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EPQ Model with Imperfect Quality Items with Allowable Shortages using Fuzzy Numbers

A. Nagoor Gani[1] and S. Raja Dharik[2]*

[1]*PG and Research Department of Mathematics,*
Jamal Mohamed College (Autonomous),
Tiruchirappalli–620020. India.
Email: ganijmc@yahoo.co.in
[2]*PG Department of Mathematics,*
Syed Hameedha Arts & Science College,
Kilakarai–623806. India.
Email: rajadhairk@gmail.com

Abstract

Investigation is initiated in the article in the production of inventory model with imperfect quality items, permitting shortages. Hundred percent screening process helps during the production stage in picking up the imperfect quality items in the sale as a single batch due to the economical scale. The main objective is to decide upon the maximum production quantity as well as shortage quantity in order to achieve the maximum of the total profit. The cost of holding, setup, rates of production, supply and defective rate are considered as triangular fuzzy numbers in developing a fuzzy model to maximize the total profit, using the method of graded mean integration to defuzzify the result. Thus, optimal lot size and optimal shortage quantities are derived at. To prove the results of the proposed model, numerical examples are supplied.

AMS Subject Classification: 03E72, 90B05

Key Words: EPQ with imperfect quality, Shortages, Triangular fuzzy numbers, Defuzzification.

1 Introduction

Usually, in the production inventory scenario, the manufactured products shall be hundred percent profit, where as imperfect quality items are also inevitable in real life. Moreover, it is the wish of the producers to supply their customers the items of perfect quality only. Hence, a screening process is followed to pick out imperfect items by removing or repairing or replacing as a rework model addressed by A.M.M. Jamal, B.R. Sarkar and S. Mondal. Many a research scholars tried their best to adopt a way or the other in their own way in this process: Y. Chiu, rework and backlogging, Salameh and Jaber incorporated some or structure of imperfect quality items and screening process by taking imperfect quality items as a random variable. Rezaei brought the concept of shortages and compared the expected profit with that to EOQ model Eroglu and Ozdemir and Wee et al independently extended the work of Salameh and Jaber to account for backorders Maddah et al considered production system where items purchased or produced are two different qualities.
Chang H.C has applied fuzzy set theory to EOQ with imperfect quality items. In our study, we assume finite rate of production of $\tilde{p}$ units and finite rate of supply of $\tilde{d}$ units per unit time. The unit production cost is Rs.c and setup cost is $\tilde{k}$ per run. Each lot produced contains $\tilde{r}$ percentage of defectives. Defective items are picked up by cent percentage of screening process at a rate x and are sold at the rate of s_p per unit prior to next production schedule shortages are permissible and backorder cost is $\tilde{b}$ per unit. q_2 is the optimum unit of backordering. It is also assumed that only good quality items are alone supplied to the customer. The behavior of inventory level is illustrated in Fig.1 where T is the total cycle length. (See [1], [2], [3], [4], [5], [6], [7], [8], [9], [10])

2 EPQ Model with Imperfect Quality Items with Allowable Shortages

2.1 Assumptions

Assume that each production run of length T consists of two parts T_1 and T_2 which are further sub-divided into two parts say T_{11} and T_{12}, T_{21} and T_{22} where

- Inventory is building up at a constant rate of $\tilde{p} - (1 - \tilde{r})\tilde{d}$ units per unit of time during time T_{11}.
- No replenishment during time T_{12} and inventory is decreasing at the rate $\tilde{d}$ per unit of time.
- Shortage is building up at a constant rate of $\tilde{d}$ per unit of time during time T_{21}.
- Shortage are being filled immediately at the rate of $\tilde{p} - (1 - \tilde{r})\tilde{d}$ units per unit of time during time T_{22}, and lead time is zero.

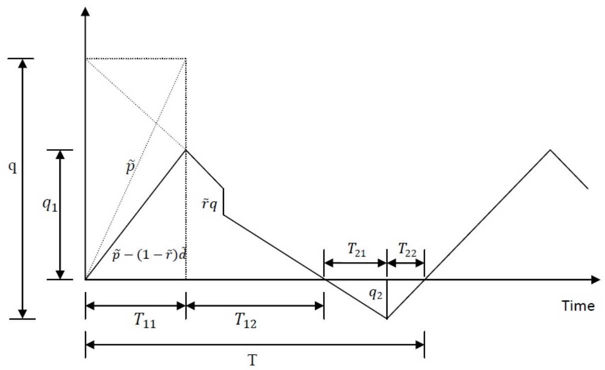
2.2 Notations

$\tilde{k}$ - Fuzzy setup cost
$\tilde{p}$ - Fuzzy production rate

$\tilde{d}$ - Fuzzy demand rate
$\tilde{r}$ - Percentage of fuzzy defective rate
$\tilde{h}$ - Fuzzy inventory holding cost per unit per time
x - Screening rate
x_c - Screening cost per item
s_p - Selling price of perfect quality item
s_i - Selling price of imperfect quality item $s_p > s_i$
q - Production quantity
q_2 - Shortage quantity
$\tilde{b}$ -Fuzzy shortage cost per unit
c - Production cost per unit
T - Cycle length
$P[\widetilde{TC}(q)]$ - Defuzzified value of $\widetilde{TC}_(q)$.

In the Fig.1 deals with at the end of T_{11} the level of inventory is q_1 and at the end of period T_{12} inventory becomes nil. Now shortages start and suppose that the shortages build up of quantity q_2 up to time T_{21} and let then these shortages be filed up during time T_{22}. Then,
$q_1 = T_{11}(\tilde{p} - (1-\tilde{r})\tilde{d})$, $q_1 = T_{12}\tilde{d}$
$q_2 = T_{21}\tilde{d}$, $q_2 = T_{22}(\tilde{p} - (1-\tilde{r})\tilde{d})$

Based on the assumptions, the situation is shown on inventory-time diagram in Fig.1.



Now if q is the lot size, then $q_1 = q - q_2 - \tilde{d}(1-\tilde{r})T_{11} - \tilde{d}(1-\tilde{r})T_{22}$
Substituting the value of T_{11} and T_{22} the above equation then we get

$$q_1 + q_2 = \frac{(\tilde{p} - (1-\tilde{r})\tilde{d})q}{\tilde{p}}$$

Production cycle T is

$$T = T_{11} + T_{12} + T_{21} + T_{22} = \frac{q_1}{\tilde{p} - (1-\tilde{r})\tilde{d}} + \frac{q_1}{\tilde{d}} + \frac{q_2}{\tilde{d}} + \frac{q_2}{\tilde{p} - (1-\tilde{r})\tilde{d}} = \frac{(\tilde{p} + \tilde{r}\tilde{d})q}{\tilde{p}\tilde{d}}$$

Average inventory= $\frac{1}{2}q_1(T_{11}+T_{12})/T$ and Average shortage= $\frac{1}{2}q_2(T_{21}+T_{22})/T$
$\widetilde{TC}$ =Setup cost + inventory holding cost + shortage cost + screening cost

$$\widetilde{TC} = \frac{\tilde{k}}{T} + \frac{q_1\tilde{h}(T_{11}+T_{12})}{2T} + \frac{q_2\tilde{b}(T_{21}+T_{22})}{2T} + \frac{\tilde{h}q^2\tilde{r}}{xT}$$

After some algebraic simplification we get

$$\widetilde{TC} = \frac{\tilde{k}\tilde{p}\tilde{d}}{(\tilde{p}+\tilde{r}\tilde{d})q} + \frac{\tilde{p}}{2q(\tilde{p}-(1-\tilde{r})\tilde{d})}\left[\tilde{h}\left(\frac{\tilde{p}-(1-\tilde{r})\tilde{d}}{\tilde{p}}q - q_2\right)^2 + \tilde{b}q_2^2\right] + \frac{\tilde{h}q\tilde{r}\tilde{p}\tilde{d}}{x(\tilde{p}+\tilde{r}\tilde{d})} \qquad (1)$$

Total selling price $\widetilde{TS} = \frac{s_pq(1-\tilde{r})+s_iq\tilde{r}-cq-x_cq}{T}$
Total profit $\widetilde{TP} = \widetilde{TS} - \widetilde{TC}$

$$\widetilde{TP} = \frac{(s_p(1-\tilde{r})+s_i\tilde{r}-c-x_c)\,\tilde{p}\tilde{d}}{(\tilde{p}+\tilde{r}\tilde{d})} - \frac{\tilde{k}\tilde{p}\tilde{d}}{(\tilde{p}+\tilde{r}\tilde{d})q} - \frac{\tilde{p}}{2q(\tilde{p}-(1-\tilde{r})\tilde{d})}\left[\tilde{h}\left(\frac{\tilde{p}-(1-\tilde{r})\tilde{d}}{\tilde{p}}q - q_2\right)^2 + \tilde{b}q_2^2\right] - \frac{\tilde{h}q\tilde{r}\tilde{p}\tilde{d}}{x(\tilde{p}+\tilde{r}\tilde{d})} \qquad (2)$$

Consider setup cost $\tilde{k}=(k_1,k_2,k_3)$, carrying cost $\tilde{h}=(h_1,h_2,h_3)$, production rate $\tilde{p}=(p_1,p_2,p_3)$, demand rate $\tilde{d}=(d_1,d_2,d_3)$, defective rate $\tilde{r}=(r_1,r_2,r_3)$ and shortage cost $\tilde{b}=(b_1,b_2,b_3)$ are triangular fuzzy numbers.

$$\begin{aligned} P[\widetilde{TC}(q)] &= \frac{1}{6}\Bigg[\{\frac{(s_p(1-r_3)+s_ir_1-c-x_c)p_1d_1}{(p_3+r_3d_3)} - \frac{k_1p_1d_1}{(p_3+r_3d_3)q} - \frac{h_1r_1p_1d_1q}{x(p_3+r_3d_3)} - \frac{p_3}{2q(p_1-(1-r_1)d_3)}\left[h_3\left(\frac{p_3-(1-r_3)d_1}{p_1}q-q_2\right)^2+b_3q_2^2\right]\} \\ &+ 4\{\frac{(s_p(1-r_2)+s_ir_2-c-x_c)p_2d_2}{(p_2+r_2d_2)} - \frac{k_2p_2d_2}{(p_2+r_2d_2)q} - \frac{h_2r_2p_2d_2q}{x(p_2+r_2d_2)} - \frac{p_2}{2q(p_2-(1-r_2)d_2)}\left[h_2\left(\frac{p_2-(1-r_2)d_2}{p_2}q-q_2\right)^2+b_2q_2^2\right]\} \\ &+ \{\frac{(s_p(1-r_1)+s_ir_3-c-x_c)p_3d_3}{(p_1+r_1d_1)} - \frac{k_3p_3d_3}{(p_1+r_1d_1)q} - \frac{h_3r_3p_3d_3q}{x(p_1+r_1d_1)} - \frac{p_1}{2q(p_3-(1-r_3)d_1)}\left[h_1\left(\frac{p_1-(1-r_1)d_3}{p_3}q-q_2\right)^2+b_1q_2^2\right]\}\Bigg] \end{aligned}$$

Partially differentiating the above equation with respect to q_2 and equating to zero we get

$$q_2 = \left[\frac{\frac{h_3(p_3-(1-r_3)d_1)p_3}{(p_1-(1-r_1)d_3)p_1} + 4h_2 + \frac{h_1(p_1-(1-r_1)d_3)p_1}{(p_3-(1-r_3)d_1)p_3}}{\frac{(b_3+h_3)p_3}{(p_1-(1-r_1)d_3)} + \frac{4(b_2+h_2)p_2}{(p_2-(1-r_2)d_2)} + \frac{(b_1+h_1)p_1}{(p_3-(1-r_3)d_1)}}\right] q = Cq(say) \qquad (3)$$

$\frac{\partial^2(P[\widetilde{TC}(q)])}{\partial q_2^2}$ is negative. Therefore $q_2^* = Cq^*$
To determine q^*, we substitute the value of q_2^* in $P[\widetilde{TC}(q)]$ then

$$\begin{aligned} P[\widetilde{TC}(q)] &= \frac{1}{6}\Bigg[\{\frac{(s_p(1-r_3)+s_ir_1-c-x_c)p_1d_1}{(p_3+r_3d_3)} - \frac{k_1p_1d_1}{(p_3+r_3d_3)q} - \frac{h_1r_1p_1d_1q}{x(p_3+r_3d_3)} - \frac{p_3}{2q(p_1-(1-r_1)d_3)}\left[h_3\left(\frac{p_3-(1-r_3)d_1}{p_1}q-Cq\right)^2+b_3C^2q^2\right]\} \\ &+ 4\{\frac{(s_p(1-r_2)+s_ir_2-c-x_c)p_2d_2}{(p_2+r_2d_2)} - \frac{k_2p_2d_2}{(p_2+r_2d_2)q} - \frac{h_2r_2p_2d_2q}{x(p_2+r_2d_2)} - \frac{p_2}{2q(p_2-(1-r_2)d_2)}\left[h_2\left(\frac{p_2-(1-r_2)d_2}{p_2}q-Cq\right)^2+b_2C^2q^2\right]\} \\ &+ \{\frac{(s_p(1-r_1)+s_ir_3-c-x_c)p_3d_3}{(p_1+r_1d_1)} - \frac{k_3p_3d_3}{(p_1+r_1d_1)q} - \frac{h_3r_3p_3d_3q}{x(p_1+r_1d_1)} - \frac{p_1}{2q(p_3-(1-r_3)d_1)}\left[h_1\left(\frac{p_1-(1-r_1)d_3}{p_3}q-Cq\right)^2+b_1C^2q^2\right]\}\Bigg] \end{aligned}$$

Differentiating the above equation with respect to q and equating to zero we get

$$q^* = \sqrt{\frac{\frac{k_1p_1d_1}{p_3+r_3d_3} + \frac{4k_2p_2d_2}{p_2+r_2d_2} + \frac{k_3p_3d_3}{p_1+r_1d_1}}{\left[\begin{array}{l} (\frac{p_3}{2(p_1-(1-r_1)d_3)})[h_3\left(\frac{p_3-(1-r_3)d_1}{p_1}-C\right)^2+b_3C^2] + 4(\frac{p_2}{2(p_2-(1-r_2)d_2)})[h_2\left(\frac{p_2-(1-r_2)d_2}{p_2}-C\right)^2+b_2C^2] \\ +(\frac{p_1}{2(p_3-(1-r_3)d_1)})[h_1\left(\frac{p_1-(1-r_1)d_3}{p_3}-C\right)^2+b_1C^2] + \frac{1}{x}[\frac{h_1r_1p_1d_1}{p_3+r_3d_3} + \frac{h_2r_2p_2d_2}{p_2+r_2d_2} + \frac{h_3r_3p_3d_3}{p_1+r_1d_1}] \end{array}\right]}} \qquad (4)$$

Validity:
If we take $k_1=k_2=k_3=k$, $p_1=p_2=p_3=p$, $d_1=d_2=d_3=d$, $r_1=r_2=r_3=r$,

$h_1 = h_2 = h_3 = h$ and $b_1 = b_2 = b_3 = b$.

Equating (3) becomes $q_2 = \frac{h(p-(1-r)d)}{(b+h)p} q$

and equation (4) becomes

$$q^* = \sqrt{\frac{kpd}{(p+rd)[\frac{hrpd}{x(p+rd)} + \frac{p-(1-r)d}{2p(b+h)} bh]}}$$

which are exactly the results of the crisp model.

3 Numerical Example

Consider a situation with the following parameters.
Production rate $p = 30000$ units per year, Supply rate $d = 16300$ units per year, Setup cost $k = Rs.1000$ per setup, Holding cost $h = Rs.3.25$ per unit per year, Shortage cost $b = Rs.30$ per unit per year, Screening rate $x = Rs.175200$ per unit per year, Screening cost $x_c = Re.0.5$ per unit, Production cost $c = Rs.25$ per unit, Selling price of good quality item $s_p = Rs.50$ per unit, Selling price of imperfect quality item $s_i = Rs.20$ per unit, Fraction of imperfect quality item $r = 0.002$

The optimum order quantity is found to be 4829 units. In fuzzy model if $\tilde{k} = (990, 1000, 1010)$, $\tilde{p} = (29950, 30000, 30050)$, $\tilde{d} = (16200, 16300, 16400)$, $\tilde{r} = (0.015, 0.02, 0.025)$, $\tilde{h} = (3.24, 3.25, 3.26)$ and $\tilde{b} = (29, 30, 31)$ then optimum order quantity $q^* = 4828$ There is a difference of only one unit between crisp and fuzzy optimal order quantity. Sensitivity analysis on the above parameters is presented in Table 1.

4 Conclusion

From Table.1, we infer the following (i) when percentage of defective items increases, then the optimum order quantity is decreases, keeping all parameters fixed (ii) when backorder cost decreases, then the optimum and shortage quantity is increases, keeping other values fixed (iii) when holding cost increases and shortage cost decreases, the optimum order quantity and shortage quantity increases (iv) when holding cost, rate of supply, percentage of defective items increase and shortage cost decreases keeping the other values fixed, the optimal order quantity and shortage quantity get almost doubled (v) when percentage of defective items and holding cost increase and shortage cost decreases, keeping the other parameters unchanged, there is considerable increase in optimum order quantity and backorder quantity. But in all the cases we see that the optimal values of order quantity and shortage are very much closer to the crisp values.

$\tilde{k}$	$\tilde{h}$	$\tilde{p}$	$\tilde{d}$	$\tilde{r}$	$\tilde{b}$	$y^*(fuzzy)$	$y^*(crisp)$	Difference	y_2^*
990,1000,1010	3.24,3.25,3.26	29950,30000,30050	16200,16300,16400	.015,.02,.025	29,30,31	4828	4829.1	1	221
990,1000,1010	3.24,3.25,3.26	29950,30000,30050	16200,16300,16400	.024,.025,.026	29,30,31	4803	4803	0	221
990,1000,1010	3.24,3.25,3.26	29950,30000,30050	16200,16300,16400	.029,.03,.031	29,30,31	4778	4779	1	221
990,1000,1010	3.24,3.25,3.26	29950,30000,30050	16200,16300,16400	.034,.035,.036	29,30,31	4753	4754	1	221
990,1000,1010	3.24,3.25,3.26	29950,30000,30050	16200,16300,16400	.034,.035,.036	14,15,16	4976	4977	1	422
990,1000,1010	3.15,3.2,3.25	29950,30000,30050	16200,16300,16400	.015,.02,.025	24,25,26	4907	4909	2	260
990,1000,1010	3.1,3.2,3.3	29950,30000,30050	16100,16200,16300	.015,.02,.025	24,25,26	4874	4878	4	260
990,1000,1010	4.9,5,5.1	29950,30000,30050	24900,25000,25100	.039,.04,.041	14,15,16	7745	7752	7	387
990,1000,1010	4.9,5,5.1	29950,30000,30050	24900,25000,25100	.039,.04,.041	19,20,21	7515	7522	7	387
990,1000,1010	3.9,4,4.1	29950,30000,30050	16400,16500,16600	.039,.04,.041	19,20,21	4487	4488	1	353

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