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4-Prime cordiality of some cycle and star related graphs

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Abstract

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with a k-prime cordial labeling is called a k-prime cordial graph. In this paper, we investigate the 4-prime cordial labelling behaviour of some cycle and star related graphs.

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Key Words and Phrases: Cycle, path, comb, corona, star.

1 Introduction

Throughout this paper we have considered only simple and undirected graph. Terms and definitions not defined here are used in the sense of Harary [3] and Gallian [2]. Let G=(V,E) be a (p,q) graph. The cardinality of V is called the order of G and the cardinality of E is called the size of G. Rosa [2] introduced graceful labelling of graphs which was the foundation of the graph labelling. Cahit [1] initiated the concept of cordial labelling of graphs. Sundaram et al. [4] introduced the concept of prime cordial labelling of graphs. Motivated by the above labelings, the notion of k-prime cordial labelling has been introduced by Ponraj et al. [5] and they studied the k-prime cordial labelling behaviour of paths, cycles, bistars of even order. Also they studied about the 3-prime cordiality of paths, cycles, corona of tree with a vertex, comb, crown, olive tree and some more graphs [5, 6]. In this paper, we investigate the 4-prime cordial labelling behaviour of some cycle and star related graphs.

2 4-prime cordial labeling

Definition 1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \dots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with a k-prime cordial labeling is called a k-prime cordial graph.

Definition 2. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a comb. The graph $C_n \odot K_1$ is called a crown.

First we investigate the 4-prime cordiality of graphs which are obtained from i copies of the cycle C_m.

Theorem 3. Let $C_m^{(i)}$ be the cycle $u_1^i u_2^i \dots u_m^i u_1^i$ and m is even. Let G be the graph with $V(G) = \bigcup_{i=1}^{n} V(C_m^{(i)}) \cup \{u\}$ and $E(G) = \bigcup_{i=1}^{n} E(C_m^{(i)}) \cup \{uu_1^{(i)} : 1 \leq i \leq m\}$. Then G is 4-prime cordial.

Proof. Clearly G has mn+1 vertices and mn+n edges. The proof is divided into two parts.

Case 1. n is even.

Assign the label 2 to the vertex u. In this case, assign the labels 2,4 alternatively to the vertices of the first copy $C_m^{(1)}$. Next assign the labels 2,4 alternatively to the next cycle. Proceeding like this until we reach $\frac{m}{2}^{th}$ copy $C_m^{\frac{n}{2}}$. Consider the next copy $C_m^{\frac{n}{2}+1}$, assign the labels 1,3 alternatively to this copy. Then assign the labels 1,3 alternatively to the next copy $C_m^{\frac{n}{2}+2}$. Proceeding like this assign the label to the vertices of the remaining copies.

Case 2. n is odd.
Assign the labels to the vertices of the cycles $C_m^{(i)}$ $(1 \leq i \leq n-1)$ and u as in case 1. Consider last copy. Assign the labels 2,4 alternatively to the first $\frac{m}{2}$ vertices $u_1^n u_2^n \ldots u_{\frac{m}{2}}^n$. Note that $u_{\frac{m}{2}}^n$ received the label 4 or 2 according as $m \equiv 0 \pmod 4$ or $m \equiv 2 \pmod 4$. Next assign the labels 1,3 alternatively to the remaining $\frac{m}{2}$ vertices of this cycle. Clearly u_m^n received the label 3 or 1 according as $m \equiv 0 \pmod 4$ or $m \equiv 2 \pmod 4$. The table 1 given below establish that the labelling f is a 4-prime cordial labelling of G.

Nature of n		$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n even		$\frac{mn}{4}$	$\frac{mn}{4}+1$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn+n}{2}$	$\frac{mn+n}{2}$
n odd	$m \equiv 0 \pmod 4$	$\frac{mn}{4}$	$\frac{mn}{4}+1$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn+n+1}{2}$	$\frac{mn+n-1}{2}$
	$m \equiv 2 \pmod 4$	$\frac{mn+6}{4}$	$\frac{mn+2}{4}$	$\frac{mn+2}{4}$	$\frac{mn+2}{4}$	$\frac{mn+n+1}{2}$	$\frac{mn+n-1}{2}$

Table 1:

□

An illustration of this graph G with m=4 and n=4 is given in Figure 1.

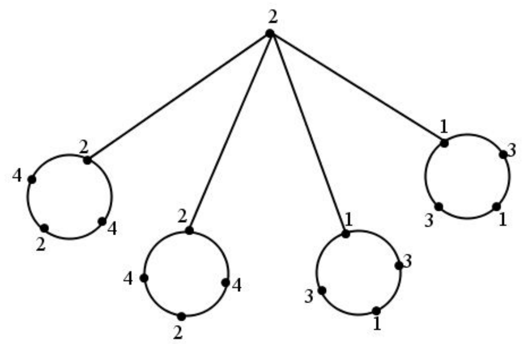

Figure 1:

Theorem 4. Let $C_m^{(i)}$ be the cycle $u_1^i u_2^i \ldots u_m^i u_1^i$. Let G be the graph where m is even with $V(G) = \bigcup_{i=1}^{n} V(C_m^{(i)})$ and $E(G) = \bigcup_{i=1}^{n} E(C_m^{(i)}) \cup \{u_1^i u_1^{i+1} : 1 \leq i \leq n-1\}$. Then G is 4-prime cordial.

Proof. Clearly the order and size of G are mn and mn+n-1 respectively.
Case 1. n is even.
Assign the labels 2,4 respectively to the vertices u_1^1 and u_1^2 respectively. Next assign the labels 2,4 to the vertices u_3^1 and u_4^2 respectively. Proceeding like assign the label in the consecutive vertices of the cycle. Next consider the second copy, assign the label to the vertices of this copy as in the 1st copy. Proceeding like this until we reach this until we reach m/2 th copy u, assign the label to the vertices of the ith copy $(2 \leq i \leq \frac{m}{2})$ as in $(i-1)^{th}$ copy. We now consider the $(\frac{m}{2}+1)^{th}$ copy. Assign the labels 1,3 respectively to the vertices $u_1^{\frac{m}{2}+1}$ and $u_2^{\frac{m}{2}+2}$. Next assign the labels

1,3 respectively to the vertices $u_3^{\frac{m}{2}+1}$ and $u_4^{\frac{m}{2}+2}$. Continuing in this pattern until we reach the last vertex. Next assign the label to the vertices of the $(\frac{m}{2}+2)^{th}$ copy as in the previous copy $(\frac{m}{2}+1)^{th}$ copy. Proceeding like this until we reach the last copy. Clearly the edges in the first $\frac{m}{2}$ cycles received the label 0 whereas the edges in the remaining $\frac{m}{2}$ edges received the label 1. $(\frac{n}{2}-1)$ edges in the path received the label 0 and $(\frac{n}{2}+1)$ edges in the path received the label 1.
Case 2. n is odd.
Assign the labels to the vertices of the first $\frac{n}{2}-1$ cycles as in case 1. Consider the $(\frac{n+1}{2})^{th}$ cycle. In this case assign the labels 2,4 to the vertices $u_1^{\frac{n+1}{2}}$, $u_2^{\frac{n+1}{2}+1}$. Then assign the labels 2,4 respectively to the next vertices $u_3^{\frac{n+1}{2}}$ and $u_4^{\frac{n+1}{2}}$. Proceeding like this until we reach the vertex $u_{\frac{m}{2}}^{\frac{n+1}{2}}$. Clearly the vertex $u_{\frac{m}{2}}^{\frac{n+1}{2}}$ received the label 4 or 2 according as $m \equiv 0 \pmod 4$ or $m \equiv 2 \pmod 4$. We now consider the next copy $C_m^{\frac{n+3}{2}}$. In this copy assign the labels 1,3 alternatively to the vertices. Continuing in this pattern until we reach the last copy that is assign the label to the vertices of the i^{th} copy as in $(i-1)^{th}$ copy $(\frac{n+3}{2} \leq i \leq n)$. Clearly this vertex labelling f is a 4-prime cordial labelling follows from the Table 2.

Nature of n		$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(1)$	$e_f(0)$
n even		$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn+n-2}{2}$	$\frac{mn+n}{2}$
n odd	$m \equiv 0 \pmod 4$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn+n+1}{2}$	$\frac{mn+n+1}{2}$
	$m \equiv 2 \pmod 4$	$\frac{mn}{4}+1$	$\frac{mn}{4}+1$	$\frac{mn}{4}$	$\frac{mn}{4}$	$\frac{mn+n-1}{2}$	$\frac{mn+n-1}{2}$

Table 2:

□

The final investigation is about the graph which is obtained from a comb.

Theorem 5. Let $P_n \odot K_1$ be the comb with vertex set $V(P_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{u_i u_{i+1} : 1 \leq i \leq n\} \cup \{u_i v_i : 1 \leq i \leq n\}$. Let G be the graph with $V(G) = V(P_n \odot K_1) \cup \{u_1^i, u_2^i, \ldots, u_m^i : 1 \leq i \leq n\}$ where m is even, and $E(G) = E(P_n \odot K_1) \cup \bigcup_{j=1}^{n} \{v_j u_1^i : 1 \leq i \leq m\}$. Then G is 4-prime cordial.

Proof. **Case 1.** n is even.
Assign the labels 2,4 respectively to the vertices u_1, u_2. Next assign the labels 2,4 to the vertices u3 and u4 respectively. Proceeding like this until we reach the vertex $u_{\frac{n}{2}}$. Obviously $u_{\frac{n}{2}}$ received the label 4 when $n \equiv 0 \pmod 4$, and 2 when $n \equiv 2 \pmod 4$. Next assign the labels 1,3 to the vertices $u_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}+2}$. Next assign the labels 1,3 to the vertices $u_{\frac{n}{2}+3}$ and $u_{\frac{n}{2}+4}$. Continuing in this pattern until we reach the vertex u_n. Note that u_n received the label 1 or 3 according as $n \equiv 2 \pmod 4$ or $n \equiv 0 \pmod 4$. Assign the label 2,4 alternatively v_1, v_2 etc until we reach the vertex $u_{\frac{n}{2}}$. Assign the label 2 to the pendent vertices whose neighbour is 2 and 4 to the pendent vertices whose neighbour is 4. Consider the vertices $v_{\frac{n}{2}+1}$. Assign the labels 3,1 to the vertices $v_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}+2}$. Next assign the labels 3,1 respectively to the vertices $v_{\frac{n}{2}+3}$ and $v_{\frac{n}{2}+4}$. Continuing in this pattern until we reach the vertex v_n. Next assign the label 1 to the pendent vertices whose neighbour received the label

3 and assign the label 3 to the pendent vertices whose neighbour received the label 1.

Case 2. n is odd.

Assign the label to the vertices u_i, v_i and all the pendent vertices adjacent to u_i, $(1 \leq i \leq \frac{n-1}{2})$ as in case 1. Consider the $(\frac{n+1}{2})^{th}$ copy. Assign the label 2 to the vertex $u_{\frac{n+1}{2}}$ and 4 to the vertex $v_{\frac{n+1}{2}}$. Assign the label 2,4 alternatively to the pendent vertices until we reach the vertex $u_{\frac{m}{2}}^{\frac{n+1}{2}}$. Next assign the 1,3 alternatively to the remaining pendent vertices of the copy. Next assign the label 1,3 alternatively to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. Assign the label 3,1 alternatively to the vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$. Finally assign the label 1 to the pendent vertices whose neighbour received the label 3 and 3 to the pendent vertices whose neighbor received the label 1. The vertex labelling f is a 4-prime cordial labeling follows from Table 3.

Nature of n		$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n even		$\frac{mn+2n}{4}$	$\frac{mn+2n}{4}$	$\frac{mn+2n}{4}$	$\frac{mn+2n}{4}$	$\frac{mn+2n-2}{2}$	$\frac{mn+2n}{2}$
n odd	$m \equiv 0 \pmod 4$	$\frac{m(n+1)+m-2}{4}$	$\frac{m(n+1)+m+2}{4}$	$\frac{m(n+1)+m-2}{4}$	$\frac{m(n+1)+m-2}{4}$	$\frac{mn+2n}{2}$	$\frac{mn+2n-2}{2}$
	$m \equiv 2 \pmod 4$	$\frac{m(n+1)+m+2}{4}$	$\frac{m(n+1)+m+2}{4}$	$\frac{m(n+1)+m+2}{4}$	$\frac{m(n+1)+m+2}{4}$	$\frac{mn+2n-2}{2}$	$\frac{mn+2n}{2}$

Table 3:

□

An illustration of graph with n=5 and m=4 is given in Figure 2.

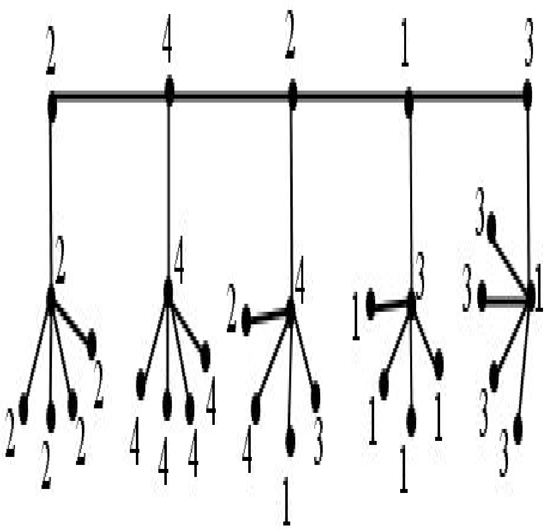

Figure 2:

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