International Journal of Pure and Applied Mathematics
Volume 118 No. 6 2018, 151-157
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

A Study on n-domination Number of a Fuzzy Graph

A. Nagoorgani [1] and S. Vasantha Gowri [2]

[1] *P.G. Research Department of Mathematics,*
Jamal Mohamed College (Autonomous),Trichy – 620020, India.
ganijmc@yahoo.co.in
[2] *Department of Mathematics,*
Vetri Vinayaha College of Engineering and Technology,
Tholurpatti, Thottiam, Trichy (Dt) – 621215, India.
srihariram2017@gmail.com

Abstract

In this paper, we define n-domination number of a fuzzy graph using strong arc. Here $\gamma_n(G)$ is the minimum cardinality of an n-dominating set of a fuzzy graph. We also discuss some definitions and theorems related to n-dominating set and total n-domination number using strong arc.

AMS Subject Classification: 03E72, 05C40, 05C72

Key Words and Phrases: Domination, n-domination, strong arc, fuzzy graph, total domination, total n-domination.

1 Introduction

Ore and Berge [1, 6] introduced the concept of dominating sets in graphs. Cockayne and Hedetniemi [3] introduced the domination number of graphs. Somasundaram and Somasundaram [7] discussed domination in fuzzy graphs using effective edges. Nagoorgani and Chandrasekaran [5] developed the concept of domination in fuzzy graph using strong arc. Kulli [4] discussed n-domination number and total n-domination number in graphs. Now we discuss n-domination number of a fuzzy graph using strong arc.

2 Preliminaries

Definition 1. A fuzzy graph $G = \langle \sigma, \mu \rangle$ is a pair of functions $\sigma : V \to (0,1)$ and $\mu : V \times V \to (0,1)$, where for all $x, y \in V$ we have $\mu(x,y) \leq \sigma(x) \cap \sigma(y)$.

Definition 2. An arc (x, y) in a fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be strong if $\mu^{\infty}(x,y) = \mu(x,y)$.

Definition 3. A set D of vertices in a graph is called a dominating set of Gif every vertex in $V - D$ is adjacent to some vertex in D.

Definition 4. Let G be a fuzzy graph. Let u and v be two $n$odes of G. We say that $u$dominates $v$if (u, v) is a strong arc. A subset D of V is called a dominating set of Gif for every $v \in V - D$, there exists $u \in D$ such that u dominates v.

Definition 5. A dominating set D is a total dominating set if the induced subgraph $\langle D \rangle$ has no isolated vertices.

3 n-domination

Definition 6 (Main definition)**.** A dominating set S in a fuzzy graph G is an n-dominating set if every vertex in $V - S$ is dominated by at least n vertices in S. i.e., between every vertex of $V - S$ and S there is at least one strong arc.

Here $|N(v) \cap S| \geq n$ and $\gamma_n(G)$ is the minimum cardinality of an n-dominating set of fuzzy graph G.

If $n = 1$, $\gamma_1(G) = \gamma(G)$.

Example 1:

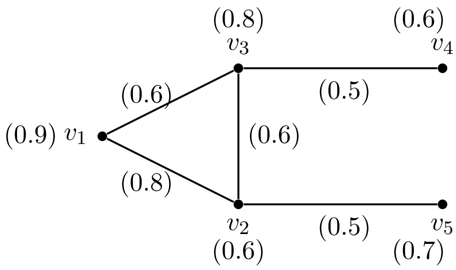

Figure 1:

Here, $S = \{v_2, v_4\}$ is a n-dominating set.

Every vertex in $V - S$ is dominated by at least one vertex in S.

i.e., $v_1 \to v_2$, $v_3 \to v_2$ and v_4, $v_5 \to v_2$.

Theorem 7. *An n-dominating set S of a fuzzy graph G is minimal iff for every vertex $v \in S$ either*

(i) $|N(v) \cap S| < n$ or

(ii) There exists a vertex $u \in V - S$ such that $|N(v) \cap S| = n$ and $u \in N(v)$.

Proof. Let S be a minimal n-dominating set of a fuzzy graph G.

Assume that there exists a vertex $u \in V - S$. Either $|N(v) \cap S| > n$ or u $\notin N(v)$

Consider $S' = S - \{v\}$ since v is adjacent to at least n vertices of S'.

$\Rightarrow$ S' is an n-dominating set which is a contradiction to the minimality of S.

Conversely, suppose S is a n-dominating set of fuzzy graph satisfying conditions (i) and (ii).

Now take the set $S' = S - \{v\}$ for any vertex $v \in S$.

If (i) holds then $|N(v) \cap S'| < n$.

$\Rightarrow$ S' is not an n-dominating set.

If (ii) holds there exists a vertex $u \in V - S$ such that $|N(v) \cap S| = n$ and u $\in N(v)$.

But in this case the set S' would not n-dominates u and hence would not be a n-dominating set of a fuzzy graph G. Thus in either case S' is not a n-dominating set of G.

$\therefore$ S is a minimal dominating set of a fuzzy graph G. □

Theorem 8. *Let G be a fuzzy graph with $\delta(G) \geq n$. If S is a minimal n-dominating set then $V - S$ contains a minimal dominating set, minimal n-dominating set and $V - S$ may itself a n-dominating set.*

Proof. Let S be a minimal n-dominating set of a fuzzy graph G. Suppose there exists a vertex $v \in S$ which dominates no vertex in $V - S$. Then v is adjacent to at least n vertices in S itself.

$\therefore$ $S - \{v\}$ is an n-dominating set which is a contradiction.

Thus every vertex in S must dominate to at least one vertex in $V - S$, which is a dominating set and it contains a minimal dominating set. Also it may contain an n-dominating set or it itself is an n-dominating set. □

Corollary 9. *If G is a fuzzy graph with $\delta(G) \geq n$, then $\gamma(G) + \gamma_n(G) \leq p$.*

Proof. The result is obvious, i.e., the sum of the domination number of a fuzzy graph and n-domination number of a fuzzy graph is either equal to p (the number of vertices) or less than p.

In Example 1, $\gamma(G) = 2$, $\gamma_n(G) = 2$, $p = 5$. $\therefore$ $2 + 2 < 5 \Rightarrow 4 < 5$. □

Theorem 10. *If G is a graph with $\Delta(G) \geq n \geq 2$, then $\gamma(G) + n - 2 \leq \gamma_n(G)$.*

Proof. Let S be a minimum n-dominating set of a fuzzy graph G.

Let $u \in V - S$ and let $v_1, v_2, \ldots, v_n$ be distinct vertices in S which dominates u.

Since $\Delta(G) \geq n \geq 2$, we know that V-S $\neq \emptyset$, there exists an n-dominating set which does not contain a vertex of degree Δ.

Since S is an n-dominating set, each vertex in $V - S$ is dominated by at least one vertex in $S - \{v_1, v_2, \ldots, v_n\}$.

$\therefore$ since u dominates each vertex in $\{v_1, v_2, \ldots, v_n\}$ then $S' = S - \{v_2, \ldots, v_n\}$ is a dominating set of G. Thus

$$\begin{aligned} \gamma(G) \leq |S'| &= \gamma_n(G) - (n-1) + 1 \\ &= \gamma_n(G) - n + 1 + 1 \\ &= \gamma_n(G) - n + 2 \\ \gamma(G) &\leq \gamma_n(G) - n + 2 \\ \Rightarrow \quad \gamma(G) + n - 2 &\leq \gamma_n(G). \end{aligned}$$

□

Corollary 11. *If G is a fuzzy graph with $\Delta(G) \geq 3$ and $n \geq 3$, then $\gamma(G) < \gamma_n(G)$.*

Theorem 12. *For any fuzzy graph G, $\frac{np}{(\Delta(G)+n)} \leq \gamma_n(G)$.*

Proof. Let S be a minimum n-dominating set of a fuzzy graph G.
Let k denote the number of edges between S and $V - S$.
Since the degree of each vertex is at most Δ,

$$k \leq \Delta(G)\gamma_n(G). \tag{1}$$

Also since each vertex in $V - S$ is adjacent to at least n vertices in S,

$$k \geq n[p - \gamma_n(G)]. \tag{2}$$

From (1) and (2),

$$\begin{aligned} \Delta(G)\gamma_n(G) &\geq k \geq n[p - \gamma_n(G)] \\ \Rightarrow \ \Delta(G)\gamma_n(G) &\geq n[p - \gamma_n(G)] \\ \Delta(G)\gamma_n(G) &\geq np - n\gamma_n(G) \\ \Rightarrow \ \Delta(G)\gamma_n(G) + n\gamma_n(G) &\geq np \\ [\Delta(G) + n]\gamma_n(G) &\geq np \\ \Rightarrow \ \gamma_n(G) &\geq \frac{np}{\Delta(G) + n} \end{aligned}$$

□

Theorem 13. *If G is a fuzzy graph with $n \leq \delta(G)$ then $\gamma_n(G) \leq \frac{np}{n+1}$.*

Proof. Let G be a fuzzy graph and S be a minimum n-dominating set of G and δ(G) be the minimum degree in G. We know that $\gamma_n(G) \geq \frac{np}{\Delta(G)+n}$. Then for minimum degree $\delta(G)$, $\gamma_n(G) \leq \frac{np}{\delta(G)+n}$. If $\delta(G) = 1$, then $\gamma_n(G) \leq \frac{np}{1+n}$. □

4 n-total Domination number

Definition 14. A set S of vertices in a fuzzy graph G is an n-total dominating set if every vertex $v \in V$ is dominated by at least n vertices of $S - \{v\}$. i.e., for every vertex $v \in V$ is adjacent to at least n vertices in S. The n-total domination number $\gamma_{t_n}(G)$ is the minimum cardinality of an n-total dominating set of a fuzzy graph G. If $n = 1$, then $\gamma_t(G) = \gamma_{t_1}(G)$.

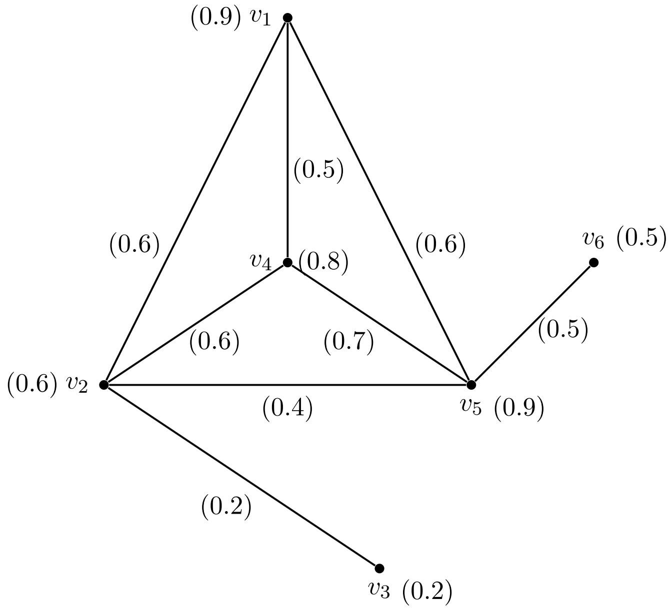

Figure 2:

Example 2:

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$
$$S = \{v_2, v_5\}.$$

Here every vertex $v \in V$ is dominated by at least n vertices in S.

Theorem 15. *If S is a γ_t set of a fuzzy graph G, then at least one vertex in V is adjacent with at most two vertices of S.*

Proof. Let S be a γ_t set of a fuzzy graph G. Assume that each vertex in v is adjacent with at least 3 vertices of S. Suppose $u \in V$. Let v and w be vertices of S which are adjacent with u.

By assumption, each vertex in V is adjacent with at least one vertex of $S - \{v, w\}$.

Thus since v and w are adjacent with u, the set $S' = [S - \{v, w\}] \cup \{u\}$ is a total dominating set in S. We know that $|S'| < |S|$, which is a $\rightarrow\leftarrow$ that S is a minimum total dominating set. □

Corollary 16. *If G is a fuzzy graph with $\Delta(G) \geq 3$ and $n \geq 3$, then*

$$\gamma_t(G) < \gamma_{t_n}(G).$$

Theorem 17. *If G is a fuzzy graph with $\Delta(G) \geq n \geq 2$, then $\gamma_t(G) + n - 1 \leq \gamma_{t_n}(G)$.*

Proof. Let S be a minimum n-total dominating set of a fuzzy graph G.

Let $u \in V$ and $\{v_1, v_2, \ldots, v_n\}$ be distinct vertices in S which dominates u.

Since $\Delta(G) \geq n \geq 2$, $V - S \neq \emptyset$, there exists an n-dominating set, which does not contain a vertex of degree Δ. Since S is an n-total dominating set, each vertex in V is dominated by at least one vertex in $S - \{v_2, \ldots, v_n\}$.

$\therefore$ since u dominates each vertex in $\{v_2, \ldots, v_n\}$, we know that $S' = S - \{v_2, \ldots, v_n\}$ is a dominating set of G.

$$\begin{aligned} \gamma_t(G) &\leq \left|S'\right| = \gamma_{t_n}(G) - [n-1] \\ &= \gamma_{t_n}(G) - n + 1 \\ \gamma_t(G) &\leq \gamma_{t_n}(G) - n + 1 \\ \Rightarrow \quad \gamma_t(G) + n - 1 &\leq \gamma_{t_n}(G). \end{aligned}$$ □

References

[1] Berge, C., Graphs and Hyper Graphs, North-Holland, Amsterdam, 309, (1973).

[2] Bhutani, K.R., and Rosenfeld, A., Strong arcs in fuzzy graphs, Information Sciences, **152** (2003) 319-322.

[3] Cockayne, E.J. and Hedetniemi, S.T., Towards a theory of domination in Graphs, Networks, **7** (1977) 247-261.

[4] Kulli, V.R., On n-domination number in graphs, In: Graph Theory, Combinatorics, Algorithms and Applications, SIAM, Philadelphia, PA (1991) 319-324.

[5] Nagoorgani, A. and Chandrasekaran, V.T., Domination in fuzzy graph, Advances in Fuzzy Sets and Systems, **1(1)** (2006) 17-26.

[6] Ore, Theory of Graphs, Amer. Math. Soc. Colloq. Publ. (Amer. Math. Soc., Providence, RI) **38**, (1962).

[7] Somasundaram, A., Somasundaram, S., Domination in fuzzy graph-I, Pattern Recognition Letters, **19** (1998) 787-791.