

# $I_{\tilde{g}}$ -Totally continuity and $I_{\tilde{g}}$ -connectedness in Ideal Topological Spaces

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## Abstract

The intention of this paper is to introduce  $I_{\tilde{g}}$ -Totally continuity and  $I_{\tilde{g}}$ -connectedness in Ideal topological spaces. It also aims to study the properties of  $I_{\tilde{g}}$ -continuous function and  $I_{\tilde{g}}$ -connectedness space.

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**Key Words and Phrases:**  $I_{\tilde{g}}$ -Totally continuous function, Totally  $I_{\tilde{g}}$ -continuous function,  $I_{\tilde{g}}$ -separated sets and  $I_{\tilde{g}}$ -connectedness space.

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## 1 Introduction

Ideal in topological spaces have been considered since 1930. This topic has won its importance by the paper of Vaidhaynathaswamy[10]. Applications to various fields were further investigated by Jankovic[4] and Hamlett[5]; Dontchev et al[1]; Mukherjee et al[7], etc. In 1960, Levin[6] introduced strongly continuous maps. In 1984, Noiri introduced and studied perfectly continuous maps.

In topology and related branches of mathematics a connected space is a topological space that cannot be requested as the union of two disjoint non-empty open sets. Connectedness is one of the principle topological spaces. In 2008, Ekici[2] introduced connectedness in ideal topological spaces.

The purpose of this paper is to introduce the notion of  $I_{\tilde{g}}$ -totally continuity, totally  $I_{\tilde{g}}$ -continuity,  $I_{\tilde{g}}$ -separated sets and  $I_{\tilde{g}}$ -connectedness in ideal topological spaces and also to investigate their properties.

## 2 Preliminaries

The present paper throughout by  $(X, \tau)$  or  $(Y, \sigma)$  denote a topological space with no separation properties assumed. For a subset  $A$  of a topological space  $(X, \tau)$ ,  $cl(A)$ , and  $Int(A)$  will denote the closure and interior of  $A$  in  $(X, \tau)$  respectively.

**Definition 1.** An ideal  $I$  on a topological space  $(X, \tau)$  is a non-empty collection of subsets of  $X$  which satisfies,

- (1)  $A \in I$  and  $B \subset A \implies B \in I$ .
- (2)  $A \in I$  and  $B \in I \implies A \cup B \in I$ .

An ideal topological space is a topological space  $(X, \tau)$  with an ideal  $I$  on  $X$  and is denoted by  $(X, \tau, I)$ .

**Definition 2.** For a subset  $A \subset X$ ,

$$A^*(I) = \{x \in X : U \cap A \notin I \text{ for every neighbourhood } U \text{ of } x\}$$

is called the local function of  $A$  with respect to  $I$  and  $\tau$ . We simple write  $A^*$  instead of  $A^*(I)$  to be brief.

**Definition 3.** For every ideal topological space  $(X, \tau, I)$  there exists a topology  $\tau^*(I)$ , finer than  $\tau$ , generated by  $\beta(I, \tau) = \{U - i : U \in \tau \text{ and } i \in I\}$ , but in general  $\beta(I, \tau)$  is not always a topology. Additionally,  $cl^*(A) = A \cup A^*$  defines a kuratowski closure operator for  $\tau^*(I)$ .

**Definition 4.** [3] A subset  $A$  of an ideal topological space  $(X, \tau, I)$  is said to be

- (1)  $I_{\tilde{g}}$ -closed if  $A^* \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $sg$ -open.
- (2)  $I_{\tilde{g}}$ -open if its complement is  $I_{\tilde{g}}$ -closed.

**Definition 5.** A function  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is said to be

- (1)  $I_{\tilde{g}}$ -continuous ( $I_{\tilde{g}}$ -continuous) if the inverse image of every closed set in  $Y$  is  $I_{\tilde{g}}$ -closed ( $I_{\tilde{g}}$ -closed) in  $X$ .
- (2) Strongly  $I_{\tilde{g}}$ -continuous if the inverse image of every  $I_{\tilde{g}}$ -closed set in  $Y$  is closed in  $X$ .
- (3) Perfectly  $I_{\tilde{g}}$ -continuous if the inverse image of every  $I_{\tilde{g}}$ -open set in  $(Y, \sigma, J)$  is both open and closed in  $(X, \tau)$ .

**lemma 6.** Every closed set in  $(X, \tau, I)$  is  $I_{\tilde{g}}$ -closed set [3].

**Definition 7.** Let  $(X, \tau, I)$  be a ideal topological space  $A \subset X$ . The intersection of all  $I_{\tilde{g}}$ -closed supersets of  $A$  is called the closure of  $A$  and is denoted by  $Cl_{I_{\tilde{g}}}(A)$ .

## 3 $I_{\tilde{g}}$ -Totally Continuous Function

**Definition 8.** A function  $f : (X, \tau, I) \rightarrow (Y, \sigma, I)$  is called  $I_{\tilde{g}}$ -totally continuous function if the inverse image of every  $I_{\tilde{g}}$ -open subset of  $Y$  is clopen in  $X$ .

**Theorem 9.** A function  $f : (X, \tau, I) \rightarrow (Y, \sigma, I)$  is  $I_{\tilde{g}}$ -totally continuous function if only if the inverse image of every  $I_{\tilde{g}}$ -closed subset of  $Y$  is clopen in  $X$ .

*Proof.* Let  $F$  be any  $I_{\tilde{g}}$ -closed set in  $Y$ . Then  $F^c$  is  $I_{\tilde{g}}$ -open in  $Y$ . By definition 8,  $f^{-1}(F^c)$  is clopen in  $X$ . But  $f^{-1}(F^c) = (f^{-1}(F))^c$ , which is clopen in  $X$  implies  $f^{-1}(F)$  is

clopen in X.

Conversely, suppose V is  $I_{\tilde{g}}$ -open in Y, then  $V^c$  is  $I_{\tilde{g}}$ -closed set in Y. By hypothesis,  $f^{-1}(V^c) = (f^{-1}(V))^c$ , which is clopen in X implies  $f^{-1}(V)$  is clopen in X. Hence the inverse image of every  $I_{\tilde{g}}$ -open set in Y is clopen in X. Thus f is  $I_{\tilde{g}}$ -totally continuous.  $\square$

**Theorem 10.** Every  $I_{\tilde{g}}$ -totally continuous functions is perfectly continuous.

The converse of the above theorem need not be true as seen from the following example.

**example 11.** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ ;  $I = \{\emptyset, \{a\}\}$ ;  $\sigma = \{\emptyset, Y, \{a\}\}$ ;  $J = \{\emptyset, \{a\}\}$ .  $I_{\tilde{g}}$ -open sets are  $X, \emptyset, \{b\}, \{c\}, \{b, c\}$ .  $I_{\tilde{g}}$ -open sets are  $Y, \emptyset, \{a\}, \{b\}, \{a, b\}$ .

Define a function  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  by  $f(a) = b; f(b) = a; f(c) = c$ . Then the inverse image of every open set in Y is clopen in X. Since  $\{a\} \in Y \implies f^{-1}(\{a\}) = \{b\}$ . Therefore  $\{b\}$  is clopen in X. Thus f is perfectly continuous but f is not  $I_{\tilde{g}}$ -totally continuous. Since the  $I_{\tilde{g}}$ -open set of Y,  $f^{-1}(\{b\}) = \{a\}$  is not clopen in X.

**Theorem 12.** Let  $f : (X, \tau, I) \rightarrow (Y, \sigma, I)$  be a function. Then the following are equivalent.

- (i) f is  $I_{\tilde{g}}$ -totally continuous.
- (ii) For each  $x \in X$  and each  $I_{\tilde{g}}$ -open set V in Y with  $f(x) \in V$ , there is a clopen set U in X such that  $x \in U$  and  $f(U) \subseteq V$ .

*Proof.* .

(i)  $\implies$  (ii)

Suppose f is  $I_{\tilde{g}}$ -totally continuous and V be any  $I_{\tilde{g}}$ -open set in Y containing  $f(x)$  so that  $x \in f^{-1}(V)$ . Since f is  $I_{\tilde{g}}$ -totally continuous,  $f^{-1}(V)$  is clopen in X. Let  $U = f^{-1}(V)$ , then U is clopen in X and  $x \in U$ . Hence  $f(U) = f(f^{-1}(V)) \subseteq V \implies f(U) \subseteq V$ .

(ii)  $\implies$  (i)

Let V be  $I_{\tilde{g}}$ -open set in Y. Let  $x \in f^{-1}(V)$  be any arbitrary point. Then  $f(x) \in V$ , there is a clopen set  $f(G) \subseteq X$  containing x such that  $f(G) \subseteq V \implies G \subseteq f^{-1}(V)$  and  $x \in G \subseteq f^{-1}(V)$ .  $f^{-1}(V)$  is neighbourhood of each of its points. Hence  $f^{-1}(V)$  is clopen set in X. Thus f is  $I_{\tilde{g}}$ -totally continuous.  $\square$

**Theorem 13.** If a function  $f : (X, \tau, I) \rightarrow (Y, \sigma, I)$  is  $I_{\tilde{g}}$ -totally continuous then it is continuous but not conversely.

**example 14.** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ ;  $I, J = \{\emptyset, \{a\}\}$ ;  $\sigma = \{\emptyset, Y, \{a\}\}$ . Define a function  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  by  $f(a) = b; f(b) = a; f(c) = c$ . Then the inverse image of every closed set in Y is closed in X. [ $f^{-1}(\{b, c\}) = \{a, c\}$ ]. Thus f is continuous but f is not  $I_{\tilde{g}}$ -totally continuous. since for the subset  $\{a\}$  is  $I_{\tilde{g}}$ -open set in Y,  $f^{-1}(\{a\}) = \{b\}$  is open in X but not closed in X.

**Theorem 15.** The composition of two  $I_{\tilde{g}}$ -totally continuous function is  $I_{\tilde{g}}$ -totally continuous.

*Proof.* Let  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  and  $g : (Y, \sigma, J) \rightarrow (Z, \eta, K)$  be any two  $I_{\tilde{g}}$ -totally continuous function. Let V be  $I_{\tilde{g}}$ -open in Z. Since g is  $I_{\tilde{g}}$ -totally continuous,  $g^{-1}(V)$  is clopen and hence open in Y. Since every open set is  $I_{\tilde{g}}$ -open,  $g^{-1}(V)$  is  $I_{\tilde{g}}$ -open in Y. Also f is  $I_{\tilde{g}}$ -totally continuous,  $f^{-1}(g^{-1}(V))$  is clopen in X. But  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ . Which is clopen in X. Hence  $g \circ f$  is  $I_{\tilde{g}}$ -totally continuous.  $\square$

**Theorem 16.** *If a function  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is  $I_{\tilde{g}}$ -totally continuous then  $f$  is strongly  $I_{\tilde{g}}$ -continuous but not conversely.*

**example 17.** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ ;  $I = \{\emptyset, \{b\}\}$ ;  $\sigma = \{\emptyset, Y, \{b\}, \{a, b\}\}$ ;  $J = \{\emptyset, \{a\}\}$ . Define a function  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  by  $f(a) = b$ ;  $f(b) = f(c) = a$ .

$I_{\tilde{g}}$ -closed sets are  $X, \emptyset, \{c\}, \{b, c\}$ ;  $I_{\tilde{g}}$ -open sets are  $X, \emptyset, \{a\}, \{a, b\}, \{b, c\}$ .

$I_{\tilde{g}}$ -closed sets are  $Y, \emptyset, \{a\}, \{c\}, \{a, c\}$ ;  $I_{\tilde{g}}$ -open sets are  $Y, \emptyset, \{b\}, \{b, c\}, \{a, b\}$ .

Then the inverse image of every  $I_{\tilde{g}}$ -open set in  $Y$  is open in  $X$ . Thus  $f$  is strongly  $I_{\tilde{g}}$ -continuous, but  $f$  is not  $I_{\tilde{g}}$ -totally continuous. Since the subset  $\{b\}$  is  $I_{\tilde{g}}$ -open in  $Y$ .  $f^{-1}(\{b\}) = \{a\}$  is open but closed in  $X$ .

**Theorem 18.** *Let  $X$  be a discrete topological space and  $Y$  be ideal space and  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  be a function where  $I = \emptyset$ . If  $f$  is strongly  $I_{\tilde{g}}$ -continuous then  $f$  is  $I_{\tilde{g}}$ -totally continuous.*

*Proof.* Let  $V$  be  $I_{\tilde{g}}$ -open in  $Y$ . since  $f$  is strongly  $I_{\tilde{g}}$ -continuous,  $f^{-1}(V)$  is open in  $Y$ . Since  $X$  is discrete space,  $f^{-1}(V)$  is both open and closed in  $X$ . Hence  $f$  is  $I_{\tilde{g}}$ -totally continuous. □

**Definition 19.** A function  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is called totally  $I_{\tilde{g}}$ -continuity if  $f^{-1}(V)$  is  $I_{\tilde{g}}$ -clopen in  $X$  for each open set  $V$  in  $(Y, \sigma, J)$ .

**Theorem 20.** *Every totally  $I_{\tilde{g}}$ -totally continuous function is  $I_{\tilde{g}}$ -continuous. The converse of the above theorem need not be true as seen from the following example.*

**example 21.** In example 17  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  by  $f(a) = b$ ;  $f(b) = a$ ,  $f(c) = a$ . then inverse image of every closed set in  $Y$  is  $I_{\tilde{g}}$ -closed in  $X$ . Thus  $f$  is  $I_{\tilde{g}}$ -continuous, but it is not totally  $I_{\tilde{g}}$ -continuous since the set  $\{b\}$  is open in  $Y$ ,  $f^{-1}(\{b\}) = \{a\}$  is  $I_{\tilde{g}}$ -open and not  $I_{\tilde{g}}$ -closed in  $X$ .

**Theorem 22.** *Every  $I_{\tilde{g}}$ -totally continuous function is totally  $I_{\tilde{g}}$ -continuous.*

*Proof.* Suppose  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is  $I_{\tilde{g}}$ -totally continuous. Let  $U$  be any open subset of  $Y$ . Then  $U$  is  $I_{\tilde{g}}$ -open in  $Y$ ,  $f^{-1}(U)$  is open in  $Y$ . Since  $f$  is  $I_{\tilde{g}}$ -totally continuous,  $f^{-1}(U)$  is clopen in  $X$  □

#### 4 $I_{\tilde{g}}$ - Connectedness

**Definition 23.** An ideal topological space  $(X, \tau, I)$  is said to be  $I_{\tilde{g}}$ -connected if  $X$  cannot be written as the disjoint union of two non-empty  $I_{\tilde{g}}$ -open sets in  $X$ . If  $X$  is not  $I_{\tilde{g}}$ -connected if it is said to be  $I_{\tilde{g}}$ -disconnected. A subset of  $X$  is  $I_{\tilde{g}}$ -connected if it is  $I_{\tilde{g}}$ -connected as a subspace of  $X$ .

**Theorem 24.** *For an ideal topological space  $(X, \tau, I)$ . Then the following are equivalent.*

- (i)  $X$  is  $I_{\tilde{g}}$ -connected.
- (ii)  $X$  and  $\emptyset$  are the only subsets of  $X$  which are both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed.

*Proof.* (i)  $\implies$  (ii)

Suppose  $X$  is  $I_{\tilde{g}}$ -connected holds. To prove  $X$  and  $\emptyset$  are the only subsets of  $X$  which are both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed. Let  $V$  be a  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed subset of  $X$ . Then  $V^c$  is

both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed subset of  $X$ . Therefore  $X = V \cup V^c$ , a disjoint union of two non-empty  $I_{\tilde{g}}$ -open sets. Which is contradiction to (i). Thus  $V = \emptyset$  or  $X$ . Hence (i)  $\implies$  (ii).

(ii)  $\implies$  (i)

Suppose (ii) holds that is  $X$  and  $\emptyset$  are the only subsets of  $X$  which are both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed. To prove:  $X$  is  $I_{\tilde{g}}$ -connected. Suppose not, then  $X = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty  $I_{\tilde{g}}$ -open subsets of  $X$ . Then  $A$  is both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed. Also  $A \neq \emptyset$  and  $A \neq X$ . Therefore  $A$  is proper non-empty subset of  $X$  which is both  $I_{\tilde{g}}$ -open and  $I_{\tilde{g}}$ -closed. Which is contradiction to our assumption. Hence (ii)  $\implies$  (i).  $\square$

**Theorem 25.** *Every  $I_{\tilde{g}}$ -connected space is connected. The converse of the above theorem need not be true as seen from the following example.*

**example 26.** Let  $X = \{a, b, c, d\}$ ;  $\tau = \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ ;  $I = \{\emptyset, \{a\}\}$   
 $I_{\tilde{g}}$ -closed sets are  $X, \emptyset, \{c\}, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}$ ;  
 $I_{\tilde{g}}$ -open sets are  $X, \emptyset, \{a\}, \{d\}, \{b, d\}, \{a, d\}, \{b, c, d\}, \{a, b, d\}$ .  
 In ideal space  $X$  is connected but not  $I_{\tilde{g}}$ -connected because the set  $X = \{a\} \cup \{b, c, d\}$  which are non-empty disjoint  $I_{\tilde{g}}$ -open sets.

**Theorem 27.** *Let  $(X, \tau, I)$  be an ideal topological space. If  $X$  is  $I_{\tilde{g}}$ -connected, then  $X$  cannot be written as union of two disjoint non-empty  $I_{\tilde{g}}$ -closed sets.*

*Proof.* Suppose  $X = A \cup B$ , where  $A$  and  $B$  are  $I_{\tilde{g}}$ -closed sets  $A \neq \emptyset, B \neq \emptyset$  and  $A \cap B = \emptyset$ . Then  $A = B^c$  and  $B = A^c$ . Since  $A$  and  $B$  are  $I_{\tilde{g}}$ -closed, then  $A$  and  $B$  are  $I_{\tilde{g}}$ -open. There exists non-empty proper subsets  $A$  and  $B$  of  $X$  which are both  $I_{\tilde{g}}$ -open. Which is contradiction to our assumption therefore  $X$  cannot be written as the union of the disjoint non-empty  $I_{\tilde{g}}$ -closed sets.  $\square$

**Theorem 28.** *Let  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  be a function. If  $X$  is  $I_{\tilde{g}}$ -connected and  $f$  is  $I_{\tilde{g}}$ -irresolute surjective, then  $Y$  is  $I_{\tilde{g}}$ -connected.*

*Proof.* suppose  $Y$  is not  $I_{\tilde{g}}$ -connected. Then  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty  $I_{\tilde{g}}$ -open sets in  $Y$ . Since  $f$  is  $I_{\tilde{g}}$ -irresolute and surjective. Then  $X = f^{-1}(A) \cup f^{-1}(B)$  and  $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\emptyset) = \emptyset$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty  $I_{\tilde{g}}$ -open in  $X$ . This contradicts to fact that  $X$  is  $I_{\tilde{g}}$ -connected. Hence  $Y$  is  $I_{\tilde{g}}$ -connected.  $\square$

**Theorem 29.** *Let  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  be a function. If  $X$  is  $I_{\tilde{g}}$ -connected and  $f$  is  $I_{\tilde{g}}$ -continuous surjective, then  $Y$  is connected.*

*Proof.* Let  $X$  be  $I_{\tilde{g}}$ -connected and  $f$  be  $I_{\tilde{g}}$ -continuous, surjective. Suppose  $Y$  is not connected. Then  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty  $I_{\tilde{g}}$ -open sets in  $Y$ . Since  $f$  is  $I_{\tilde{g}}$ -continuous and surjective. Then  $X = f^{-1}(A) \cup f^{-1}(B)$  and  $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\emptyset) = \emptyset$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty  $I_{\tilde{g}}$ -open in  $X$ . This contradicts to fact that  $X$  is  $I_{\tilde{g}}$ -connected. Hence  $Y$  is connected.  $\square$

**Theorem 30.** *If a surjective map  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is strongly  $I_{\tilde{g}}$ -continuous and  $X$  is connected space then  $Y$  is  $I_{\tilde{g}}$ -connected.*

*Proof.* Suppose  $Y$  is not  $I_{\tilde{g}}$ -connected. Then  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty  $I_{\tilde{g}}$ -open sets in  $Y$ . Since  $f$  is strongly  $I_{\tilde{g}}$ -continuous and surjective map. Then  $X = f^{-1}(A) \cup f^{-1}(B)$  and  $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\emptyset) = \emptyset$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are

disjoint non-empty open in X. This contradicts to fact that X is connected. Hence Y is  $I_{\tilde{g}}$ -connected.  $\square$

**Theorem 31.** *If  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is  $I_{\tilde{g}}$ -totally continuous, surjective and X is connected space then Y is  $I_{\tilde{g}}$ -connected.*

*Proof.* Suppose Y is not  $I_{\tilde{g}}$ -connected. Then  $Y = A \cup B$ , where A and B are disjoint non-empty  $I_{\tilde{g}}$ -open sets in Y. Therefore  $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$  and  $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\emptyset) = \emptyset$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty clopen in X (since f is  $I_{\tilde{g}}$ -totally continuous) implies X is not connected. This contradicts to fact that X is connected. Hence Y is  $I_{\tilde{g}}$ -connected.  $\square$

**Definition 32.** An ideal topological space  $(X, \tau, I)$  is said to be  $I_{\tilde{g}}$ -separated if there exists two non-empty subsets A and B such that  $cl_{I_{\tilde{g}}}(A) \cap B = \emptyset = A \cap cl_{I_{\tilde{g}}}(B)$ .

**Theorem 33.** *Let  $(X, \tau, I)$  be an ideal topological space and A be an open subset of X. If A is  $I_{\tilde{g}}$ -connected subset of X and H, G are  $I_{\tilde{g}}$ -separated subset of X with  $A \subseteq H \cup G$  then either  $A \subseteq H$  or  $A \subseteq G$ .*

*Proof.* Suppose  $A \subseteq H \cup G$ , where H and G are  $I_{\tilde{g}}$ -separated sets. Then  $H \cap cl_{I_{\tilde{g}}}(G) = \emptyset = cl_{I_{\tilde{g}}}(H) \cap G$ . Now  $A \subseteq H \cup G$  implies  $A = A \cap (H \cup G) = (A \cap H) \cup (A \cap G)$ .  $cl_{I_{\tilde{g}}}(A \cap H) \cap (A \cap G) \subseteq (cl_{I_{\tilde{g}}}(A \cap H) \cap cl_{I_{\tilde{g}}}(H)) \cap (A \cap G) = (cl_{I_{\tilde{g}}}(A) \cap A) \cap (cl_{I_{\tilde{g}}}(H) \cap G) = \emptyset$ . Similarly  $(A \cap H) \cap cl_{I_{\tilde{g}}}(A \cap G) = \emptyset$ . Hence  $A \cap H$  and  $A \cap G$  are  $I_{\tilde{g}}$ -separated sets but A is connected one of them and their must be empty that is either  $A \cap H = \emptyset$  or  $A \cap G = \emptyset$ .  $A \cap G = \emptyset \implies A \subseteq H$  or  $A \cap H = \emptyset \implies A \subseteq G$ .  $\square$

**Theorem 34.** *Let A and B are subsets of a connected space  $(X, \tau, I)$  such that  $A \subseteq B \subseteq cl_{I_{\tilde{g}}}(A)$ . If A is  $I_{\tilde{g}}$ -connected then B is  $I_{\tilde{g}}$ -connected.*

*Proof.* Given that  $A \subseteq B \subseteq cl_{I_{\tilde{g}}}(A)$  and A is  $I_{\tilde{g}}$ -connected. To prove: B is  $I_{\tilde{g}}$ -connected. Suppose not, there exists non-empty  $I_{\tilde{g}}$ -separated sets G and H such that  $B = G \cup H$ . Now  $A \subseteq B = G \cup H \implies A \subseteq G \cup H$ . Since A is  $I_{\tilde{g}}$ -connected by theorem either  $A \subseteq G$  or  $A \subseteq H$ . Let  $A \subseteq G \implies cl_{I_{\tilde{g}}}(A) \subseteq cl_{I_{\tilde{g}}}(G) \implies cl_{I_{\tilde{g}}}(A) \cap H \subseteq cl_{I_{\tilde{g}}}(G) \cap H = \emptyset$ .  $\implies cl_{I_{\tilde{g}}}(A) \cap H = \emptyset$ .  $G \cup H = B \subseteq cl_{I_{\tilde{g}}}(A) \implies H \subseteq B \subseteq cl_{I_{\tilde{g}}}(A)$ .  $cl_{I_{\tilde{g}}}(A) \cap H = H = \emptyset$ . Which is a contradiction. Therefore B is  $I_{\tilde{g}}$ -connected.  $\square$

### 5 Conclusion

In this paper we introduced  $I_{\tilde{g}}$ -totally continuity, totally  $I_{\tilde{g}}$ -continuity,  $I_{\tilde{g}}$ -separated sets and  $I_{\tilde{g}}$ -connectedness and their properties were investigated. Further we proposed to introduced  $I_{\tilde{g}}$ -compactness,  $I_{\tilde{g}}$ -separated axioms, Higher separation axioms in ideal topological spaces.

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