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Lagrange's quadratic functional equation linked to random homomorphisms and random derivations in random Banach algebras

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Abstract

In this paper, we discuss the generalized Ulam-Hyers stability of Lagrange's quadratic functional equation associated to random homomorphisms and random derivations in random Banach algebras using both direct and fixed point methods.

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1 Introduction

The stability problem of functional equations originated from a question of S.M. Ulam [21] concerning the stability of group homomorphism. Last seven decades the stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [1, 2, 5, 8, 13, 17, 18]). Recently John M. Rassias, M. Arunkumar and S. Karthikeyan introduced some quadratic functional equations namely, Brahmagupta [3], Lagrange [14], Euler [15], Degen-Graves-Cayley-Eight Squares [16] and established Ulam-Hyers stability results in various normed algebras. Theory of random normed spaces were first introduced by A. N. Sherstnev in 1963 (see [20]) which was generalized in [4, 6, 7, 19]. We apply the definition of random normed spaces and random Banach algebras briefly as given in [10, 11].

In this paper, the authors discuss the generalized Ulam-Hyers stability of Lagrange's quadratic functional equation

$$\left(\sum_{i=1}^{n} f(x_i)\right)\left(\sum_{i=1}^{n} f(y_i)\right) = f\left(\sum_{i=1}^{n} x_i y_i\right) + \sum_{1\le i<j\le n} f\left(x_i y_j - x_j y_i\right) \tag{1}$$

associated to random homomorphisms and random derivations in random Banach algebras using both direct and fixed point methods.

Throughout this paper, let us consider $(\mathcal{X}, \mu, T_M)$ is a random normed algebra and $(\mathcal{Y}, \mu, T_M)$ is a random Banach algebra. For a given mapping $f : \mathcal{X} \to \mathcal{Y}$, we define

$$Df(x_1, y_1, \cdots, x_n, y_n) = \left(\sum_{i=1}^{n} f(x_i)\right)\left(\sum_{i=1}^{n} f(y_i)\right) - f\left(\sum_{i=1}^{n} x_i y_i\right) - \sum_{1\le i<j\le n} f\left(x_i y_j - x_j y_i\right)$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$.

2 Stability results: Direct method

In this section, using Hyers direct method, we discuss the generalized Ulam- Hyers stability of random homomorphisms and random derivations connected to the Lagrange's quadratic functional equation (1) in random Banach algebras.

2.1 Stability of random homomorphisms via direct method

Definition 1. An $\mathbb{R}$-linear mapping $f : \mathcal{X} \to \mathcal{Y}$ is called a quadratic random homomorphism in random Banach algebras if $f(xy) = f(x)f(y)$ for all $x, y \in \mathcal{X}$.

Theorem 2. *Let $j = \pm 1$. Let $f : \mathcal{X} \to \mathcal{Y}$ be a mapping for which there exist a function $\eta : \mathcal{X}^{2n} \to D^+$ with the condition*

$$\lim_{k\to\infty} T_{i=0}^{\infty}\left(\eta_{n^{kj}x_1, n^{kj}y_1, \cdots, n^{kj}x_n, n^{kj}y_n}\left(n^{2kj}t\right)\right) = 1 = \lim_{k\to\infty} \eta_{n^{kj}x_1, n^{kj}y_1, \cdots, n^{kj}x_n, n^{kj}y_n}\left(n^{2kj}t\right) \tag{2}$$

such that the functional inequalities

$$\mu_{Df(x_1, y_1, \cdots, x_n, y_n)}(t) \ge \eta_{x_1, y_1, \cdots, x_n, y_n}(t) \tag{3}$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$ and all $t > 0$ and

$$\mu_{f(xy)-f(x)f(y)}(t) \ge \eta_{x,y,0,\cdots,0,0}(t) \tag{4}$$

for all $x, y \in \mathcal{X}$ and all $t > 0$. Then there exists a unique quadratic random homomorphism $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ satisfying the functional equation (1) and

$$\mu_{\mathcal{H}(x)-f(x)}(t) \ge T_{i=0}^{\infty} \eta_{\sqrt{n^{ij}}x, \sqrt{n^{ij}}x, \cdots, \sqrt{n^{ij}}x, \sqrt{n^{ij}}x}\left(n^{2(i+1)j}t\right) \tag{5}$$

for all $x \in \mathcal{X}$ and all $t > 0$. The mapping $\mathcal{H}(x)$ is defined by

$$\mu_{\mathcal{H}(x)}(t) = \lim_{k\to\infty} \mu_{\frac{f(n^{kj}x)}{n^{2kj}}}(t) \tag{6}$$

for all $x \in X$ and all $t > 0$.

Proof. Assume $j = 1$. Replacing $(x_1, y_1, \cdots, x_n, y_n)$ by $(\sqrt{x}, \sqrt{x}, \cdots, \sqrt{x}, \sqrt{x})$ and using random normed space definition in (3), we get

$$\mu_{\frac{f(nx)}{n^2} - f(x)}(t) \geq \eta_{\sqrt{x},\sqrt{x},\cdots,\sqrt{x},\sqrt{x}}(n^2 t) \tag{7}$$

for all $x \in \mathcal{X}$ and all $t > 0$. Replacing x by $n^k x$ in (7), we arrive

$$\mu_{\frac{f(n^{k+1}x)}{n^{2(k+1)}} - \frac{f(n^k x)}{n^{2k}}}(t) \geq \eta_{\sqrt{n^k x},\sqrt{n^k x},\ldots,\sqrt{n^k x},\sqrt{n^k x}}(n^{2(k+1)} t) \tag{8}$$

for all $x \in \mathcal{X}$ and all $t > 0$. It is easy to see that

$$\frac{f(n^k x)}{n^{2k}} - f(x) = \sum_{i=0}^{k-1} \frac{f(n^{i+1}x)}{n^{2(i+1)}} - \frac{f(n^i x)}{n^{2i}} \tag{9}$$

for all $x \in \mathcal{X}$. From equations (8) and (9), we have

$$\begin{aligned} \mu_{\frac{f(n^k x)}{n^{2k}} - f(x)}(t) = \mu_{\sum_{i=0}^{k-1} \frac{f(n^{i+1}x)}{n^{2(i+1)}} - \frac{f(n^i x)}{n^{2i}}}(t) &\geq T_{i=0}^{k-1} \mu_{\frac{f(n^{i+1}x)}{n^{2(i+1)}} - \frac{f(n^i x)}{n^{2i}}}(t) \\ &\geq T_{i=0}^{k-1} \eta_{\sqrt{n^i x},\sqrt{n^i x},\cdots,\sqrt{n^i x},\sqrt{n^i x}}(n^{2(i+1)} t) \end{aligned} \tag{10}$$

for all $x \in \mathcal{X}$ and all $t > 0$. In order to prove the convergence of the sequence $\left\{\frac{f(n^k x)}{n^{2k}}\right\}$, we replace x by $n^m x$ in (10), for any $m > n > 0$, we arrive

$$\begin{aligned} \mu_{\frac{f(n^{k+m}x)}{n^{2(k+m)}} - \frac{f(n^m x)}{n^{2m}}}(t) &\geq T_{i=0}^{k-1} \eta_{\sqrt{n^{i+m}x},\sqrt{n^{i+m}x},\cdots,\sqrt{n^{i+m}x},\sqrt{n^{i+m}x}}(n^{2(i+m+1)} t) \\ &= T_{i=m}^{m+k-1} \eta_{\sqrt{n^i x},\sqrt{n^i x},\cdots,\sqrt{n^i x},\sqrt{n^i x}}(n^{2(i+1)} t) \\ &\rightarrow \quad 1 \quad as \quad m \quad \rightarrow \quad \infty \end{aligned}$$

for all $x \in \mathcal{X}$ and all $t > 0$. Hence the sequence $\left\{\dfrac{f(n^k x)}{n^{2k}}\right\}$ is a Cauchy sequence. Since $\mathcal{Y}$ is complete, there exists a mapping $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ such that $\mu_{\mathcal{H}(x)}(t) = \lim_{k\to\infty} \mu_{\frac{f(n^{kj}x)}{n^{2kj}}}(t)$ for all $x \in \mathcal{X}$ and all $t > 0$. Letting $m = 0$ and $k \to \infty$ in (10), we arrive (5) for all $x \in \mathcal{X}$ and all $t > 0$. Now, we have to show that $\mathcal{H}$ satisfies (1), replacing $(x_1, y_1, \cdots, x_n, y_n)$ by $(n^k x_1, n^k y_1, \cdots, n^k x_n, n^k y_n)$ in (3), we have

$$\begin{aligned} \mu_{\frac{1}{n^{2k}} Df(n^k x_1, n^k y_1, \cdots, n^k x_n, n^k y_n)}(t) &\geq \eta_{n^k x_1, n^k y_1, \cdots, n^k x_n, n^k y_n}(n^{2k}\ t) \\ &= T_{i=m}^{m+k-1} \left(\eta_{\sqrt{n^i x},\sqrt{n^i x},\cdots,\sqrt{n^i x},\sqrt{n^i x}}\right)(n^{2(i+1)} t) \end{aligned} \tag{11}$$

for all $x \in \mathcal{X}$ and all $t > 0$. Taking $k \to \infty$ both sides in (11) and using the definition of $\mathcal{H}(x)$, we arrive $\mu_{\mathcal{H}(x_1,y_1,\cdots,x_n,y_n)}(t) \geq 1$. Hence $\mathcal{H}$ satisfies (1) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$. Therefore the mapping $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ is quadratic. Also

$$\mu_{\mathcal{H}(xy)-\mathcal{H}(x)\mathcal{H}(y)}(t) = \lim_{k\to\infty} \mu_{\frac{1}{n^{4k}}\left(f(n^k x n^k y)-f(n^k x)f(n^k y)\right)}(t) = 1$$

for all $x \in \mathcal{X}$ and all $t > 0$. Thus, we have $\mathcal{H}(xy) = \mathcal{H}(x)\mathcal{H}(y)$ for all $x, y \in \mathcal{X}$. Therefore $\mathcal{H}$ is a quadratic random homomorphism. Finally, to prove the uniqueness of the quadratic function $\mathcal{H}$ subject to (6), let us assume that there exist a quadratic function $\mathcal{H}'$ which satisfies (5) and (6). It follows from (6) that

$$\mu_{\mathcal{H}(x)-\mathcal{H}'(x)}(2t) = \mu_{\mathcal{H}(n^k x)-\mathcal{H}'(n^k x)}(2n^{2k}t) \geq T\left(\mu_{\mathcal{H}(n^k x)-f(n^k x)}(n^{2k}t), \mu_{f(n^k x)-\mathcal{H}'(n^k x)}(n^{2k}t)\right)$$
$$= T\left(T_{i=0}^{k-1}\left(\eta_{\sqrt{n^i x},\sqrt{n^i x},\cdots,\sqrt{n^i x},\sqrt{n^i x}}\right)(n^{2(i+1)}t), T_{i=0}^{k-1}\left(\eta_{\sqrt{n^i x},\sqrt{n^i x},\cdots,\sqrt{n^i x},\sqrt{n^i x}}\right)(n^{2(i+1)}t)\right)$$
$$\to \quad 1 \quad as \quad k \quad \to \quad \infty$$

for all $x \in \mathcal{X}$ and all $t > 0$. Hence $\mathcal{H}$ is unique.

For $j = -1$, we can prove a similar stability result. This completes the proof. □

The following corollary is an immediate consequence of Theorem 2 concerning the stability of (1).

Corollary 3. *Let λ and s be nonnegative real numbers. Let a function $f : \mathcal{X} \to \mathcal{Y}$ satisfies the inequalities*

$$\mu_{Df(x_1,y_1,\cdots,x_n,y_n)}(t) \quad \geq \begin{cases} \eta_\lambda(t), \\ \eta_{\lambda \sum_{i=1}^{n}\{||x_i||^s+||y_i||^s\}}(t), & s \neq 4; \\ \eta_{\lambda\left\{\prod_{i=1}^{n}||x_i||^s||y_i||^s+\sum_{i=1}^{n}(||x_i||^{2ns}+||y_i||^{2ns})\right\}}(t), & s \neq \frac{2}{n}; \end{cases} \tag{12}$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$ and all $t > 0$ and

$$\mu_{f(xy)-f(x)f(y)}(t) \quad \geq \begin{cases} \eta_\lambda(t), \\ \eta_{\lambda\{||x||^s+||y||^s\}}(t), \\ \eta_{\lambda\{||x||^s||y||^s+(||x||^{2s}+||y||^{2s})\}}(t) \end{cases} \tag{13}$$

for all $x, y \in \mathcal{X}$. Then there exists a unique quadratic homomorphism $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ such that

$$\mu_{\mathcal{H}(x)-f(x)}(t) \leq \begin{cases} \eta_{\frac{\lambda}{|n^2-1|}}(t), \\ \eta_{\frac{2\lambda||x||^{s/2}}{|n^2-n^{s/2}|}}(t), \\ \eta_{\frac{2\lambda||x||^{ns}}{|n^2-n^{ns}|}}(t) \end{cases} \tag{14}$$

for all $x \in \mathcal{X}$ and all $t > 0$.

2.2 Stability of random derivations via direct method

Definition 4. An $\mathbb{R}$-linear mapping $f : \mathcal{X} \to \mathcal{X}$ is called a quadratic random derivation in random Banach algebras if $f(xy) = f(x)y^2 + x^2 f(y)$ for all $x, y \in \mathcal{X}$.

Theorem 5. *Let $j = \pm 1$. Let $f : \mathcal{X} \to \mathcal{X}$ be a mapping for which there exist a function $\eta : \mathcal{X}^{2n} \to D^+$ with the condition (2) such that the functional inequalities (3) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$ and all $t > 0$ and*

$$\mu_{f(xy)-f(x)y^2-x^2f(y)}(t) \geq \eta_{x,y,0,\cdots,0,0}(t) \tag{15}$$

for all $x, y \in \mathcal{X}$ and all $t > 0$. Then there exists a unique quadratic random derivation $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ satisfying the functional equation (1) and

$$\mu_{\mathcal{D}(x)-f(x)}(t) \geq T_{i=0}^{\infty} \eta_{\sqrt{n^{ij}}x,\sqrt{n^{ij}}x,\cdots,\sqrt{n^{ij}}x,\sqrt{n^{ij}}x}\left(n^{2(i+1)j}t\right) \tag{16}$$

for all $x \in \mathcal{X}$ and all $t > 0$. The mapping $\mathcal{D}(x)$ is defined by

$$\mu_{\mathcal{D}(x)}(t) = \lim_{k\to\infty} \mu_{\frac{f(n^{kj}x)}{n^{2kj}}}(t) \tag{17}$$

for all $x \in X$ and all $t > 0$.

Proof. Assume $j = 1$. By the same reasoning as that in the proof of the Theorem 2, there exist a unique quadratic mapping $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ satisfying (16). The mapping $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ given by $\mathcal{D}(x) = \lim_{k\to\infty} \dfrac{f(n^k x)}{n^{2k}}$. It follows from (3) that

$$\mu_{\mathcal{D}(xy)-\mathcal{D}(x)y^2-x^2\mathcal{D}(y)}(t) = \lim_{k\to\infty} \mu_{\frac{1}{n^{4k}}\left(\mathcal{D}(n^kxn^ky)-\mathcal{D}(n^kx)\left(n^ky\right)^2-\left(n^kx\right)^2\mathcal{D}(n^ky)\right)}(t) = 1$$

for all $x \in \mathcal{X}$ and all $t > 0$. Therefore $\mathcal{D} : \mathcal{X} \to \mathcal{D}$ is a quadratic random derivation satisfying (16). □

The following corollary is an immediate consequence of Theorem 5 concerning the stability of (1).

Corollary 6. *Let λ and s be nonnegative real numbers. Let a function $f : \mathcal{X} \to \mathcal{X}$ satisfies the inequalities (12) for all $x_1, y_1, \cdots, x_n, y_n \in X$ and all $t > 0$ and*

$$\mu_{f(xy)-f(x)y^2-x^2f(y)}(t) \quad \geq \begin{cases} \eta_{\lambda}(t), \\ \eta_{\lambda\{||x||^s+||y||^s\}}(t), \\ \eta_{\lambda\{||x||^s||y||^s+(||x||^{2s}+||y||^{2s})\}}(t) \end{cases} \tag{18}$$

for all $x, y \in \mathcal{X}$. Then there exists a unique quadratic derivation $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ such that

$$\mu_{\mathcal{D}(x)-f(x)}(t) \leq \begin{cases} \eta_{\frac{\lambda}{|n^2-1|}}(t), \\ \eta_{\frac{2\lambda||x||^{s/2}}{|n^2-n^{s/2}|}}(t), \\ \eta_{\frac{2\lambda||x||^{ns}}{|n^2-n^{ns}|}}(t) \end{cases} \tag{19}$$

for all $x \in \mathcal{X}$ and all $t > 0$.

3 Stability Results: Fixed Point Method

In this section, using Radu's fixed point method, we discuss the generalized Ulam-Hyers stability of random homomorphisms and random derivations associated with the Lagrange's quadratic functional equation (1) in random Banach algebras. For more details about the alternative of fixed point one can refer to [9, 12].

3.1 Stability of random homomorphisms via fixed point method

Theorem 7. *Let $f : \mathcal{X} \to \mathcal{Y}$ be a mapping for which there exist a function $\eta : \mathcal{X}^{2n} \to D^+$ with the condition*

$$\lim_{k\to\infty} \eta_{\rho_i^k x_1,\rho_i^k y_1,\cdots,\rho_i^k x_n,\rho_i^k y_n}\left(\rho_i^{2k} t\right) = 1, \quad \forall\, x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}, t > 0, \tag{20}$$

where $\rho_i = n$ if $i = 0$ and $\rho_i = \frac{1}{n}$ if $i = 1$ such that the functional inequalities (3) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}, t > 0$ and (4) for all $x, y \in \mathcal{X}, t > 0$. If there exists L such that the function

$$x \to \beta(x,t) = \eta_{\sqrt{\frac{x}{n}},\sqrt{\frac{x}{n}},\ldots,\sqrt{\frac{x}{n}},\sqrt{\frac{x}{n}}}(t), \tag{21}$$

has the property

$$\beta(x,t) \le L\frac{1}{\rho_i^2}\beta\left(\rho_i x, t\right), \ \forall\, x \in \mathcal{X}, t > 0. \tag{22}$$

Then there exists a unique quadratic random homomorphiosm $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ satisfying the functional equation (1) and

$$\mu_{\mathcal{H}(x)-f(x)}\left(\frac{L^{1-i}}{1-L}t\right) \ge \beta(x,t), \ \forall\, x \in \mathcal{X}, t > 0. \tag{23}$$

Proof. Let d be a general metric on Ω, such that

$$d(g,h) = inf\left\{K \in (0,\infty) | \mu_{g(x)-h(x)}(Kt) \ge \beta(x,t), x \in \mathcal{X}, t > 0\right\}.$$

It is easy to see that (Ω, d) is complete. Define $T : \Omega \to \Omega$ by $Tg(x) = \dfrac{1}{\rho_i^2}g(\rho_i x)$, for all $x \in \mathcal{X}$. One can show that $d\left(Tg, Th\right) \le \frac{1}{\rho_i^2}d(g,h) = Ld(g,h)$ for all $g, h \in \Omega$. There fore T is strictly contractive mapping on Ω with Lipschitz constant L. Replacing $(x_1, y_1, \cdots, x_n, y_n)$ by $(\sqrt{x}, \sqrt{x}, \cdots, \sqrt{x}, \sqrt{x})$ in (3) and using random normed space definition, we get

$$\mu_{\frac{f(nx)}{n^2}-f(x)}\left(\frac{t}{n^2}\right) \ge \eta_{\sqrt{x},\sqrt{x},\cdots,\sqrt{x},\sqrt{x}}(t) \tag{24}$$

for all $x \in \mathcal{X}, t > 0$, with the help of (22) when $i = 0$, it follows from (24), we get

$$\Rightarrow \qquad \mu_{\frac{f(nx)}{n^2}-f(x)}\left(\frac{t}{n^2}\right) \ge \beta(x,t) \qquad \Rightarrow \qquad d(Tf, f) \le L = L^{1-0} < \infty. \tag{25}$$

Again replacing x by $\frac{x}{n}$ in (3), we obtain

$$\mu_{f(x)-n^2 f\left(\frac{x}{n}\right)}(t) \ge \eta_{\sqrt{\frac{x}{n}},\sqrt{\frac{x}{n}},\ldots,\sqrt{\frac{x}{n}},\sqrt{\frac{x}{n}}}(t) \tag{26}$$

for all $x \in \mathcal{X}, t > 0$ with the help of (22) when $i = 1$, it follows from (26), we get

$$\Rightarrow \qquad \mu_{f(x)-n^2 f\left(\frac{x}{n}\right)}(t) \ge \beta(x,t) \qquad \Rightarrow \qquad d(f, Tf) \le 1 = L^0 = L^{1-i} \tag{27}$$

for all $x \in \mathcal{X}, t > 0$. Then from (25) and (27), we can conclude

$$d(f, Tf) \leq L^{1-i}.$$

Now from the fixed point alternative in both cases, it follows that there exists a fixed point $\mathcal{H}$ of T in Ω such that

$$\mu_{\mathcal{H}(x)}(t) = \lim_{k\to\infty} \frac{\mu_{f(\rho_i^k x)}}{\rho_i^{2k}}(t), \qquad \forall x \in \mathcal{X}, t > 0. \tag{28}$$

By proceeding the same procedure as in the Theorem 2, we can prove the function, $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ satisfies the functional equation (1) and one can show that $\mathcal{H}$ is quadratic random homomorphism.

By fixed point alternative, since $\mathcal{H}$ is unique fixed point of T in the set $\Delta = \{f \in \Omega | d(f, \mathcal{H}) < \infty\}$, therefore $\mathcal{H}$ is a uniqe function such that

$$\mu_{f(x)-\mathcal{H}(x)}(Kt) \geq \beta(x,t)\ \forall x \in \mathcal{X}, t > 0, K > 0 \qquad \Rightarrow d(f, \mathcal{H}) \leq \frac{1}{1-L} d(f, Tf)$$
$$\Rightarrow d(f, \mathcal{H}) \leq \frac{L^{1-i}}{1-L} \qquad \Rightarrow \mu_{f(x)-\mathcal{H}(x)}\left(\frac{L^{1-i}}{1-L}t\right) \geq \beta(x,t) \tag{29}$$

for all $x \in \mathcal{X}$ and $t > 0$. This completes the proof. □

From Theorem 7, we obtain the following corollary concerning the stability for the functional equation (1).

Corollary 8. *Let λ and s be nonnegative real numbers. Let a function $f : \mathcal{X} \to \mathcal{Y}$ satisfies the inequalities (12) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$ and all $t > 0$ and (13) for all $x, y \in \mathcal{X}$. Then there exists a unique quadratic homomorphism $\mathcal{H} : \mathcal{X} \to \mathcal{Y}$ such that the inequality (14) holds for all $x \in \mathcal{X}$ and all $t > 0$.*

3.2 Stability of random derivations via fixed point method

Theorem 9. *Let $f : \mathcal{X} \to \mathcal{X}$ be a mapping for which there exist a function $\eta : \mathcal{X}^{2n} \to D^+$ with the condition (20) such that the functional inequalities (3) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}, t > 0$ and (15) for all $x, y \in \mathcal{X}, t > 0$. If there exists L such that the function (21) has the property (22) for all $x \in \mathcal{X}, t > 0$. Then there exists a unique quadratic random derivation $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ satisfying the functional equation (1) and*

$$\mu_{\mathcal{D}(x)-f(x)}\left(\frac{L^{1-i}}{1-L}t\right) \geq \beta(x,t),\ \forall\ x \in \mathcal{X}, t > 0. \tag{30}$$

Proof. By a similar method to the proof of Theorems 5, 7, one can show that the quadratic mapping $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ is a quadratic random derivation satisfying the functional equation (1) and the inequality (30), as desired. □

Corollary 10. *Let λ and s be nonnegative real numbers. Let a function $f : \mathcal{X} \to \mathcal{X}$ satisfies the inequalities (12) for all $x_1, y_1, \cdots, x_n, y_n \in \mathcal{X}$ and all $t > 0$ and (18) for all $x, y \in \mathcal{X}$. Then there exists a unique quadratic derivation $\mathcal{D} : \mathcal{X} \to \mathcal{X}$ such that the inequality (19) holds for all $x \in \mathcal{X}$ and all $t > 0$.*

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