International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 11-18
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Labelings for some Octopus Graph

A.Edward Samuel[1] and S. Kalaivani[2]

[1]*Ramanujan Research Centre,P.G. & Research Department of Mathematics, GAC(A), Kumbakonam, TN, India.*
aedward74_thrc@yahoo.co.in
[2]*Ramanujan Research Centre, P.G. & Research Department of Mathematics, GAC(A), Kumbakonam, TN, India.*
vanikalai.248@gmail.com

Abstract

In this paper, we investigate octopus graph satisfying the conditions of some labelings. We discuss octopus graph in the context of some graph labelings namely absolute differences of the differences of the cubes of the vertices and the differences of the squares of the vertices, square difference labeling, cube difference labeling, lucky edge labeling and its lucky number and also apply coloring condition satisfying to the octopus graph for some labelings.

AMS Subject Classification: 05C78

Key Words and Phrases: Octopus graph; Differences of cubic and squared difference labeling; Square difference labeling; Cube difference labeling; Lucky edge labeling; Lucky number; Coloring.

1 Introduction

In this paper, we consider only simple, finite, undirected and non - trivial graph $G = (V(G), E(G))$ with the vertex set V(G) and the edge set E(G). If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling. A dynamic survey on graph labeling is regularly updated by Gallian (see [8])and it is published by Electronic Journal of Combinatory. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. For any labeling problems following three characteristics are really note-worthy : A set of numbers from which vertex labelings are chosen; A rule that assigns a value to each edge; A condition that these values must satisfy. For all other terminology and notations I follow Harary (see [7]).

Here brief summary of definitions which is used for the present investigations are given. In Edward Samuel .A and Kalaivani .S (see [4]) have proved that the Prime labeling for some planter related graphs. In Edward Samuel .A and Kalaivani .S (see [5])have proved that the Prime labeling for some vanessa related graphs. In Edward Samuel .A and Kalaivani .S (see [6])have proved that the Square sum labeling for some lilly related graphs.

1.1 Definition(see [9])

Let G=(V(G),E(G))be a graph G. A graph G is said to be absolute differences of the differences of the cubes of the vertices and the differences of the squares of the vertices, if there exist a bijection f from V(G)to $(1, 2, ..., p)$ such that the induced function f^* from E(G)to multiples of 2 is given by $f^*(uv) = ||[f(u)]^3 - [f(v)]^3| - |[f(u)]^2 - [f(v)]^2||$. ie. $f^*(uv) = |[f^{'}(uv) - f^{''}(uv)]|$ is injective. Where, $f^{'}(uv) = |[f(u)]^3 - [f(v)]^3|$ and $f^{''}(uv) = |[f(u)]^2 - [f(v)]^2|$.

1.2 Definition(see [11])

Let G=(V(G),E(G)) be a graph. G is said to be square difference labeling if there exist a bijection f from V(G)to $\{0, 1, 2, ..., p-1\}$ such that the induced function f^* from E(G)to N given by $f^*(uv) = |[f(u)]^2 - [f(v)]^2|$ is injective.
A square difference graph with weights as odd numbers are called Odd square difference graph.
A square difference graph with weights as perfect square numbers are called Perfect square difference graph.

1.3 Definition(see [1])

Let G be a simple graph with vertex set V(G) and edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper coloring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. A graph which admits lucky edge labeling is the lucky edge labeled graph. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, ..., k\}$ is the lucky number of G denoted by $\eta(G)$.

1.4 Definition(see [10])

Let G=(V(G),E(G)) be a graph. G is said to be cube difference labeling if there exist a injection f from V(G)to $\{0, 1, 2, ..., p-1\}$ such that the induced function f^* from E(G)to N given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ is injective. A graph which satisfies the cube difference labeling is called the cube difference graph.

1.5 Definition(see [2])

A coloring is proper if adjacent vertices have different colors. A graph is k - colorable if there is a proper k - coloring. The chromatic number $\eta(G)$ of a graph G is the minimum k such that G is k - colorable.

2 Octopus Graph Of Some Labelings

2.1 Octopus Graph(see [3])

An Octopus graph $O_n, (n \geq 2)$ can be constructed by a fan graph $F_n, (n \geq 2)$ joining a star graph $K_{1,n}$ with sharing a common vertex, where n is any positive integer. i.e., O_n =F_n +$K_{1,n}$.
Refer Figure 2.1. An octopus graph O_2

Theorem 1. *An octopus graph O_n satisfies the differences of cubic and squared difference graph, where n is any natural number.*

Proof. Let G be a graph of octopus O_n. Let $|V(O_n)| = 2n + 1$ and $|E(O_n)| = 3n-1$. We define a labeling f from $V(O_n)$ to $\{1, 2, ..., 2n+1\}$ as follows the function.
$f(u_i)$ =i for 1 ≤ i ≤ $2n + 1$
$f(e_i)$ =$i^3 + 2i^2 + i$ for 1 ≤ i ≤ $2n$
$f(e_i^{'})$ =$3(i + 1)^2 + i + 1$ for 1 ≤ i ≤ $n - 1$
The edges of the octopus graph have distinct values which are multiples of 2 and it is in the form of increasing order. The graph O_n is the differences of cubic and squared difference graph.

Refer Figure 1. Differences of cubic and squared difference graph for O_3. □

Theorem 2. *The graph octopus O_n is a square difference graph for $n \geq 2$.*

Proof. Let G be an octopus graph O_n. Denote the vertices of O_n be $\{u_1, u_2, ..., u_(2n+1)\}$. Let $|V(O_n)| = 2n + 1$ and $|E(O_n)| = 3n - 1$. Define a labeling function f from $V(O_n)$ to $\{0, 1, 2, ..., p - 1\}$ as follows.
$f(u_i)$ =i-1 for 1 ≤ i ≤ $2n + 1$
$f(e_i)$ =i^2 for 1 ≤ i ≤ $2n$
$f(e_i^{'})$ =$2i + 1$ for 1 ≤ i ≤ $n - 1$
The induced function f^* from E(G)to N is defined by $f^*(uv) = |[f(u)]^2 - [f(v)]^2|$ is in increasing function and one-to-one and also it is an injective function. The weights of the edges are distinct also perfect squares as $1, 4, 9, 16, ..., (2n)$ In this labeling one interesting thing is the weights are odd numbers, starting from 3 it goes on increasing as 3,5,7,...
Hence the octopus graph O_n can be perfect square difference graph and also odd square difference graph.

Refer figure 2. Square difference labeling in O_4 . □

Theorem 3. *An octopus graph O_n admits a cube difference graph.*

Proof. Denote the graph G $= O_n$ and the vertices consecutively be $\{u_1, u_2, ..., u_(2n+1)\}$ Let $|V(O_n)| = 2n+1$ and $|E(O_n)| = 3n-1$. The mapping f from $V(O_n)$ to $\{0, 1, 2, ..., p-1\}$ is defined by $f(u_i)$=i-1 for $1 \leq$ i $\leq 2n+1$ and the induced function f^* from E(G)to N is defined by $f^*(u_1u_i) = |[f(u_1)]^3 - [f(u_i)]^3|$ for $1 \leq$ i $\leq 2n$. Also, $f^*(u_{i+1}u_{i+2}) = |[f(u_{2i})]^3 - [f(u_{2i+1})]^3| = 3i^2 + 3i + 1$ for $1 \leq$ i $\leq n-1$. Here the edge sets are,$E_1 = \{i^3\ /\ 0 \leq$ i $\leq 2n\}$ and E_2=$\{3i^2 + 3i + 1/1\leq$i$\leq n-1\}$ Here the edge values are distinct and is in the increasing function and one-to-one and also it is an injective function. Hence f admits cube difference labeling. Thus the octopus graph O_n is a cube difference graph.

Refer figure 3. Cube difference graph for O_5. □

Theorem 4. *The lucky number of an octopus graph O_n is $\eta(O_n) = 2n+2$*

Proof. Let G be a graph of octopus O_n . Let $|V(O_n)| = 2n+1$ and $|E(O_n)| = 3n-1$. Denote the middle vertex of the join of two fan graphs $2F_n$ with star graph $K_{1,n}$ be u_1 . Let the remaining vertices be $u_2, u_3, ..., u_{2n+1}$. Label the vertices u_i with labels i (i= 1 to 2n+1). Define a labeling f from $V(O_n)$ to $\{1, 2, ..., 2n+1\}$ as follows. $f(u_i)$ =i for $1 \leq$ i $\leq 2n+1$
This induces a lucky edge labeling of an octopus graph O_n with lucky number is given by $\eta(O_n) = 2n+2$. Therefore, O_n is a lucky edge labeled graph.

Refer figure 4. Lucky edge labeled graph for O_6 and its lucky number i.e. $\eta(O_6) = 14$. □

3 Conclusion

The main aim of this paper is to be satisfied the various graph labeling conditions like Differences of the cubic and squared difference labeling, Square difference labeling, Cube difference labeling and lucky edge labeling and also its lucky number in an octopus graph. This paper also discussed about the coloring condition satisfied to the octopus graph.

4 Acknowledgements

The authors thanks to the anonymous reviewer for his suggestions which have led to an improvement in both quality and clarity of the paper.

References

[1] A. Aishwarya, Lucky edge labeling of different graphs, *International Journal of Innovative Research in Science, Engineering and Technology*, **4**, Issue 2 (2015), 469-471.

[2] R. L. Brooks, On coloring the vertices of a graph, *Proc. Cambridge Phil. Soc.*,**37** (1941), 194-197.

[3] A. Edward Samuel and S. Kalaivani, Prime labeling for some octopus related graphs,*International Organisation of Scientific Research*, **12**(6), (2016), 57-64.

[4] A. Edward Samuel and S. Kalaivani, Prime labeling for some planter related graphs,*International Journal of Mathematics Research* , **8**(3), (2016), 221-231.

[5] A. Edward Samuel and S. Kalaivani, Prime labeling for some vanessa related graphs,*Indian Journal of Applied Research* , **7**(4), (2017), 136-145.

[6] A. Edward Samuel and S. Kalaivani, Square sum labeling for some lilly related graphs,*International Journal of Advanced Technology and Engineering Exploration* , **4**(29), (2017), 68-72.

[7] Frank Harary, *Graph theory*, Narosa Publishing House-New Delhi, (2001).

[8] J.A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinations*,**16**(6), 2009.

[9] P. Shalini, D. Paul Dhayabaran, An absolute differences of cubic and square difference labeling, *International Journal of Advanced Scientific and Technical Research*,**3** (5) (2015), 1-8.

[10] J.Shiama, Cube difference labeling of some graphs, *International Journal of Engineering Science and Innovative Technology(IJESIT)*,**2**(6) (2013), 200-205.

[11] J. Shiama, Square difference labeling for some graphs, *International Journal of Computer Application*,**44** (4), (2012), 30-33.

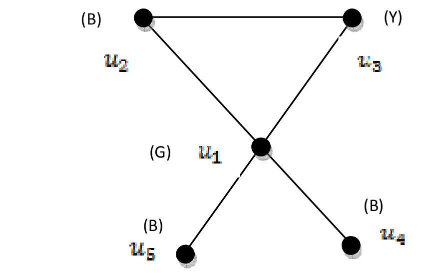

Figure 3.1. An octopus graph O_2.

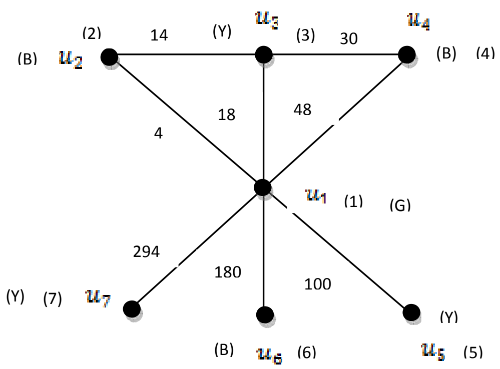

Figure 1. Differences of cubic and squared difference graph for O_3.

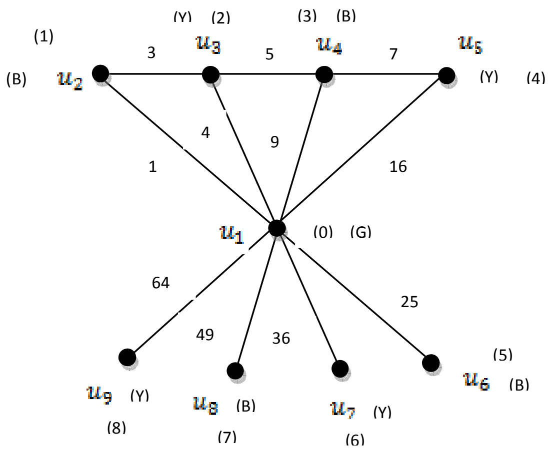

Figure 2. Square difference labeling in O_4.

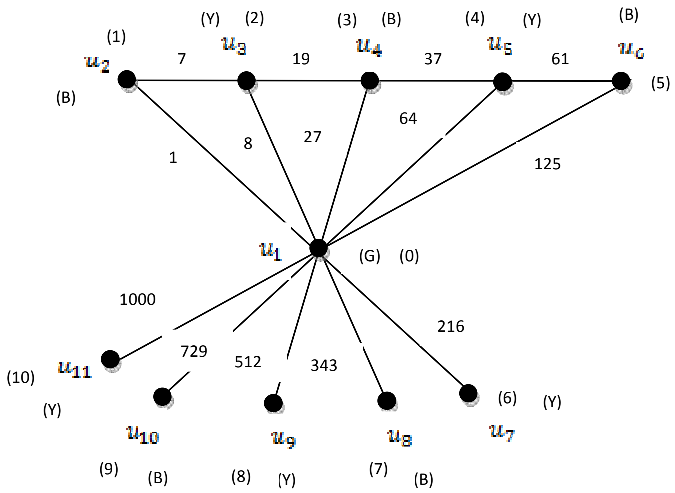

Figure 3. Cube difference graph for O_5.

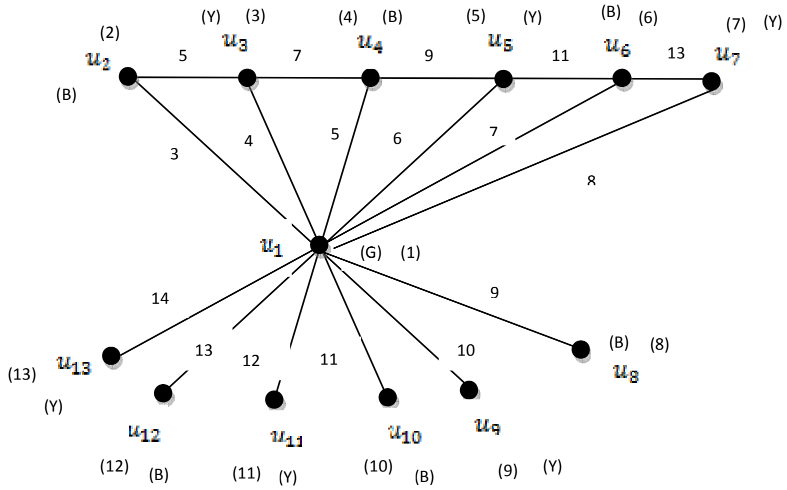

Figure 4. Lucky edge labeled graph for O_6 and its lucky number i.e. $\eta(O_6) = 14$.