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A Study on Solving Octagonal Fuzzy Numbers Using Modified Vogel's Approximation Method

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Abstract

In this paper, focuses on solving a problem involving octagonal fuzzy numbers. The problems discussed here are solved by octagonal fuzzy transportation problems under fuzzy environment. In octagonal fuzzy transportation problem whose cost, supply and demand are all octagonal fuzzy numbers. Also it is solved by using modified Vogel's approximation method.

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1 Introduction

The transportation problem is one of the oldest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [6]. Efficient methods of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [3] and then by Charnes and Cooper [2]. The transportation problem can be modeled as a standard linear programming problem that can be solved by the simplex method. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row Minima, Column Minima, Matrix Minima or the Vogel's Approximation Method. To get an optimal solution for the transportation problem, MODI method is used (Modified Distribution Method). Charnes and Cooper [2] developed the Stepping Stone Method, which provides an alternative way of determining the optimal solution.

In this paper, we propose an algorithm namely, Octagonal Fuzzy numbers using Modified Vogel's Approximation Method is proposed for solving fuzzy transportation problems which is more efficient than other existing algorithms as it requires least iterations to reach optimality. The procedure for the solution is illustrated with a numerical example.

2 Preliminaries

In this section, some basic definition, arithmetic operations and an existing method for comparing generalized octagonal fuzzy numbers are presented.

Definition 1. A real fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is a fuzzy subset from the real line $\mathbb{R}$ with the membership function $\mu_{\tilde{a}}(a)$ satisfying the following conditions:

1. $\mu_{\tilde{a}}(a)$ is a continuous mapping from R to the closed interval defined by [0, 1]
2. $\mu_{\tilde{a}}(a) = 0$ for every $a \in (-\infty, a_1]$
3. $\mu_{\tilde{a}}(a)$ is strictly increasing and continuous on $[a_1, a_2]$
4. $\mu_{\tilde{a}}(a) = 0.5$ for every $a \in [a_2, a_3]$
5. $\mu_{\tilde{a}}(a)$ is strictly increasing and continuous on $[a_3, a_4]$
6. $\mu_{\tilde{a}}(a) = 1$ for every $a \in [a_4, a_5]$
7. $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_5, a_6]$
8. $\mu_{\tilde{a}}(a) = 0.5$ for every $a \in [a_6, a_7]$
9. $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_7, a_8]$
10. $\mu_{\tilde{a}}(a) = 0$ for every $a \in [a_8, \infty)$

Definition 2. A real fuzzy number $\tilde{a}$ is a octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ and a_8 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$$\mu_{\tilde{A}}(X) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ 0.5, & a_2 \leq x \leq a_3 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_4-a_2}\right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq\leq a_5 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ 0.5, & a_6 \leq x \leq_7 \\ \frac{1}{2}\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \leq x \leq a_8 \\ 0, & x \geq a_8 \end{cases}$$

Definition 3 (Arithmetic Operations). Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two octagonal fuzzy numbers. Then,

1. $\tilde{A} + \tilde{B} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \oplus (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$

$$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$$

2. $\tilde{A} - \tilde{B} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)_{-}(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$

$$= (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1)$$

3. $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \otimes (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) = (T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8)$

$$T_1 = \min\{a_1b_1, a_1b_8, a_8b_1, a_8b_8\}, \qquad T_2 = \min\{a_2b_2, a_2b_7, a_7b_2, a_7b_7\}$$

$$T_3 = \min\{a_3b_3, a_3b_6, a_6b_3, a_6b_6\}, \qquad T_4 = \min\{a_4b_4, a_4b_5, a_5b_4, a_5b_5\}$$

$$T_5 = \max\{a_4b_4, a_4b_5, a_5b_4, a_5b_5\}, \qquad T_6 = \max\{a_3b_3, a_3b_6, a_6b_3, a_6b_6\}$$

$$T_7 = \max\{a_2b_2, a_2b_7, a_7b_2, a_7b_7\}, \qquad T_8 = \max\{a_1b_1, a_1b_8, a_8b_1, a_8b_8\}$$

4. $k(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) = (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8)$ for $k \geq 0$

5. $k(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) = (ka_8, ka_7, ka_6, ka_5, ka_4, ka_3, ka_2, ka_1)$ for $k < 0$

3 Ranking Function

For an octagonal fuzzy number $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ a ranking method is devised based on the following formula $A = \frac{1}{8}(2a_1 - a_2 + a_3 + 2a_4 + 2a_5 + a_6 - a_7 + 2a_8)$

In the above formula when $a_2 = a_3$ and $a_6 = a_7$ the formula coincides with the formula of trapezoidal fuzzy number framed by Hadi.

4 Octagonal Fuzzy Numbers Using Modified Vogel's Approximation Method (OFMVAM)

OFMVAM was improved by using Total opportunity Cost (TOC) matrix by considering alternative allocation costs. The TOC matrix is obtained by adding the "row opportunity cost matrix" (row opportunity cost matrix: for each row, the smallest cost of that row is subtracted from each element of the same row) and the "column opportunity cost matrix" (column opportunity cost matrix: for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each element of the same column) [4, 5, 7]. Proposed algorithm is applied to the TOC matrix that considers highest three penalty costs and calculates alternative allocation costs in the VAM procedure. Then it selects the minimum among them. Detailed processes are given below: Step 1:

1. Balance the given octagonal fuzzy transportation problem if either (total supply>total demand) or (total supply<total demand).
2. Obtain the TOC matrix for octagonal fuzzy number.
3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.
4. Select the rows or columns with the highest three penalty costs (breaking ties arbitrarily or choosing the lowest-cost cell).
5. Compute three octagonal fuzzy transportation costs for selected three rows or columns in step 4 by allocating as much as possible to the feasible cell with the lowest transportation cost.

6. Select minimum octagonal fuzzy transportation cost of three allocations in step 5 (breaking ties arbitrarily or choosing the lowest-cost cell).

7. Repeat steps 3-6 until all requirements have been met.

8. Compute total octagonal fuzzy transportation cost for the feasible allocations using the original balanced-transportation cost matrix.

5 Important Remarks [6, 7]

The Algorithm will be improved if we add the following two additional steps for breaking ties

1. If there is a tie in penalty or minimum octagonal fuzzy transportation cost, choose the largest penalty for allocation.

2. If there is a tie in penalty and minimum octagonal fuzzy transportation cost, then calculate their corresponding row opportunity cost value column opportunity cost value, and select the one with maximum.

6 Numerical Example

Four whole sale distributors of an industry and there are three supplier of the products for three factories. The unit cost of transportation from whole sale distributors to factories has been given in table. The capacity of the whole sale distributors to supply and the requirements of the factories are also given

Table 1:

	S_l	S_2	S_3	S_4	SUPPLY
D_l	(1,2,3,4, 5,6,7,8)	(1,3,4,6,7, 9, 10,12)	(8,9,11,13,14, 15,16,17)	(5,6,8,9,11, ,12,14,15)	(2,6,8,9,11, 12,14,15)
D_2	(0,1,2,4,5, 5,6,7,9)	(-2,-1,0,2 2,4,5,6,7)	(5,6,7,8,9, 11,12,15)	(0,1,2,4, 7,8,9, 10)	(4,7,9,11,12, 13,15,16)
D_3	(3,4,5,6,7, 7,8,9, 10)	(12,14,16,17, (12,14,16,17,	(7,8,9,11, 12,14,15,16)	(7,8,10,13,14, 15,18,19)	(4,5,7,9, 12,13,14,17)
DEMAND	(5,6,8,9,10, 11,12,13)	(1,5,6,8,9, 10,12,14)	(1,3,4,5, 7,9,10,13)	(1,3,4,7,8 9, 11,15)	

Table 2:

	S_l	S_2	S_3
D_l	$(1,2,3,4,5,6,7,8)$	$(1,3,4,6,7,9,10,12)$	$(-23,-10,-2,3,10,15,21,24)$ $(8,9,11,13,14,15,16,17)$
D_2	$(0,1,2,4,5,6,7,9)$	$(1,5,6,8,9,10,12,14)$ $(-2,-1,0,2,4,5,6,7)$	$(5,6,7,8,9,11,12,15)$
D_3	$(5,6,8,9,10,11,12,13)$ $(3,4,5,6,7,8,9,10)$	$(12,14,16,17,18,20,21,22)$	$(-9,-7,-4,-2,3,5,8,12)$ $(7,8,9,11,12,14,15,16)$
DEMAND	$(5,6,8,9,10,11,12,13)$	$(1,5,6,8,9,10,12,14)$	$(1,3,4,5,7,9,10,13)$

	S_4	SUPPLY
D_l	$(-14,-7,-3,1,6,10,16,25)$ $(5,6,8,9,11,12,14,15)$	$(2,6,8,9,11,12,14,15)$
D_2	$(-10,-5,-1,2,4,7,10,15)$ $(0,1,2,4,7,8,9,10)$	$(4,7,9,11,12,13,15,16)$
D_3	$(7,8,10,13,14,15,18,19)$	$(4,5,7,9,12,13,14,17)$
DEMAND	$(1,3,4,7,8,9,11,15)$	

The Optimal Octagonal Fuzzy Transportation cost is

$$\begin{aligned}
=&(-23,-10,-2,3,10,15,21,24)(8,9,11,13,14,15,16,17)\\
&+(-14,-7,-3,1,6,10,16,25)(5,6,8,9,11,12,14,15)\\
&+(1,5,6,8,9,10,12,14)(-2,-1,0,2,4,5,6,7)\\
&+(-10,-5,-1,2,4,7,10,15)(0,1,2,4,7,8,9,10)\\
&+(5,6,8,9,10,11,12,13)\ (3,4,5,6,7,8,9,10)\\
&+(-9,-7,-4,-2,3,5,8,12)\ (7,8,9,11,12,14,15,16)\\
=&(-1716,-396,-90,204,752,609,950,\ 2706)
\end{aligned}$$

The Ranking function is R(A) = 1911.

7 Conclusion

The proposed method namely, octagonal fuzzy Modified VAM'S method has the following major advantages,

1. Very easy to understand
2. Better than the existing methods and
3. The solution is very nearer to optimum solution.

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