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A Study on Constant Intuitionistic Fuzzy Graphs of Second Type

S. Sheik Dhavudh[1] and R. Srinivasan[2]

[1]*Department of Mathematics, Islamiah College(Autonomous), Vaniyambadi - 635 752, Tamil Nadu, India.*
sheikdhavudh01@gmail.com
[2]*Department of Mathematics, Islamiah College(Autonomous), Vaniyambadi - 635 752, Tamil Nadu, India.*
srinivasanmaths@yahoo.com

Abstract

In this paper, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type with examples. Also establish some of their properties.

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Key Words : Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, degree, order, size, constant, totally constant.

1 Introduction

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets [IFS]. He has also intrduced the extensions of IFS namely Intuitionistic Fuzzy sets of second type [IFSST], Intuitionistic L-Fuzzy sets [ILFS] and Temporal Intuitionistic Fuzzy sets [TIFS]. R. Parvathi and M. G. Karunambigai [3, 5] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and analyzed its components also introduced constant and totally constant intuitionistic fuzzy graphs. Further A. Nagoor Gani and S. Shajitha Begum [4] introduced the degree of IFG and studied their properties. The present authors [7, 8, 9] introduced the extension of IFG namely Intuitionistic Fuzzy Graphs of Second Type [IFGST] and defined the degree, order and size of IFGST. In section 2, we give some basic definitions and in section 3, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type. Also we establish some of their properties. The paper is concluded in section 4 .

2 Preliminaries

In this section, we give some basic definitions.

Definition 2.1. [5] An Intuitionistic Fuzzy Graph [IFG] is of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \longrightarrow [0, 1]$ and $\nu_1 : V \longrightarrow [0, 1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and
$0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$
for every $v_i \in V$, $(i = 1, 2, ...n)$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \longrightarrow [0, 1]$ and $\nu_2 : V \times V \longrightarrow [0, 1]$ are such that
$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,
$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i), \nu_1(v_j)]$
and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$
for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ...n)$.

Definition 2.2. [7] An Intuitionistic Fuzzy Graphs of Second Type [IFGST] is of the form $G = [V, E]$ where
(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \longrightarrow [0, 1]$ and $\nu_1 : V \longrightarrow [0, 1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and
$0 \leq \mu_1(v_i)^2 + \nu_1(v_i)^2 \leq 1$
for every $v_i \in V$, $(i = 1, 2, ...n)$,
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \longrightarrow [0, 1]$ and $\nu_2 : V \times V \longrightarrow [0, 1]$ are such that
$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$,
$\nu_2(v_i, v_j) \leq \max[\nu_1(v_i)^2, \nu_1(v_j)^2]$
and $0 \leq \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \leq 1$
for every $(v_i, v_j) \in E$, $(i, j = 1, 2, ...n)$.

Definition 2.3. [8] Let $G = [V, E]$ is an IFGST then the degree of a vertex in G is denoted by $d(v)$ and defined as, $d(v) = [d_\mu(v), d_\nu(v)]$ where $d_\mu(v) = \sum_{u \neq v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \neq v} \nu_2(v, u)$ for all $u, v \in V$

Definition 2.4. [8] Let $G = [V, E]$ is an IFGST then the minimum degree of G is denoted by $\delta(G)$ and defined as, $\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$ where
$\delta_\mu(G) = \min\{d_\mu(v)/v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v)/v \in V\}$

Definition 2.5. [8] Let $G = [V, E]$ is an IFGST then the maximum degree of G is denoted by $\Delta(G)$ and defined as, $\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where
$\Delta_\mu(G) = \max\{d_\mu(v)/v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v)/v \in V\}$

Definition 2.6. [9] Let $G = [V, E]$ be an IFGST. Then the order of G is defined by $O(G) = (O_\mu(G), O_\nu(G))$ where $O_\mu(G) = \sum_{v \in V} \mu_1(v)$ and $O_\nu(G) = \sum_{v \in V} \nu_1(v)$

Definition 2.7. [9] Let $G = [V, E]$ be an IFGST. Then the size of G is defined by $S(G) = (S_\mu(G), S_\nu(G))$ where $S_\mu(G) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $S_\nu(G) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$

Definition 2.8. [3] Let $G : [(\mu_{1i}, \nu_{1i}), (\mu_{2ij}, \nu_{2ij})]$ be an IFG on $G^* : [V, E]$. If $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i, v_j \in V$ that is called as (k_i, k_j) - constant IFG (or) Constant IFG of degree (k_i, k_j)

Definition 2.9. [3] Let G be an IFG. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[\sum_{v_1 v_2 \in E} d_{\mu_2}(v) + \mu_1(v), \sum_{v_1 v_2 \in E} d_{\nu_2}(v) + \nu_1(v) \right]$$

If each vertex of G has the same total degree (r_1, r_2), then G is said to be an IFG of total degree (r_1, r_2) or (r_1, r_2) - totally constant IFG.

3 Constant Intuitionistic Fuzzy Graphs of Second Type

In this section, we define the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type. Also establish some of their properties.

Definition 3.1. Let $G = [V, E]$ is an IFGST. Suppose $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i, v_j \in V$ then the graph G is called as constant IFGST of degree (k_i, k_j) or (k_i, k_j) - constant IFGST.

Example 3.1.

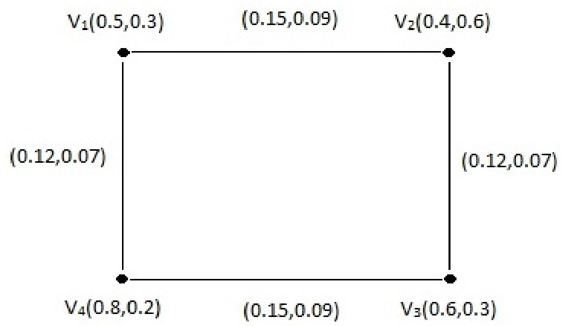

Fig. 1. Constant IFGST

Remark 3.1. Suppose G is a constant IFGST of degree (k_i, k_j) iff $\delta = \Delta = (k_i, k_j)$

Definition 3.2. Let $G = [V, E]$ be an IFGST. The total degree of a vertex $v \in V$ is defined as

$$td(v) = \left[\sum_{v_1 v_2 \in E} d_{\mu_2}(v) + \mu_1(v), \sum_{v_1 v_2 \in E} d_{\nu_2}(v) + \nu_1(v) \right]$$

Suppose that each vertex of G has the same total degree (r_1, r_2), then G is said to be a totally constant IFGST of degree (r_1, r_2) or (r_1, r_2) - totally constant IFGST.

Example 3.2.

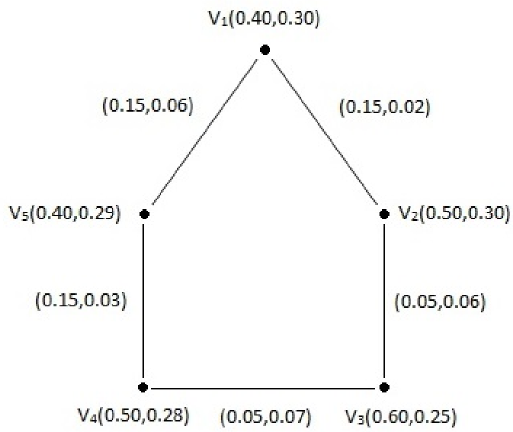

Fig. 2. Totally constant IFGST

Definition 3.3. Let $G = [V, E]$ is an IFGST then (μ_1, ν_1) is said to be constant function if each vertices of G having the same pair of membership and non-membership degrees.

Theorem 3.1. *Let $G = [V, E]$ be an IFGST. If (μ_1, ν_1) is a constant function iff the following are equivalent*
(i) G is a constant IFGST.
(ii) G is a totally constant IFGST.

Proof. Suppose that (μ_1, ν_1) is a constant function.
Let $\mu_1(v_i) = c_1$ and $\nu_1(v_i) = c_2$ for all $v_i \in V$ where c_1, c_2 are constants.
Assume that $G = [V, E]$ is a constant IFGST of degree (k_1, k_2) then we have, $d_\mu(v_i) = k_1$ and $d_\nu(v_i) = k_2$ for all $v_i \in V$
But by the definition of totally constant IFGST we have,
$td_\mu(v_i) = d_\mu(v_i) + \mu_1(v_i)$, $td_\mu(v_i) = d_\nu(v_i) + \nu_1(v_i)$ for all $v_i \in V$
$\Rightarrow td_\mu(v_i) = k_1 + c_1,\ td_\nu(v_i) = k_2 + c_2$ for all $v_i \in V$
Therefore G is a totally constant IFGST
Hence (i) $\Rightarrow$ (ii) is proved.
Now assume that G is totally constant IFGST of degree (r_1, r_2) then we have,
$td_\mu(v_i) = r_1$, $td_\nu(v_i) = r_2$ for all $v_i \in V$
$\Rightarrow d_\mu(v_i) + \mu_1(v_i) = r_1,\ d_\nu(v_i) + \nu_1(v_i) = r_2$
$\Rightarrow d_\mu(v_i) = r_1 - c_1,\ d_\nu(v_i) = r_2 - c_2$
Therefore G is a constant IFGST
Hence (ii) $\Rightarrow$ (i) is proved.
Therefore (i) and (ii) are equivalent.
Conversely, now assume that (i) and (ii) are equivalent
We will prove that (μ_1, ν_1) is a constant function.
Suppose that it is not true then at least one pair of vertices satisfies

$\mu_1(v_1) \neq \mu_1(v_2),\ \nu_1(v_1) \neq \nu_1(v_2)$ for some $v_1, v_2 \in V$
Let G be a constant IFGST of degree (k_1, k_2) then
$d_\mu(v_1) = d_\mu(v_2) = k_1$ and $d_\nu(v_1) = d_\nu(v_2) = k_2$ for some $v_1, v_2 \in V$
So $td_\mu(v_1) = d_\mu(v_1) + \mu_1(v_1),\ td_\mu(v_2) = d_\mu(v_2) + \mu_1(v_2)$
$\Rightarrow td_\mu(v_1) = k_1 + \mu_1(v_1), td_\mu(v_2) = k_1 + \mu_1(v_2)$
Similarly we have $td_\nu(v_1) = k_2 + \nu_1(v_1), td_\nu(v_2) = k_2 + \nu_1(v_2)$
So $\mu_1(v_1) \neq \mu_1(v_2)$ and $\nu_1(v_1) \neq \nu_1(v_2)$
Finally we have G is not a totally constant and which is a contradiction to our assumption.
Hence (μ_1, ν_1) is a constant function. □

Theorem 3.2. *Let $G = [V, E]$ be an IFGST. If G is both constant and totally constant then (μ_1, ν_1) is a constant function*

Proof. Let G be a constant IFGST of degree (k_1, k_2) and totally constant IFGST of degree (r_1, r_2)
we have $d_\mu(v_i) = k_1$ and $d_\nu(v_i) = k_2$ for all $v_i \in V$ (1)
Also, $td_\mu(v_i) = r_1$ and $td_\nu(v_i) = r_2$ for all $v_i \in V$
$\Rightarrow d_\mu(v_i) + \mu_1(v_i) = r_1$ and $d_\nu(v_i) + \nu_1(v_i) = r_2$ (2)
Substitute (1) in (2) we have $\mu_1(v_i) = r_1 - k_1$ and $\nu_1(v_i) = r_2 - k_2$ for all $v_i \in V$
Hence (μ_1, ν_1) is a constant function. □

Example 3.3.

The Fig. 3. shows that G is both constant and totally constant IFGST then (μ_1, ν_1) is a constant function.

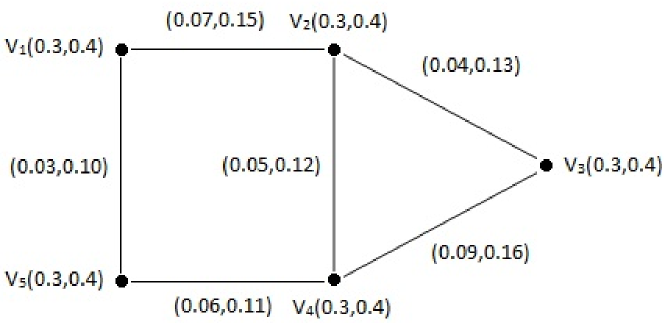

Fig. 3

Theorem 3.3. *The size of a constant IFGST of degree (k_1, k_2) is $(\frac{pk_1}{2}, \frac{pk_2}{2})$ where p is the number of vertices of a graph.*

Proof. We know that the size of IFGST $G = [V, E]$ is

$$S(G) = \left[\sum_{v_i, v_j \in E} \mu_2(v_i v_j), \sum_{v_i, v_j \in E} \nu_2(v_i v_j) \right] \text{ for all } v_i, v_j \in V$$

Since G is constant IFGST of degree (k_1, k_2) so we have $d_\mu(v_i) = k_1$ and $d_\nu(v_j) = k_2$ for all $v_i, v_j \in V$

But $\sum_{v_i \in V} d_G(v_i) = 2\left[\sum_{v_i, v_j \in E} \mu_2(v_i v_j), \sum_{v_i, v_j \in E} \nu_2(v_i v_j)\right]$

$$\left[\sum_{v_i \in V} k_1, \sum_{v_j \in V} k_2\right] = 2S(G)$$

$$[pk_1, pk_2] = 2S(G)$$

$$\left[\frac{pk_1}{2}, \frac{pk_2}{2}\right] = S(G)$$

Hence the proof. □

Theorem 3.4. *Let $G = [V, E]$ be an IFGST. If G is a totally constant IFGST of degree (r_1, r_2) then $2S(G) + O(G) = (pr_1, pr_2)$ where p is the number of vertices of a graph.*

Proof. From the given information we have
$td_\mu(v_i) = d_\mu(v_i) + \mu_1(v_i)$ and $td_\nu(v_i) = d_\nu(v_i) + \nu_1(v_i)$ for all $v_i \in V$
$\Rightarrow \sum_{v_i \in V} td_\mu(v_i) = \sum_{v_i \in V} d_\mu(v_i) + \sum_{v_i \in V} \mu_1(v_i)$ and $\sum_{v_i \in V} td_\nu(v_i) = \sum_{v_i \in V} d_\nu(v_i) + \sum_{v_i \in V} \nu_1(v_i)$
$\Rightarrow pr_1 = 2S_\mu(G) + O_\mu(G)$ and $pr_2 = 2S_\nu(G) + O_\nu(G)$
$\Rightarrow (pr_1 + pr_2) = 2[S_\mu(G) + S_\nu(G)] + O_\mu(G) + O_\nu(G) \Rightarrow (pr_1 + pr_2) = 2S(G) + O(G)$
Hence the proof. □

Theorem 3.5. *If G is a constant IFGST of degree (k_1, k_2) and totally IFGST of degree (r_1, r_2) then $O_\mu(G) = p(r_1 - k_1)$ and $O_\nu(G) = p(r_2 - k_2)$*

Proof. From theorem 3.3 we have $2S_\mu(G) = pk_1$, $2S_\nu(G) = pk_2$ and from theorem 3.4 we have $2S_\mu(G) + O_\mu(G) = pr_1$ and $2S_\nu(G) + O_\nu(G) = pr_2$
$\Rightarrow O_\mu(G) = pr_1 - 2S_\mu(G)$ and $O_\nu(G) = pr_2 - 2S_\nu(G)$
$\Rightarrow O_\mu(G) = pr_1 - pk_1$ and $O_\nu(G) = pr_2 - pk_2$
$\Rightarrow O_\mu(G) = p(r_1 - k_1)$ and $O_\nu(G) = p(r_2 - k_2)$
Hence the proof. □

4 Conclusion

In this paper, we have defined the Constant and Totally constant Intuitionistic Fuzzy Graphs of Second Type and established some of their properties. In future we will study some more properties and applications of IFGST.

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