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Newton's Divided Difference Interpolating Polynomial Approach for Multi-choice Indefinite Quadratic Transportation Problem

Merline Vinotha J [1] and Saranya A[2]

[1,2]*PG & Research Department of Mathematics,*
Holy Cross College,
Trichy 620 002, India
[1]*merlinejoe@ymail.com* and [2]*saran.arumugam90@gmail.com*

Abstract

This paper deals with the multi-choice indefinite quadratic transportation problem (MIQTP) in which parameters of the MIQTP such as the variable cost and damage cost of the objective function, supplies and demands of the constraints are all followed multi-choice parameters . Newtons divided difference interpolating polynomial approach is proposed for selecting an appropriate choice from the multi-choice for the variable cost and damage cost of the objective function, supplies and demands of the constraints in MIQTP which is better than the Lagranges interpolating polynomial approach for solving multi-choice transportation problem. A numerical example is presented to show the efficiency of the proposed method and solved using LINGO 17.0 software package.

AMS Subject Classification: 90B06
Key words: Indefinite quadratic transportation problem , Multi-choice programming, Newtons divided difference interpolating polynomial

1 Introduction

The classical transportation problem (CTP) is a particular class of linear programming problem (LPP) and deals with the distribution of goods (or products) from sources (plants or suppliers) to destinations (warehouses or customers). The main objective in CTP is to determine the amount of goods to be shipped from each source to each destination so as to satisfy the demand and supply requirements at the minimum total transportation cost.The linear functions are the most useful and widely used in operational research. A fair number of functional relationships occurring in the real world are truly quadratic.A bibliography of Quadratic programming problems can be found in [10].Indefinite quadratic programming problems and Interval Transportation Problem have been extensively studied for several decades. In [5], using fuzzy technique, a new method is proposed for interval transportation problems by considering the right bound and midpoint of interval.

Due to increase of the fuel price, road tax, market fluctuation and other considerable factors,this paper considers the cost coefficients of the objective function,supply and demand parameters based on multi-choice for the real-life transportation problem. Healy [7] first developed a procedure for linear programming problem with zero-one variables, which is called multiple choice programming. A method for modeling the multi-choice programming problem, using binary variables was presented by Chang [3].Again Chang [4] proposed a revised method of multi-choice goal programming model.Biswal and Acharya [1] formulated the interpolating polynomials to handle the multi-choice parameters included in the linear programming problem in which the constraints were associated with some multi-choice parameters. Roy et al.[12] presented the procedures to solve the multi-choice stochastic transportation problem where the authors converted the multi-choice transportation problem into a standard mathematical programming through the selection of binary variables, bounds for binary codes and restriction of binary codes using auxiliary constraints.

Shankar Kumar Roy [13] was solved multi choice transportation problem using Lagranges interpolating polynomial in which adding more data points would require re-doing the whole problem.But in the Newton's divided difference method, more data points can be added, for improved accuracy, without re-doing the whole problem. The terms based on the previous data points can continue to be used.Moreover the Lagrange approach amounts to diagonalizing the problem of finding the coefficients, so it takes only linear time to find the coefficients. In Newtons divided , we get the coefficients reasonably fast (ie. in quadratic time), the evaluation is much more stable the evaluation can be done quickly and straightforwardly

To the best of our knowledge none of the researchers studied the transportation problem with multi-choice parameter by Newtons divided interpolating polynomial method. In this paper we proposed Newtons divided difference interpolating polynomial approach for selecting an appropriate choice from the multi-choice parameters of MIQTP.A numerical example is presented to show the efficiency of the proposed method and solved using LINGO 17.0 software package.

The remainder is organized as follows. In section 2, Mathematical formulation of MCQTP is given. Section 3 deals with the Newton's divided difference approach in order to select an appropriate choice from the multi-choice parameters. A numerical example has been solved in Section 4. Concluding remarks are presented in Section 5 with scope of future research.

2 Mathematical Model of MIQTP

Consider the problem of transporting goods from various sources to different destinations. Let c_{ij} and d_{ij} be the transportation cost and damage cost of one unit from i^{th} source to j^{th} destination. Let the supplies $(a_1, a_2, \ldots, a_m)$ be the quantity of homogeneous product, which are to transport from m origins to n destinations and satisfy the demands $(b_1, b_2, ..., b_n)$ respectively. The quantity of damaged goods may be some fraction of the goods transported. We are interested in minimizing both the cost of transportation and the cost of damaged goods simultaneously. Moreover, in practical situations, the two costs, i.e. the cost of transportation and damage cost are always interdependent. Therefore, the objective function of the problem under consideration should be the product of two cost functions so that both of them are minimized simultaneously, and their interdependence is justified. Again assuming that $c_{ij} \in \{c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^{s_{ij}}\}$, $d_{ij} \in \{d_{ij}^1, d_{ij}^2, \ldots, d_{ij}^{t_{ij}}\}$, $a_i \in \{a_i^1, a_i^2, \ldots, a_i^{o_i}\}$ and $b_{ij} \in \{b_j^1, b_j^2, \ldots, b_j^{d_j}\}$ are multi-choice parameters. The mathematical model of the multi-choice transportation problem is presented as follows:

Model 1

$$min \quad z = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \{c_{ij}^1, c_{ij}^2, \ldots, c_{ij}^{s_{ij}}\} x_{ij} \right) \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \{d_{ij}^1, d_{ij}^2, \ldots, d_{ij}^{t_{ij}}\} x_{ij} \right) \tag{1}$$

subject to

$$\begin{aligned} \sum_{i=1}^{m} x_{ij} &\leq \{a_i^1, a_i^2, \ldots, a_i^{o_i}\}, \quad i = 1, 2, \ldots m \\ \sum_{j=1}^{n} x_{ij} &\geq \left\{b_j^1, b_j^2, \ldots, b_j^{d_j}\right\}, \quad j = 1, 2, \ldots n \\ x_{ij} &\geq 0, \qquad \forall i \quad and \quad j \end{aligned}$$

where

$$\sum_{i=1}^{m} max\, \{a_i^1, a_i^2, \ldots a_i^{o_i}\} \leq \sum_{j=1}^{m} max\, \left\{b_j^1, b_j^2, \ldots b_j^{d_j}\right\} (\text{ Feasibility Condition})$$

The following cases are to be considered

1. When $c_{ij}\,(i = 1, 2, \ldots m, j = 1, 2, \ldots n)$ follows multi-choice parameter.

2. When $d_{ij}\,(i = 1, 2, \ldots m, j = 1, 2, \ldots n)$ follows multi-choice parameter.

3. When $a_i\,(i = 1, 2, \ldots m)$ follows multi-choice parameter.

4. When $b_j\,(j = 1, 2, \ldots n)$ follows multi-choice parameter.

3 Newton's divided Difference Interpolating Polynomial approach for MIQTP

Newtons Divided Difference interpolating polynomial is constructed for the multi-choice parameters taking some integral values for the nodal points. Each node corresponds to exactly one functional value of a multi-choice parameter. Exactly o_i or d_j or s_{ij} or t_{ij} number of nodal points are needed if a parameter has o_i or d_j or s_{ij} or t_{ij} number of choices. The multi-choice parameters are replaced by Newtons Divided Difference interpolating polynomial and using some standard numerical methods which selects a parameter from multi-choice parameters.

3.1 When $c_{ij}\,(i = 1, 2, \ldots m, j = 1, 2, \ldots n)$ follows multi-choice parameter

Let $0, 1, ...(s_{ij} - 1)$ be s_{ij} number of node points where $c^1_{ij}, c^2_{ij}, \ldots, c^{s_{ij}}_{ij}$ are the associated functional values of the interpolation polynomial at s_{ij} different node points.
Table 1 : Transportation costs at the node points for TP

Node points	z_{ij}	0	1	2	...	$s_{ij} - 1$
Transportation Cost	$f(z_{ij}) = c_{ij}$	c^1_{ij}	c^2_{ij}	c^3_{ij}	...	$c^{s_{ij}}_{ij}$

Newtons Divided Difference interpolating polynomial $P_{s_{ij-1}}(z_{ij})$ of degree (s_{ij-1}) which interpolates the data from Table 1 has been derived as follows:

$$P_{(s_{ij}-1)}(s) = c^{s+1}_{ij}, s = 0, 1, ...(s_{ij} - 1), i = 1, 2, ...m \quad and \quad j = 1, 2, ..n. \tag{2}$$

Then Newtons Divided Difference interpolating polynomial for i j-th multi-choice parameters has been formulated as follows :

$$\begin{aligned} P(z_{ij}) = c^1_{ij} + z_{ij}(c^2_{ij} - c^1_{ij}) + z_{ij}(z_{ij} - 1)\frac{c^3_{ij} - 2c^2_{ij} + c^1_{ij}}{2!} + \ldots \\ + z_{ij}(z_{ij} - 1)(z_{ij} - 2)\ldots(z_{ij} - s_{ij} + 1)\frac{(c^{s_{ij}-1}_{ij} - nC_1c^{s_{ij-2}}_{ij} + nC_2c^{s_{ij-3}}_{ij} \ldots + (-1)^n c^1_{ij})}{n!} \end{aligned} \tag{3}$$

$\forall i \quad and \quad j, \quad n = 0, 1, 2, ..., (s_{ij} - 1)$

3.2 When $d_{ij}\,(i = 1, 2, \ldots m, j = 1, 2, \ldots n)$ follows multi-choice parameter

Let $0, 1, ...(t_{ij} - 1)$ be t_{ij} number of node points $d^1_{ij}, d^2_{ij}, \ldots, d^{t_{ij}}_{ij}$ are the associated functional values of the interpolation polynomial at t_{ij} different node points.
Table 2 : Damage costs at the node points for TP

Node points	w_{ij}	0	1	2	...	$t_{ij} - 1$
Damage Cost	$f(w_{ij}) = d_{ij}$	d^1_{ij}	d^2_{ij}	d^3_{ij}	...	d^1_{ij}

By Using above method , Newtons Divided Difference interpolating polynomial for i j-th multi-choice parameters has been formulated as follows :

$$\begin{aligned} P(w_{ij}) = d^1_{ij} + w_{ij}(d^2_{ij} - d^1_{ij}) + w_{ij}(w_{ij} - 1)\frac{d^3_{ij} - 2d^2_{ij} + d^1_{ij}}{2!} + \ldots \\ + w_{ij}(w_{ij} - 1)(w_{ij} - 2)\ldots(w_{ij} - d_{ij} + 1)\frac{(d^{t_{ij}-1}_{ij} - nC_1d^{t_{ij-2}}_{ij} + nC_2d^{t_{ij-3}}_{ij} \ldots + (-1)^n d^1_{ij})}{n!} \end{aligned} \tag{4}$$

$\forall i \quad and \quad j, \quad n = 0, 1, 2, ..., (s_{ij} - 1)$

3.3 When $a_i\,(i=1,2,\dots m)$follows multi-choice parameter

Let $0,1,...(o_i-1)$ be o_i number of node points $a_i^1,a_i^2,\dots,a_i^{o_i}$ are the associated functional values of the interpolation polynomial at o_i different node points.
Table 3: Supply values at the node points for TP

Node points	y_i	0	1	2	...	o_i-1
Supply Values	$f(y_i)=a_i$	a_i^1	a_i^2	a_i^3	...	$a_i^{o_i}$

Newtons Divided Difference interpolating polynomial $P_{o_{i-1}}(y_i)$ of degree(o_i-1) which interpolates the data from Table 3 has been derived as follows:

$$P_{(o_i-1)}(r)=a_i^{r+1},r=0,1,...(o_i-1),i=1,2,...m \tag{5}$$

Then Newtons Divided Difference interpolating polynomial for i -th multi-choice parameters has been formulated as follows :

$$\begin{aligned}P_{o_{i-1}}(y_i)&=a_i^1+y_i(a_i^2-a_i^1)+y_i(y_i-1)\frac{a_i^3-2a_i^2+a_i^1}{2!}+\dots\\&+y_i(y_i-1)(y_i-2)\dots(y_i-o_i+1)\frac{(a_i^{o_i-1}-nC_1a_i^{o_{i-2}}+nC_2a_i^{o_{i-3})}\dots+(-1)^na_i^1)}{n!}\end{aligned} \tag{6}$$

$\forall i, \quad n=0,1,2,...,(o_i-1)$

3.4 When $b_j\,(j=1,2,\dots n)$ follows multi-choice parameter

Let $0,1,...(d_j-1)$ be d_j number of node points $b_j^1,b_j^2,\dots,b_j^{d_j}$ are the associated functional values of the interpolation polynomial at d_j different node points.
Table 4: Demand values at the node points for TP

Node points	z_j	0	1	2	...	d_j-1
Demand Values	$f(z_j)=b_j$	b_j^1	b_j^2	b_j^3	...	$b_j^{d_j}$

Similarly, we have Newtons Divided Difference interpolating polynomial $P_{d_{j-1}}(z_j)$ of degree (d_j-1) which interpolates the data from Table 4 has been derived as follows:

$$\begin{aligned}P_{d_{j-1}}(z_j)&=b_j^1+z_j(b_j^2-b_j^1)+z_i(z_j-1)\frac{b_j^3-2b_j^2+b_j^1}{2!}+\dots\\&+z_i(z_j-1)(z_j-2)\dots(z_j-d_j+1)\frac{(b_j^{d_j-1}-nC_1b_j^{d_{j-2}}+nC_2a_i^{d_{i-3}}\dots+(-1)^na_i^1}{n!}\end{aligned} \tag{7}$$

$\forall j, \quad n=0,1,2,...,(d_j-1)$

Model 2

Model 1 can be redefined by considering the equations(3),(4),(6) and (7) as follows

$$min \quad z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij}^1 + z_{ij}(c_{ij}^2 - c_{ij}^1) + z_{ij}(z_{ij}-1) + + \frac{c_{ij}^3 - 2c_{ij}^2 + c_{ij}^1}{2!} + \dots\right.$$
$$\left. + z_{ij}(z_{ij}-1)(z_{ij}-2)\dots(z_{ij}-s_{ij}+1)\frac{(c_{ij}^{s_{ij}-1} - nC_1 c_{ij}^{s_{ij}-2} + nC_2 c_{ij}^{s_{ij}-3}\dots + (-1)^n c_{ij}^1)}{n!}\right)$$
$$\times \left(d_{ij}^1 + w_{ij}(d_{ij}^2 - d_{ij}^1) + w_{ij}(w_{ij}-1)\frac{d_{ij}^3 - 2d_{ij}^2 + d_{ij}^1}{2!} + \dots\right.$$
$$\left. + w_{ij}(w_{ij}-1)(w_{ij}-2)\dots(w_{ij}-t_{ij}+1)\frac{(d_{ij}^{t_{ij}-1} - nC_1 d_{ij}^{t_{ij}-2} + nC_2 c_{ij}^{t_{ij}-3}\dots + (-1)^n d_{ij}^1)}{n!}\right)$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i^1 + y_i(a_i^2 + a_i^1) + y_i(y_i-1)\frac{a_i^3 - 2a_i^2 + a_i^1}{2!} + \dots$$
$$+ y_i(y_i-1)(y_i-2)\dots(y_i-o_i+1)\frac{(a_i^{o_i-1} - nC_1 a_i^{o_i-2} + nC_2 a_i^{o_i-3}\dots + (-1)^n a_i^1)}{n!}$$

$$\sum_{i=1}^{m} x_{ij} \le b_j^1 + z_j(b_j^2 - b_j^1) + z_i(z_j-1)\frac{b_j^3 - 2b_j^2 + b_j^1}{2!} + \dots$$
$$+ z_i(z_j-1)(z_j-2)\dots(z_j-d_j+1)\frac{(b_j^{d_j-1} - nC_1 b_j^{d_j-2} + nC_2 a_i^{d_i-3}\dots + (-1)^n a_i^1)}{n!}$$

$$\begin{aligned} x_{ij} &\ge 0, \quad i = 1,2,...m \quad and \quad j = 1,2,....n \\ z_{ij} &= 0,1,...(s_{ij}-1), \quad i = 1,2,...m \quad and \quad j = 1,2,....n \\ w_{ij} &= 0,1,...(t_{ij}-1), \quad i = 1,2,...m \quad and \quad j = 1,2,....n \\ y_i &= 0,1,...(o_i-1), \quad i = 1,2,...m \\ z_j &= 0,1,...(d_j-1), \quad j = 1,2,....n \end{aligned}$$

4 Numerical Example

Consider a 2×3 indefinite quadratic transportation problem with transportation cost, damaged cost ,supply and demand are given as multi-choice parameters. The following table gives the values of c_{ij}, d_{ij}, a_i and b_j for i = 1, 2,3 and j = 1, 2, 3, 4.

O/D	D_1	D_2	D_3	D_4	Supply
O_1	(1,1.5,2,2.5)	(2,3,4,5)	(1,1.5,2,3)	(3,3.5,4,5)	
	(1,2,2.5,3)	(1,1.5,2,3)	(3,4,4.5,5)	(1,2,1.5,3)	(18,20,23,24)
O_2	(0,1,2,3)	(2,3,5,6)	(1,2,3.5,4.5)	(3,3.5,4,4.5)	
	(0,1,2,3)	(1,3,4,5)	(2,2.5,3,3.5)	(3,4,4.5,5)	(10,12,15,17)
O_3	(0,0.5,2,3)	(1,1.5,2,2.5)	(3,4.5,5,5.5)	(2,3,4,4.5)	
	(0,2,3,5)	(0,0.5,1,1.5)	(1,2.5,3,4)	(2,3,4,5)	(20,22,23,26)
Demand	(10,12,15,20)	(7,10,12,13)	(16,17,20,21)	(15,17,18,20)	

Note : Values in the upper left corners are $c'_{ij}s$ and values in the lower left corners are $d'_{ij}s$ for i = 1,2,3 and j = 1,2,3,4 .

Result Using proposed method and Lingo 17.0 package the above model is solved and the optimal solution is obtained as: $x_{13} = 3, x_{14} = 15, x_{21} = 2, x_{23} = 13, x_{31} = 13, x_{32} = 7$ and rest of the decision variables are zero. The minimum cost of the objective function is (in Rupees) 3400.00. The selected parameters for cost coefficients of objective function, supply and demand that provide optimal solution to the problem are as follows:

$$c_{11} = 1.5, c_{12} = 3, c_{13} = 1, c_{14} = 3, c_{21} = 0, c_{22} = 2, c_{23} = 1, c_{24} = 3.5$$
$$c_{31} = 0, c_{31} = 1, c_{32} = 3, c_{34} = 2, d_{11} = 2, d_{12} = 1.5, d_{13} = 3, d_{14} = 1$$
$$d_{21} = 0, d_{22} = 3, d_{23} = 2, d_{24} = 4, d_{31} = 0, d_{31} = 0, d_{32} = 1, d_{34} = 2$$
$$a_1 = 18, a_2 = 12, a_3 = 20, b_1 = 15, b_2 = 7, b_3 = 16, b_4 = 15$$

5 Conclusion

In this paper the solution procedure for indefinite quadratic transportation problem with the consideration of all the parameters which are multi-choice types is discussed by using Newtons Divided Difference interpolating polynomial. The main advantages of Newtons Divided Difference interpolating polynomial are (i) more points can be added for improved accuracy without re-doing the whole problem(ii) the coefficients can be obtained in quadratic time (iii) the evaluation can be done quickly and straightforwardly.

Finally, the proposed model is highly applicable for solving the real-life transportation problem and by following this model, the decision maker will be more benefited to take right decision for giving more information. In future the proposed methodology of this paper can be extended to solve multi-objective multi-choice transportation problem.Also this methodology may be useful to extract better solution of decision making problems in supply chain management.

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