{'}(uv) + \sum_{uv \in E_1 \cap E_2} \mu_2(uv) \vee \mu_2^{'}(uv) \\ &= \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E_2} \mu_2^{'}(uv) - \sum_{uv \in E_1 \cap E_2} \mu_2(uv) \wedge \mu_2^{'}(uv) \\ &= d_{\mu(G_1)}(u) + d_{\mu(G_2)}(u) - \sum_{uv \in E_1 \cap E_2} \mu_2(uv) \wedge \mu_2^{'}(uv) \end{aligned}$$

Similarly,

$$d_{\nu(G_1 \cup G_2)}(u) = d_{\nu(G_1)}(u) + d_{\nu(G_2)}(u) - \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2^{'}(uv)$$

3 DEGREE OF A VERTEX IN JOIN

Here$V_1 \cap V_2 = \emptyset$, Hence$E_1 \cap E_2 = \emptyset$. So, $(\mu_2 \cup \mu_2^{'})(uv) = \{ \begin{array}{l} \mu_2(uv),\ if\ uv \in E_1 \\ \mu_2^{'}(uv),\ if\ uv \in E_2 \end{array}$.

By definition, $d_{\mu(G_1+G_2)}(u) = \sum_{uv \in E}(\mu_2 + \mu_2^{'})(uv)$

$$= \sum_{uv \in E_1 \cup E_2} (\mu_2 \cup \mu_2^{'})(uv) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1^{'}(v)$$

For any $u \in V_1$,

$$\begin{aligned} d_{\mu(G_1+G_2)}(u) &= \sum_{uv \in E_1} \mu_2(uv) + \sum_{uv \in E'} \mu_1(u) \wedge \mu_1^{'}(v) \\ &= d_{\mu(G_1)}(u) + \sum_{v \in V_2} \mu_1(u) \wedge \mu_1^{'}(v) \end{aligned} \tag{1}$$

Similarly, for any $u \in V_2$,

$$d_{\mu(G_1 \cup G_2)}(u) = d_{\mu(G_2)}(u) + \sum_{v \in V_1} \mu_1(u) \wedge \mu_1^{'}(v) \tag{2}$$

Now, $(\nu_2 \cup \nu_2^{'})(uv) = \{ \begin{array}{l} \nu_2(uv),\ if\ uv \in E_1 \\ \nu_2^{'}(uv),\ if\ uv \in E_2 \end{array}$.

By definition, $d_{\nu(G_1+G_2)}(u) = \sum_{uv \in E}(\mu_2 + \mu_2^{'})(uv)$

$$= \sum_{uv \in E_1 \cup E_2} (\nu_2 \cup \nu_2^{'})(uv) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1^{'}(v)$$

For any $u \in V_1$,

$$\begin{aligned} d_{\nu(G_1+G_2)}(u) &= \sum_{uv \in E_1} \nu_2(uv) + \sum_{uv \in E'} \nu_1(u) \vee \nu_1^{'}(v) \\ &= d_{\nu(G_1)}(u) + \sum_{v \in V_2} \nu_1(u) \vee \nu_1^{'}(v) \end{aligned} \tag{3}$$

Similarly, for any $u \in V_2$,

$$d_{\nu(G_1 \cup G_2)}(u) = d_{\nu(G_2)}(u) + \sum_{v \in V_1} \nu_1(u) \vee \nu_1^{'}(v) \tag{4}$$

Theorem 1. *Let G_1 and G_2 be two intuitionistic fuzzy graphs such that $\mu_1 \wedge \mu_1^{'}$ and $\nu_1 \vee \nu_1^{'}$ are constant functions for all $u \in V_1$ and $v \in V_2$. Then*

(i) $d_{\mu(G_1+G_2)}(u) = \{ \begin{array}{l} d_{\mu(G_1)}(u) + c_1 p_2,\ if u \in V_1 \\ d_{\mu(G_2)}(u) + c_1 p_1,\ if u \in V_2 \end{array}$.

(ii) $d_{\nu(G_1+G_2)}(u) = \{ \begin{array}{l} d_{\nu(G_1)}(u) + c_2 p_2,\ if u \in V_1 \\ d_{\nu(G_2)}(u) + c_2 p_1,\ if u \in V_2 \end{array}$.

Where $c_1$and c_2 are constant values of $\mu_1 \wedge \mu_1^{'}$ and $\nu_1 \vee \nu_1^{'}$ respectively.

Proof. Let $\mu_1(u) \wedge \mu_1'(v) = c_1$ and $\nu_1(u) \vee \nu_1'(v) = c_2$ are constants, for all $u \in V_1$and $v \in V_2$.

From (1), for any $u \in V_1$,

$$d_{\mu(G_1+G_2)}(u) = \sum_{uv\in E_1} \mu_2(uv) + \sum_{v\in V_2} \mu_1(u) \wedge \mu_1'(v) = d_{\mu(G_1)}(u) + \sum_{v\in V_2} c_1 = d_{\mu(G_1)}(u) + c_1 p_2$$

From (2), for any $u \in V_2$,

$$d_{\mu(G_1+G_2)}(u) = \sum_{uv\in E_2} \mu_2'(uv) + \sum_{v\in V_1} \mu_1(u) \wedge \mu_1'(v) = d_{\mu(G_2)}(u) + \sum_{v\in V_1} c_1 = d_{\mu(G_2)}(u) + c_1 p_1$$

From (3), for any $u \in V_1$,

$$d_{\nu(G_1+G_2)}(u) = \sum_{uv\in E_1} \nu_2(uv) + \sum_{v\in V_2} \nu_1(u) \vee \nu_1'(v) = d_{\nu(G_1)}(u) + \sum_{v\in V_2} c_2 = d_{\nu(G_1)}(u) + c_2 p_2$$

From (4), for any $u \in V_2$,

$$d_{\nu(G_1+G_2)}(u) = \sum_{uv\in E_2} \nu_2'(uv) + \sum_{v\in V_1} \nu_1(u) \vee \nu_1'(v) = d_{\nu(G_2)}(u) + \sum_{v\in V_1} c_2 = d_{\nu(G_2)}(u) + c_2 p_1$$

□

4 DEGREE OF A VERTEX IN CARTESIAN PRODUCT

By definition, for any $(u_1, u_2) \in V_1 \times V_2$,

$$\begin{aligned} d_{\mu(G_1\times G_2)}(u_1,u_2) &= \sum_{(u_1,u_2)(v_1,v_2)\in E} (\mu_2 \times \mu_2')((u_1,u_2)(v_1,v_2)) \\ &= \sum_{u_1=v_1, u_2v_2\in E_2} \mu_1(u_1) \wedge \mu_2'(u_2v_2) + \sum_{u_2=v_2,\ u_1v_1\in E_1} \mu_1'(u_2) \wedge \mu_2(u_1v_1) \end{aligned} \tag{5}$$

$$\begin{aligned} d_{\nu(G_1\times G_2)}(u_1,u_2) &= \sum_{(u_1,u_2)(v_1,v_2)\in E} (\nu_2 \times \nu_2')((u_1,u_2)(v_1,v_2)) \\ &= \sum_{u_1=v_1, u_2v_2\in E_2} \nu_1(u_1) \vee \nu_2'(u_2v_2) + \sum_{u_2=v_2, u_1v_1\in E_1} \nu_1'(u_2) \vee \nu_2(u_1v_1) \end{aligned} \tag{6}$$

In the following theorems we find the degree of (u_1, u_2) in $G_1 \times G_2$ in terms of those in G_1 and G_2 in some particular cases.

Theorem 2. *Let G_1 and G_2 be two intuitionistic fuzzy graphs.*

1. *If $\mu_1 \geq \mu_2'$ and $\mu_1' \geq \mu_2$ then $d_{\mu(G_1\times G_2)}(u_1, u_2) = d_{\mu(G_1)}(u_1) + d_{\mu(G_2)}(u_2)$*
2. *If $\nu_1 \leq \nu_2'$ and $\nu_1' \leq \nu_2$ then $d_{\nu(G_1\times G_2)}(u_1, u_2) = d_{\nu(G_1)}(u_1) + d_{\nu(G_2)}(u_2)$*

Proof. From (5),

$$d_{\mu(G_1\times G_2)}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \mu_1(u_1)\wedge\mu_2'(u_2v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \mu_1'(u_2)\wedge\mu_2(u_1v_1)$$
$$= \sum_{u_2v_2\in E_2} \mu_2'(u_2v_2) + \sum_{u_1v_1\in E_1} \mu_2(u_1v_1) \qquad [since\ \mu_1\geq\mu_2'\ and \mu_1'\geq\mu_2\ ]$$
$$d_{\mu(G_1\times G_2)}(u_1,u_2) = d_{\mu(G_2)}(u_2)+d_{\mu(G_1)}(u_1) \tag{7}$$

. From (6),

$$d_{\nu(G_1\times G_2)}(u_1,u_2) = \sum_{u_1=\ v_1,\ u_2v_2\in E_2} \nu_1(u_1)\vee\nu_2'(u_2v_2) + \sum_{u_2=\ v_2\ ,\ u_1v_1\in E_1} \nu_1'(u_2)\vee\nu_2(u_1v_1)$$
$$= \sum_{u_2v_2\in E_2} \nu_2'(u_2v_2) + \sum_{u_1v_1\in E_1} \nu_2(u_1v_1) \qquad [since\ \nu_1\leq\nu_2'\ and\nu_1'\ \leq\nu_2\ ]$$
$$d_{\nu(G_1\times G_2)}(u_1,u_2) = d_{\nu(G_2)}(u_2)+d_{\nu(G_1)}(u_1)$$

□

5 DEGREE OF A VERTEX IN COMPOSITION

By definition, for any $(u_1,u_2)\in V_1\times V_2$,

$$d_{\mu(G_1[G_2])}(u_1,u_2) = \sum_{(u_1,u_2)(v_1,v_2)\in E} ((\mu_2\circ\mu_2')(u_1,u_2)(v_1,v_2))$$
$$= \sum_{u_1=v_1,u_2v_2\in E_2} \mu_1(u_1)\wedge\mu_2'(u_2v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \mu_1'(u_2)\wedge\mu_2(u_1v_1)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \mu_1'(u_2)\wedge\mu_2(u_1v_1) \tag{8}$$

$$d_{\nu(G_1[G_2])}(u_1,u_2) = \sum_{(u_1,u_2)(v_1,v_2)\in E} ((\nu_2\circ\nu_2')(u_1,u_2)(v_1,v_2))$$
$$= \sum_{u_1=v_1,u_2v_2\in E_2} \nu_1(u_1)\vee\nu_2'(u_2v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \nu_1'(u_2)\vee\nu_2(u_1v_1)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \nu_1'(u_2)\vee\nu_2(u_1v_1) \tag{9}$$

Theorem 3. *Let G_1 and G_2 be two intuitionistic fuzzy graphs.*
(i) If $\mu_1\geq\mu_2'$ and $\mu_1'\geq\mu_2$, then $d_{\mu(G_1\ [\ G_2])}(u_1,u_2) = d_{\mu G_2}(u_2)+p_2\ d_{\mu G_1}(u_1)$
(ii) If $\nu_1\leq\nu_2'$ and $\nu_1'\leq\nu_2$, then $d_{\nu(G_1\ [\ G_2])}(u_1,u_2) = d_{\nu G_2}(u_2)+p_2\ d_{\nu G_1}(u_1)$
Where$p_2=|V_2|$, V_2 is the set of all nodes inG_1.

Proof. From (8),

$$\begin{aligned}
d_{\mu(G_1[\ G_2])}(u_1,u_2) &= \sum_{u_1=\ v_1,\ u_2v_2\in E_2} \mu_1(u_1)\wedge\mu_2'(u_2v_2) + \sum_{u_2\ =\ v_2,\ u_1v_1\in E_1} \mu_1'(u_2)\wedge\mu_2(u_1v_1) \\
&+ \sum_{u_2\neq v_2,\ u_1v_1\in E_1} \mu_1'(u_2)\wedge\mu_2(u_1v_1) \\
&= d_{\mu(G_2)}(u_2) + |V_2| \sum_{u_1v_1\in E_1} \mu_2(u_1v_1) \\
&= d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1).
\end{aligned}$$

From (9),

$$\begin{aligned}
d_{\nu(G_1[G_2])}(u_1,u_2) &= \sum_{u_1=v_1,u_2v_2\in E_2} \nu_1(u_1)\vee\nu_2'(u_2v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \nu_1'(u_2)\vee\nu_2(u_1v_1) \\
&+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \nu_1'(u_2)\vee\nu_2(u_1v_1) \\
&= d_{\nu(G_2)}(u_2) + |V_2| \sum_{u_1v_1\in E_1} \nu_2(u_1v_1) \\
&= d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1)
\end{aligned}$$

□

6 Conclusion

In this paper we have found the degree of vertices in $G_1 \cup G_2$ in terms of those in G_1& G_2 and the degree of vertices in $G_1 + G_2$,$G_1 \times G_2$, and $G_1[G_2]$ in terms of the degree of vertices in G_1 and G_2under some conditions and illustrated them through examples. They will be helpful especially when the graphs are very large. Also they will be useful in studying various properties of Union, Join, Cartesian product and Composition of intuitionistic fuzzy graphs.

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