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Fuzzy Critical Path with Area Measure

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Abstract

Network diagram plays a vital role to determine project completion time. Network analysis is a technique which determines the various sequences of activities concerning a project and the project completion time. The popular method of this technique is widely used as the critical path method. In this paper, we find the fuzzy critical path in a acyclic project network using area measure to identify the fuzzy critical path from type-2 trapezoidal fuzzy numbers. An illustrative example is also included to demonstrate our proposed approach.

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Key Words: Fuzzy critical path, type-2 trapezoidal fuzzy numbers, acyclic project network, area measure.

1 Introduction

Network Scheduling is a technique used for planning and scheduling large projects in the various fields such as construction, fabrication, purchasing etc. The main aim of government agencies and Industrial organizations is to plan their project in order to maximize resource utility and minimize over all cost. This type of management problem can be very well tackled using the network techniques called critical path method. Since the activities in the network can be carried out is parallel, the minimum time to complete the project is the length of the longest path from the start of project to its finish. The longest path is the critical path of the network. The critical path method (CPM) is to identify critical activities on the Critical path.

This paper analyze the critical path in a general project network with fuzzy activity times. The fuzzy measures were introduced by Sugeno. We propose area measure for fuzzy numbers to a critical path method in a fuzzy project network, where the duration time of each activity in a fuzzy project network is represented by a type-2 trapezoidal fuzzy number.

The structure of this paper is as follows: In section 2, we have some basic concepts. Section 3 contains some properties regarding calculation of the total slack fuzzy time. Section 4 gives the network terminology. Section 5 gives an algorithm to find the critical path combined with type-2 trapezoidal fuzzy number using area measure method. To illustrate the proposed algorithm the numerical example is solved in section 6.

2 Basic concepts

2.1 Type-2 fuzzy number

Let X be a type-2 fuzzy set defined in the universe of discourse R, if the following conditions are satisfied, then X is called a type-2 fuzzy number.

- X is normal.
- X is a convex set.
- The support of X is closed and bounded.

2.2 Normal type-2 fuzzy number

A type-2 fuzzy number(T2fs) X is said to be normal if its Foot of Uncertainty (FOU) is normal interval type-2 fuzzy number (IT2FS) and it has a primary membership function.

2.3 Addition on type-2 fuzzy numbers

Let

$$\begin{aligned}
\tilde{A} &= \cup_{\forall\tilde{\alpha}}\tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}}) = (A^L, A^M, A^N, A^U) \\
&= ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M), (a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U)) \\
\tilde{B} &= \cup_{\forall\tilde{\alpha}}\tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}}) = (B^L, B^M, B^N, B^U) \\
&= ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M), (b_1^N, b_2^N, b_3^N, b_4^N), (b_1^U, b_2^U, b_3^U, b_4^U))
\end{aligned}$$

be two normal type-2 fuzzy numbers. By using extension principle, we have

$$\begin{aligned}
\tilde{A} + \tilde{B} &= \cup_{\forall\tilde{\alpha}}\tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}}) + \cup_{\forall\tilde{\alpha}}\tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}}) \\
&= (A^L + B^L, A^M + B^M, A^N + B^N, A^U + B^U) \\
&= ((a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M, a_4^M + b_4^M), \\
&\quad (a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N, a_4^N + b_4^N), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U))
\end{aligned}$$

2.4 Ranking Function on type-2 Fuzzy number

Let $\tilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M), (a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U))$
be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as,

$$R(\tilde{A}) = \left(\frac{(a_1^L + 2a_2^L + 2a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + 2a_4^N + a_1^U + 2a_2^U + 2a_3^U + a_4^U)}{36}\right)$$

2.5 Area Measure

Let $\tilde{A} = (a, b, c, d : \lambda)$ be a level λ trapezoidal fuzzy number such that $a < b < c < d$, $0 < \lambda \leqslant 1$. If$(c - b) \neq \lambda$ the area measure of $\tilde{A} = \dfrac{\lambda(c - b - a + d)}{2}$ and if $(c - b) = \lambda$ the area measure of $\tilde{A} = \dfrac{\lambda(b - a + 2\lambda + d - c)}{2}$.

2.6 Notations

t_{ij} = The activity between node i and j.
ESF_j = The earliest starting fuzzy time of node j.
LFF_i = The latest finishing fuzzy time of node i.
TSF_{ij} = The total slack fuzzy time of t_{ij}.
p_n = the n^{th} fuzzy path.
P = the set of all fuzzy paths in a project network.
$F(p_n)$ = The total slack fuzzy time of path p_n in a project network.

3 Properties

3.1 Property: Forward pass calculation

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie)$ESF_1 = (0.0, 0.0, 0.0, 0.0)$
$ESF_j = max_i\{ESF_j + TSF_{ij}\}, j \neq i, j \in N, i =$number of preceding nodes.(ESF_j) = The earliest starting fuzzy time of node j).
Ranking value is utilized to identify the maximum value. Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

3.2 Property: Backward pass calculation

To calculate the latest finishing time in the project network set $LFF_n = ESF_n$
$LFF_j = min_j\{LFF_j(-)SET_{ij}\}, i \neq n, i \in N, j =$ number of succeeding nodes. Ranking value is utilized to identify the minimum value. Latest starting fuzzy time= Latest finishing Fuzzy time (−) Fuzzy activity time.

3.3 Property

For the activity $t_{ij}, i < j$
Total fuzzy slack: $SFT_{ij} = LFF_j(-)(ESF_i(+)SFT_{ij})$ or $(LFF_j(-)SFT_{ij})(-)ESF_i$,
$1 \leq i \leq j \leq n, i, j \in N$

3.4 Property

$F(P_n) = \sum_{1\leq i\leq j\leq n, i,j\in P_k} SFT_{ij}, p_k \in P, P_n$ is the possible paths in a network from source node to the destination node, $k = 1$ to m.

4 Network terminology

Consider a directed acyclic project network $G(V, E)$ consisting of a finite set of nodes $V = 1, 2, ..n$ and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i, j) where $i, j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path p_{ij} as a sequence $p_{ij} = [i = i_1, (i_1, i_2), i_2, .i_{i-1}, i_l = j]$ of alternating nodes and edges. The existence of at least one path p_{si} in $G(V, E)$ is assumed for every node $i \in V - S$. $\tilde{d_{ij}}$ denotes a trapezoidal type-2 fuzzy number associated with the edge (i, j), corresponding to the length necessary to transverse (i, j) from i to j. The fuzzy length along the path P is denoted by $d(\tilde{P})$ and is defined as $d(\tilde{P}) = \sum_{(i,j\in p)} \tilde{d_{ij}}$.

5 Algorithm (for finding critical path)

Step 1: Estimate the fuzzy activity time with respect to each activity.
Step 2: Let $ESF_1 = (0.0, 0.0, 0.0, 0.0)$ calculate $ESF_j, j = 2, 3, ...n$ by using property 1.
Step 3: Let $LFF_n = ESF_n$ and calculate $LFF_i, i = n-1, n-2, ..2, 1$. By using property 2.
Step 4: Calculate SFT_{ij} with respect to each activity in a project network by using property 3.
Step 5: Calculate area measure for each activity using definition 2.5.
Step 6: If area measure = 0, those activities are called as fuzzy critical activity and the corresponding path is fuzzy critical path.

6 Numerical example

The problem is to find the critical path and critical path length between source node to destination node in the network having 6 vertices 7 edges with type-2 fuzzy number.
Solution : The edge lengths are
$\tilde{P} = ((1.5, 1.8, 2.0, 2.5), (1.2, 1.5, 2.0, 2.2), (1.1, 1.3, 1.5, 2.0), (1.0, 1.3, 1.5, 2.0))$
$\tilde{Q} = ((1.6, 1.7, 1.8, 2.0), (1.5, 1.6, 2.0, 2.5), (1.3, 1.5, 2.0, 2.3), (1.2, 1.4, 1.5, 1.8))$
$\tilde{R} = ((1.8, 1.9, 2.2, 2.6), (1.6, 1.9, 2.2, 2.6), (1.5, 1.7, 2.1, 2.3), (1.2, 1.8, 2.3, 2.6))$
$\tilde{S} = ((1.3, 1.5, 1.9, 2.2), (1.3, 1.4, 2.3, 2.4), (1.2, 1.6, 2.3, 2.8), (1.0, 1.5, 1.9, 2.3))$
$\tilde{T} = ((1.8, 1.9, 2.3, 2.5), (1.6, 1.8, 2.3, 2.4), (1.5, 1.6, 1.9, 2.3), (1.3, 1.6, 1.9, 2.5))$
$\tilde{U} = ((1.5, 1.7, 2.0, 2.5), (1.3, 1.6, 2.2, 2.5), (1.2, 1.6, 2.3, 2.6), (1.1, 1.5, 1.8, 2.0))$
$\tilde{V} = ((1.9, 2.0, 2.3, 2.4), (1.8, 2.1, 2.3, 2.5), (1.5, 1.7, 2.0, 2.3), (1.4, 1.7, 2.0, 2.5))$

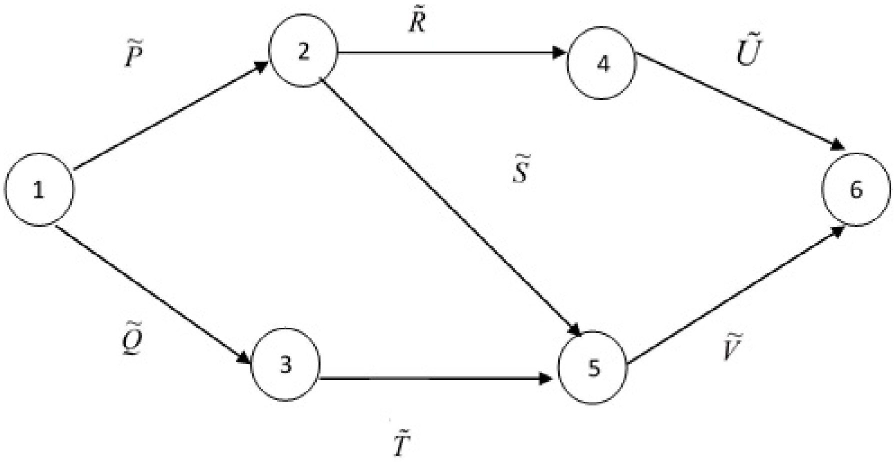

Fig: 6.1

Activity$(i-j)i<j$	Fuzzy activity time	Defuzzified activity time	Total float
1-2	((1.5,1.8,2.0,2.5),(1.2,1.5,2.0,2.2), (1.1,1.3,1.5,2.0),(1.0,1.3,1.5,2.0))	-0.1	0.05
1-3	((1.6,1.7,1.8,2.0),(1.5,1.6,2.0,2.5), (1.3,1.5,2.0,2.3),(1.2,1.4,1.5,1.8))	0.075	0
2-4	((1.8,1.9,2.2,2.6),(1.6,1.9,2.2,2.6), (1.5,1.7,2.1,2.3),(1.2,1.8,2.3,2.6))	0.175	0.2
2-5	((1.3,1.5,1.9,2.2),(1.3,1.4,2.3,2.4), (1.2,1.6,2.3,2.8),(1.0,1.5,1.9,2.3))	0.175	0.05
3-5	((1.8,1.9,2.3,2.5),(1.6,1.8,2.3,2.4), (1.5,1.6,1.9,2.3),(1.3,1.6,1.9,2.5))	0.05	0
4-6	((1.5,1.7,2.0,2.5),(1.3,1.6,2.2,2.5), (1.2,1.6,2.3,2.6),(1.1,1.5,1.8,2.0))	0.05	0
5-6	((1.9,2.0,2.3,2.4),(1.8,2.1,2.3,2.5) (1.5,1.7,2.0,2.3),(1.4,1.7,2.0,2.5))	0.2	0

Table:1 Activities, fuzzy durations and total slack fuzzy time for each activity.

From the table we observe that

- The expected time in terms of trapezoidal fuzzy numbers are defuzzified using definition 2.5($\lambda = 1$) for all activities in the given acyclic project network.
- By using property 3.4 all possible paths $P = \{(1-2-4-6), (1-2-5-6), (1-3-5-6)\}$ are found.
- Fuzzy critical path is identified with the help of area measure.
- The path $(1-3-5-6)$ is the fuzzy critical path in a given acyclic fuzzy project network.

7 Conclusion

In this paper an attempt is made to find fuzzy critical path in a acyclic project network using type-2 trapezoidal fuzzy numbers with the help of area measure.

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