A Study on Constant Intuitionistic Fuzzy Graphs of Second Type

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Abstract
In this paper, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type with examples. Also establish some of their properties.

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1 Introduction

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets [IFS]. He has also introduced the extensions of IFS namely Intuitionistic Fuzzy sets of second type [IFSST], Intuitionistic L-Fuzzy sets [ILFS] and Temporal Intuitionistic Fuzzy sets [TIFS]. R. Parvathi and M. G. Karumambigai [3, 5] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and analyzed its components also introduced constant and totally constant intuitionistic fuzzy graphs. Further A. Nagoor Gani and S. Shajitha Begum [4] introduced the degree of IFG and studied their properties. The present authors [7, 8, 9] introduced the extension of IFG namely Intuitionistic Fuzzy Graphs of Second Type [IFGST] and defined the degree, order and size of IFGST. In section 2, we give some basic definitions and in section 3, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type. Also we establish some of their properties. The paper is concluded in section 4.
2 Preliminaries

In this section, we give some basic definitions.

(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$ denote the
degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$
for every $v_i \in V$, $i = 1, 2, ..., n$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$ are such that
$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,
$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$
and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$
for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ..., n)$.

Definition 2.2. [7] An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is
of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$ denote the
degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \nu_1(v_i)^2 \leq 1$
for every $v_i \in V$, $i = 1, 2, ..., n$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$ are such that
$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$
$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i)^2, \nu_1(v_j)^2]$ and
$0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1$
for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ..., n)$.

Definition 2.3. [8] Let $G = [V, E]$ is an IFGST then the degree of a vertex in $G$ is denoted by $d(v)$ and defined as, $d(v) = [d_\mu(v), d_\nu(v)]$ where $d_\mu(v) = \sum_{u \neq v} \mu_2(v, u)$
and $d_\nu(v) = \sum_{u \neq v} \nu_2(v, u)$ for all $v \in V$

Definition 2.4. [8] Let $G = [V, E]$ is an IFGST then the minimum degree of $G$ is denoted by $\delta(G)$ and defined as, $\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$ where $\delta_\mu(G) = \min\{d_\mu(v) : v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v) : v \in V\}$

Definition 2.5. [8] Let $G = [V, E]$ is an IFGST then the maximum degree of $G$ is denoted by $\Delta(G)$ and defined as, $\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where $\Delta_\mu(G) = \max\{d_\mu(v) : v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v) : v \in V\}$

Definition 2.6. [9] Let $G = [V, E]$ be an IFGST. Then the order of $G$ is defined
by $O(G) = (O_\mu(G), O_\nu(G))$ where $O_\mu(G) = \sum_{v \in V} \mu_1(v)$ and $O_\nu(G) = \sum_{v \in V} \nu_1(v)$

Definition 2.7. [9] Let $G = [V, E]$ be an IFGST. Then the size of $G$ is defined
by $S(G) = (S_\mu(G), S_\nu(G))$ where $S_\mu(G) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $S_\nu(G) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$
Definition 2.8. [3] Let $G : [(\mu_{1i}, \nu_{1i}), (\mu_{2ij}, \nu_{2ij})]$ be an IFG on $G^* : [V, E]$. If $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i, v_j \in V$ that is called as $(k_i, k_j)$ - constant IFG (or) Constant IFG of degree $(k_i, k_j)$

Definition 2.9. [3] Let $G$ be an IFG. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[ \sum_{v_1, v_2 \in E} d_\mu_2(v) + \mu_1(v), \sum_{v_1, v_2 \in E} d_\nu_2(v) + \nu_1(v) \right]$$

If each vertex of $G$ has the same total degree $(r_1, r_2)$, then $G$ is said to be an IFG of total degree $(r_1, r_2)$ or $(r_1, r_2)$ - totally constant IFG.

3 Constant Intuitionistic Fuzzy Graphs of Second Type

In this section, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type. Also establish some of their properties.

Definition 3.1. Let $G = [V, E]$ is an IFGST. Suppose $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i, v_j \in V$ then the graph $G$ is called as constant IFGST of degree $(k_i, k_j)$ or $(k_i, k_j)$ - constant IFGST.

Example 3.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Constant IFGST}
\end{figure}

Remark 3.1. Suppose $G$ is a constant IFGST of degree $(k_i, k_j)$ iff $\delta = \Delta = (k_i, k_j)$

Definition 3.2. Let $G = [V, E]$ be an IFGST. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[ \sum_{v_1, v_2 \in E} d_\mu_2(v) + \mu_1(v), \sum_{v_1, v_2 \in E} d_\nu_2(v) + \nu_1(v) \right]$$

Suppose that each vertex of $G$ has the same total degree $(r_1, r_2)$, then $G$ is said to be a totally constant IFGST of degree $(r_1, r_2)$ or $(r_1, r_2)$ - totally constant IFGST.
Definition 3.3. Let $G = [V,E]$ be an IFGST then $(\mu_1, \nu_1)$ is said to be constant function if each vertices of $G$ having the same pair of membership and non-membership degrees.

Theorem 3.1. Let $G = [V,E]$ be an IFGST. If $(\mu_1, \nu_1)$ is a constant function iff the following are equivalent
(i) $G$ is a constant IFGST.
(ii) $G$ is a totally constant IFGST.

Proof. Suppose that $(\mu_1, \nu_1)$ is a constant function.
Let $\mu_1(v_i) = c_1$ and $\nu_1(v_i) = c_2$ for all $v_i \in V$ where $c_1, c_2$ are constants.
Assume that $G = [V,E]$ is a constant IFGST of degree $(k_1, k_2)$ then we have,
$$d_\mu(v_i) = k_1 \quad \text{and} \quad d_\nu(v_i) = k_2 \quad \text{for all} \quad v_i \in V$$
But by the definition of totally constant IFGST we have,
$$td_\mu(v_i) = d_\mu(v_i) + \mu_1(v_i), \quad td_\nu(v_i) = d_\nu(v_i) + \nu_1(v_i) \quad \text{for all} \quad v_i \in V$$
$$\Rightarrow td_\mu(v_i) = k_1 + c_1, \quad td_\nu(v_i) = k_2 + c_2 \quad \text{for all} \quad v_i \in V$$
Therefore $G$ is a totally constant IFGST.
Hence (i) $\Rightarrow$ (ii) is proved.
Now assume that $G$ is totally constant IFGST of degree $(r_1, r_2)$ then we have,
$$td_\mu(v_i) = r_1, \quad td_\nu(v_i) = r_2 \quad \text{for all} \quad v_i \in V$$
$$\Rightarrow d_\mu(v_i) + \mu_1(v_i) = r_1, \quad d_\nu(v_i) + \nu_1(v_i) = r_2$$
$$\Rightarrow d_\mu(v_i) = r_1 - c_1, \quad d_\nu(v_i) = r_2 - c_2$$
Therefore $G$ is a constant IFGST.
Hence (ii) $\Rightarrow$ (i) is proved.
Therefore (i) and (ii) are equivalent.
Conversely, now assume that (i) and (ii) are equivalent.
We will prove that $(\mu_1, \nu_1)$ is a constant function.
Suppose that it is not true then at least one pair of vertices satisfies
$\mu_1(v_1) \neq \mu_1(v_2), \nu_1(v_1) \neq \nu_1(v_2)$ for some $v_1, v_2 \in V$

Let $G$ be a constant IFGST of degree $(k_1, k_2)$ then

$d_\mu(v_1) = d_\mu(v_2) = k_1$ and $d_\nu(v_1) = d_\nu(v_2) = k_2$ for some $v_1, v_2 \in V$

So $td_\mu(v_1) = d_\mu(v_1) + \mu_1(v_1), td_\mu(v_2) = d_\mu(v_2) + \mu_1(v_2)$

$\Rightarrow td_\mu(v_1) = k_1 + \mu_1(v_1), td_\nu(v_2) = k_1 + \mu_1(v_2)$

Similarly we have $td_\mu(v_1) = k_2 + \nu_1(v_1), td_\nu(v_2) = k_2 + \nu_1(v_2)$

So $\mu_1(v_1) \neq \mu_1(v_2)$ and $\nu_1(v_1) \neq \nu_1(v_2)$

Finally we have $G$ is not a totally constant and which is a contradiction to our assumption.

Hence $(\mu_1, \nu_1)$ is a constant function.

\[ \square \]

**Theorem 3.2.** Let $G = [V, E]$ be an IFGST. If $G$ is both constant and totally constant then $(\mu_1, \nu_1)$ is a constant function

**Proof.** Let $G$ be a constant IFGST of degree $(k_1, k_2)$ and totally constant IFGST of degree $(r_1, r_2)$

we have $d_\mu(v_i) = k_1$ and $d_\nu(v_i) = k_2$ for all $v_i \in V$ (1)

Also, $td_\mu(v_i) = r_1$ and $td_\nu(v_i) = r_2$ for all $v_i \in V$

$\Rightarrow d_\mu(v_i) + \mu_1(v_i) = r_1$ and $d_\nu(v_i) + \nu_1(v_i) = r_2$ (2)

Substitute (1) in (2) we have $\mu_1(v_i) = r_1 - k_1$ and $\nu_1(v_i) = r_2 - k_2$ for all $v_i \in V$

Hence $(\mu_1, \nu_1)$ is a constant function. \[ \square \]

**Example 3.3.**

The Fig. 3. shows that $G$ is both constant and totally constant IFGST then $(\mu_1, \nu_1)$ is a constant function.

\[ \text{Fig. 3} \]

**Theorem 3.3.** The size of a constant IFGST of degree $(k_1, k_2)$ is $(\frac{pk_1^2}{2}, \frac{pk_2^2}{2})$

where $p$ is the number of vertices of a graph.

**Proof.** We know that the size of IFGST $G = [V, E]$ is

$S(G) = \left[ \sum_{v_i, v_j \in E} \mu_2(v_i, v_j), \sum_{v_i, v_j \in E} \nu_2(v_i, v_j) \right]$ for all $v_i, v_j \in V$
Since $G$ is constant IFGST of degree $(k_1, k_2)$ so we have $d_\mu(v_i) = k_1$ and $d_\nu(v_j) = k_2$
for all $v_i, v_j \in V$
But
$$\sum_{v_i \in V} d_G(v_i) = 2 \left[ \sum_{v_i, v_j \in E} \mu_2(v_i v_j), \sum_{v_i, v_j \in E} \nu_2(v_i v_j) \right]$$
$$\left[ \sum_{v_i \in V} k_1, \sum_{v_j \in V} k_2 \right] = 2S(G)$$
$$[pk_1, pk_2] = 2S(G)$$
$$\left[ \frac{pk_1}{2}, \frac{pk_2}{2} \right] = S(G)$$
Hence the proof. \[ \square \]

**Theorem 3.4.** Let $G = [V, E]$ be an IFGST. If $G$ is a totally constant IFGST of degree $(r_1, r_2)$ then $2S(G) + O(G) = (pr_1, pr_2)$ where $p$ is the number of vertices of a graph.

**Proof.** From the given information we have
$$td_\mu(v_i) = d_\mu(v_i) + \mu_1(v_i) \quad \text{and} \quad td_\nu(v_i) = d_\nu(v_i) + \nu_1(v_i) \quad \text{for all} \quad v_i \in V$$
$$\Rightarrow \sum_{v_i \in V} td_\mu(v_i) = \sum_{v_i \in V} d_\mu(v_i) + \sum_{v_i \in V} \mu_1(v_i) \quad \text{and} \quad \sum_{v_i \in V} td_\nu(v_i) = \sum_{v_i \in V} d_\nu(v_i) + \sum_{v_i \in V} \nu_1(v_i)$$
$$\Rightarrow pr_1 = 2S_\mu(G) + O_\mu(G) \quad \text{and} \quad pr_2 = 2S_\nu(G) + O_\nu(G)$$
$$\Rightarrow (pr_1 + pr_2) = 2[S_\mu(G) + S_\nu(G)] + O_\mu(G) + O_\nu(G) \Rightarrow (pr_1 + pr_2) = 2S(G) + O(G)$$
Hence the proof. \[ \square \]

**Theorem 3.5.** If $G$ is a constant IFGST of degree $(k_1, k_2)$ and totally IFGST of degree $(r_1, r_2)$ then $O_\mu(G) = p(r_1 - k_1)$ and $O_\nu(G) = p(r_2 - k_2)$

**Proof.** From theorem 3.3 we have $2S_\mu(G) = pk_1$, $2S_\nu(G) = pk_2$ and from theorem 3.4 we have $2S_\mu(G) + O_\mu(G) = pr_1$ and $2S_\nu(G) + O_\nu(G) = pr_2$
$$\Rightarrow O_\mu(G) = pr_1 - 2S_\mu(G) \quad \text{and} \quad O_\nu(G) = pr_2 - 2S_\nu(G)$$
$$\Rightarrow O_\mu(G) = pr_1 - pk_1 \quad \text{and} \quad O_\nu(G) = pr_2 - pk_2$$
$$\Rightarrow O_\mu(G) = p(r_1 - k_1) \quad \text{and} \quad O_\nu(G) = p(r_2 - k_2)$$
Hence the proof. \[ \square \]

4 Conclusion

In this paper, we have defined the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type and established some of their properties. In future we will study some more properties and applications of IFGST.

References


