A Study of Diabetes Using Hexagonal Fuzzy Number Matrices

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Abstract

The important and difficult process of diabetes confirmation has promoted attempts to model it with the use of hexagonal fuzzy number. In this paper, we calculate the four different indications using occurrence relation ($R_o$) and conformability relation ($R_c$) are determined from expert medical documentation and observation of related patients with diabetes. Also this paper used to formulate the problem by comparing the max-min and proposed operations of hexagonal fuzzy numbers.

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1 Introduction

The idea of fuzzy concept was first used in a scientific sense by the computer scientist Lotfi A. Zadeh\textsuperscript{[11]} in 1965. Some models to understand and teach process of medical diagnosis using fuzzy set theory vary in the degree, to which they attempt to deal with different complication aspects as, 1. Relative important of symptoms, 2. Symptom patterns of disease stage, 3. Relation between disease themselves, 4. Stages of hypothesis formation, 5. Preliminary and final diagnosis within diagnosis process. These models also form the basis for computerized medical expert system, which is useful for physician in the diagnosis of some specified category of disease, inspires us to make a model related to disease DIABETES, which is commonly observed in india. The fuzzy matrices were introduced first time by Thomason\textsuperscript{[9]},
who discussed the convergence of powers of fuzzy matrix. Shyamal and Pal [4] introduced
the concept triangular fuzzy matrix. Stephen Dinagar and Latha [7], proposed
Type-2 Triangular fuzzy matrices in Medical Diagnosis. Sophia Porchelvi et al. [5]
introduced the concept of an application of fuzzy matrices in Medical Diagnosis.
Vivek Raich et al. [10] established the application of fuzzy matrix in the study of
diabetes. Stephen Dinagar and Harinarayana [8], proposed the modified form of
hexagonal fuzzy number for convexity condition which were developed using matrix
Stephen Dinagar and Anbalagan [6] presented Type-2 fuzzy numbers in medical di-
agnosis. In this paper, we give some basic definitions recall hexagonal fuzzy number
and its operations. In section 3, we have reviewed the definition of hexagonal fuzzy
matrix and its operations. In section 4, overview of diabetes in medical science.
In section 5, diabetes diagnosis problem using hexagonal fuzzy matrix relations are
discussed. Finally section 6, we present the conclusion of this work.

2 Hexagonal Fuzzy Number

Definition 1. (Hexagonal Fuzzy Number)
A fuzzy number on \( \tilde{A}_h \) is hexagonal fuzzy number denoted by \( \tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \),
where \( a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \) are real number satisfying
\( a_2 - a_1 \leq a_3 - a_2 \) and \( a_5 - a_4 \geq a_6 - a_5 \) and its membership function \( \mu_{\tilde{A}_h} (x) \) is given by

\[
\mu_{\tilde{A}_h} (x) = \begin{cases} 
0 & \text{if } x \leq a_1 \\
\frac{1}{2} \left( \frac{x - a_1}{a_2 - a_1} \right) & \text{if } a_1 \leq x \leq a_2 \\
\frac{1}{2} + \frac{1}{2} \left( \frac{x - a_3}{a_3 - a_2} \right) & \text{if } a_1 \leq x \leq a_2 \\
1 & \text{if } a_1 \leq x \leq a_2 \\
1 - \frac{1}{2} \left( \frac{x - a_5}{a_6 - a_5} \right) & \text{if } a_1 \leq x \leq a_2 \\
\frac{1}{2} \left( \frac{a_6 - x}{a_6 - a_5} \right) & \text{if } a_1 \leq x \leq a_2 \\
0 & \text{if } x \leq a_1.
\end{cases}
\]

Remark 2. The hexagonal fuzzy number \( \tilde{A}_h \) becomes trapezoidal fuzzy number
if \( a_2 - a_1 = a_3 - a_2 \) and \( a_5 - a_4 = a_6 - a_5 \).
The hexagonal fuzzy number \( \tilde{A}_h \) becomes non-convex fuzzy number if
\( a_2 - a_1 > a_3 - a_2 \) and \( a_5 - a_4 < a_6 - a_5 \).

2.1 Arithmetic Operations On Hexagonal Fuzzy Numbers (HFNs)
The arithmetic operations between hexagonal fuzzy numbers (HFNs) are proposed
given below.

2.2 Min-Max Arithmetic Operations
Let us consider \( \tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \) and \( \tilde{B}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \) be two
hexagonal fuzzy numbers then,
2.3 Our Proposed Arithmetic Operations

Let us consider \( \tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \) and \( \tilde{B}_h = (a_1, a_2, a_3, a_4, a_5, a_6) \) be two hexagonal fuzzy numbers then,

(i) Addition:
\[
\tilde{A}_h (+) \tilde{B}_h = \max (a_i, b_i), \text{forall} i = 1, 2 \ldots 6
\]

(ii) Subtraction:
\[
\tilde{A}_h (-) \tilde{B}_h = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1). 
\]

(iii) Multiplication:
\[
\tilde{A}_h (\times) \tilde{B}_h = \min (a_i, b_i), \text{forall} i = 1, 2 \ldots 6 
\]

(iv) Division:
\[
\tilde{A}_h (\div) \tilde{B}_h = \left( \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}, \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} \right).
\]

(v) Scalar Multiplication:
\[
k\tilde{A}_h = \left\{ \begin{array}{ll}
(ka_1, ka_2, ka_3, ka_4, ka_5, ka_6) & \text{if} k \geq 0 \\
(ka_6, ka_5, ka_4, ka_3, ka_2, ka_1) & \text{if} k \leq 0 
\end{array} \right.
\]

Definition 3. (Ranking Function)
We define a ranking function \( \tilde{R} : F(R) \to R \) which maps each fuzzy numbers to real line \( F(R) \) represented the set of all hexagonal fuzzy numbers. If \( R \) be any linear ranking functions,
\[
\tilde{R} (\tilde{A}_h) = \left( \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} \right).
\]
3 Hexagonal Fuzzy Matrices (HFMs)

Definition 4. A hexagonal fuzzy matrix of order $m \times n$ is defined as $\hat{A} = (\tilde{a}_{hij})_{m \times n}$ where $(\tilde{a}_{hij}) = (\tilde{a}_{ij1}, \tilde{a}_{ij2}, \tilde{a}_{ij3}, \tilde{a}_{ij4}, \tilde{a}_{ij5}, \tilde{a}_{ij6})$ is the $ij^{th}$ element of $\hat{A}$.

3.1 Operations on Hexagonal Fuzzy Matrices (HFMs)

Let $\hat{A} = (\tilde{a}_{hij})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hij})_{m \times n}$ be two HFMs of same order. Then we have the following:

1. $\hat{A} + \hat{B} = (\tilde{a}_{hij})_{m \times n} = \left((\tilde{a}_{hij}) + (\tilde{b}_{hij})\right)$

2. $\hat{A} - \hat{B} = (\tilde{a}_{hij})_{m \times n} = \left((\tilde{a}_{hij}) - (\tilde{b}_{hij})\right)$

3. For $\hat{A} = (\tilde{a}_{hij})_{m \times n}$ and $\hat{B} = (\tilde{b}_{hij})_{m \times n}$ then $\hat{A}\hat{B} = (\tilde{c}_{hij})_{m \times k}$ where $(\tilde{c}_{hij})_{m \times k} = \sum_{p=1}^{n} (\tilde{a}_{hjp})(\tilde{b}_{hpj})$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, k$

4. $\hat{A}^T$ or $\hat{A}' = (\tilde{a}_{hji})$

5. $k\hat{A} = (k(\tilde{a}_{hij}))$, where $k$ is scalar.

4 Survey of Diabetes in Medical Science

4.1 Diabetes Overview

People with diabetes either do not produce enough insulin (type-1 diabetes) or cannot use insulin properly (type-2 diabetes) or both. In diabetes, glucose in the blood cannot move into cells and stays in the blood. This not only harms the cells that need the glucose for fuel, but also harms certain organs and tissues exposed to high glucose levels.

Type-I Diabetes

The body stops producing insulin or produce too little insulin to regulate body glucose level. Type-1 diabetes is typically recognized in childhood or adolescence.

Type-II Diabetes

The pancreas secretes insulin, but the body is partially or completely unable to use the insulin. This is sometimes referred to as insulin resistance by secreting more and more insulin people with insulin resistance develop diabetes type-II when they do not continue to secrete enough insulin to cope with the higher demands.

At least 90% of patients with diabetes have type-II diabetes. It is typically recognized in adulthood, usually after age 45 years. It used to be called adult-onset diabetes mellitus or non-insulin-dependent diabetes mellitus. More than half of all people with type-II diabetes require insulin to control their blood sugar at same point in the course of their illness. Beside this there are other forms of diabetes like Gestational diabetes, Pre-Adiabetes etc.
4.2 Complications of diabetes

Both forms of diabetes ultimately lead to high blood sugar levels, a condition called hyperglycemia. Over a long period of time, hyperglycemia damages the retina of the eyes, the kidneys, the nerves, and the blood vessels. The body tries to get rid of the excess blood sugar by eliminating it in the urine. This increases the amount of urine significantly and often leads to dehydration, so severe that it can cause seizures, coma, even death. This syndrome typically occurs in people with type-II diabetes who are not controlling their blood sugar or have become dehydrated or have stress, injury, stroke or medication like steroids. In the short run, diabetes can contribute to a number of acute (short-lived) medical problems.

In this paper, four types of diabetic diseases are taken and using the hexagonal fuzzy matrix relations how a patient can be diagnosed is explained in the next section.

5 Case Study in Confirmation of the Diabetes Using Hexagonal Fuzzy Relational Matrices

In this section, we compare the diabetes confirmation due to the operations of 2.6.1 and 2.6.2. Also, the results happen in various analysis, the patient who affects the diabetes confirmation for the same symptoms deduct it and not affect the diabetes but the general symptom spread on the patients.

The model proposes two types of relations to exist between symptoms and disease. An occurrence relation $R_o$ provides knowledge about the tendency or frequency of appearance of symptoms when the specific disease is present, i.e., how often does the symptoms occur with disease. A conformability relation $R_c$ describes the discriminating power of the symptoms to confirm the presence of the disease, i.e., how strongly does the symptoms confirm disease. The distinction between occurrence and conformability is important because a symptom may occur with given disease but may be commonly occur with several other diseases.

5.1 Occurrence and Conformability Using Hexagonal Fuzzy Matrix Relation

The study of occurrence relation $R_o$ and conformability $R_c$ in medical science using hexagonal fuzzy matrices is useful because a symptom may likely to occur with a given disease but may also commonly occur with several other diseases, therefore limiting its power as a discriminating factor among them is important on the other hand, another symptom may be relatively rare with a given disease, but its presence may never certainly confirm the presence of the disease.

5.2 Confirmation of the Diagnosis Using Hexagonal Fuzzy Matrix Relation

Here we can explain with an example given below. Let us take, $S=$ Crisp set of all symptoms, $D=$ Set of all diseases, $P=$ Set of all patients. Here, $S= \{s_1, s_2, s_3, s_4\}$ where,
\[ s_1 = \text{Fatigue, weight loss, polydipsia, polyuria, irritations, extreme lethargy, weight around waist, high triglyceride (> 150), low level HDL (< 40 \text{formen, < 50 for women})}, \]
\[ s_2 = \text{Polyphagia (Excessive eating), Nausea and vomiting, Dehydration, abdominal pain, Low Blood pressure}, \]
\[ s_3 = \text{Loss of appetite, increased heart rate, High blood pressure, Blurry vision and } s_4 = \text{Loss of appetite, increased heart rate, High blood pressure, Blurry vision}. \]

\[ D = \{d_1, d_2, d_3, d_4\}, \]

where
\[ d_1 = \text{Diabetic general}, d_2 = \text{DKA (diabetes Ketoacidosis)}, d_3 = \text{Diabetic Nephropathy} \]
and \[ d_4 = \text{Diabetic retinopathy}. \]

\[ P = \{p_1, p_2, p_3, p_4\} \]
respectively.

Hexagonal fuzzy matrix for occurrence relation is \( \hat{R}_o = S \times D. \)

\[
\begin{align*}
\hat{R}_o &= \\
&= \begin{bmatrix}
-1,0,1,2,4,6 & 2,4,6,8,10,12 & -1,0,1,2,4,6 & -1,0,1,2,4,6 \\
-2,1,0,1,2,6 & -3,2,1,1,2,3 & -1,0,1,2,4,6 & -1,0,1,2,4,6 \\
-2,1,0,1,2,6 & -1,1,2,4,8,10 & 10,10,10,10,10,10 & -2,1,0,1,2,6 \\
-2,1,0,1,2,6 & -2,1,0,1,2,6 & -3,2,1,1,2,3 & 10,10,10,10,10,10
\end{bmatrix}
\end{align*}
\]

Hexagonal fuzzy matrix for conformative relation is \( \hat{R}_c = S \times D \)
\[
\hat{R}_c = \\
= \begin{bmatrix}
-2,1,0,1,2,6 & 1,3,4,5,8,9 & -2,1,0,1,2,6 & -1,0,1,2,4,6 \\
-1,0,1,2,4,6 & -2,1,0,1,2,6 & -2,1,0,1,2,6 & -1,0,1,2,4,6 \\
-2,1,0,1,2,6 & -1,0,1,2,4,6 & 10,10,10,10,10,10 & -2,1,0,1,2,6 \\
-2,1,0,1,2,6 & -2,1,0,1,2,6 & -1,0,1,2,4,6 & 10,10,10,10,10,10
\end{bmatrix}
\]

Now assume a HFM relation \( \hat{R}_s \) specifying the degree of presence of symptoms \( s_1, s_2, s_3 \) and \( s_4 \) for four patients \( p_1, p_2, p_3 \) and \( p_4 \) as follows: this indicate the degree to which the symptoms is present in patient P.

\[
\hat{R}_s = \\
= \begin{bmatrix}
-2,1,0,1,2,6 & -2,1,0,1,2,6 & 2,4,6,8,10,12 & -1,0,1,2,4,6 \\
-2,1,0,1,2,6 & -3,2,1,1,2,3 & -2,1,0,1,2,6 & -2,1,0,1,2,6 \\
-1,0,1,2,4,6 & -3,2,1,1,2,3 & 10,10,10,10,10,10 & -2,1,0,1,2,6 \\
-2,1,0,1,2,6 & -2,1,0,1,2,6 & -3,2,1,1,2,3 & 10,10,10,10,10,10
\end{bmatrix}
\]

Case(i) Using hexagonal fuzzy matrix relation \( \hat{R}_s, \hat{R}_o \) and \( \hat{R}_c \) can be calculated four different indication relations as below with the aid of 2.6.1 arithmetic operations of HFNs

The occurrence indication relation: \( \hat{R}_1 \) calculated by \( \hat{R}_1 = \hat{R}_s \ast \hat{R}_o \)

By using 2.6.1 arithmetic operations of hexagonal fuzzy number we get,
\[
\hat{R}_1 = \\
\text{Using the ranking function the above } \hat{R}_1 \text{ is}
\]
\[
\hat{R}_1 = \\
\text{The conformability indication relation: } \hat{R}_2 \text{ is calculated by } \hat{R}_2 = \hat{R}_s \ast \hat{R}_c
\]
\[
\hat{R}_2 = \\
\text{Using the ranking function the above } \hat{R}_2 \text{ is}
\]

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The non-occurrence indication relation: $\hat{R}_3$ is calculated by $\hat{R}_3 = \hat{R}_s \ast (\hat{C} - \hat{R}_o)$

Where $\hat{C}$ is the constant hexagonal fuzzy matrix of order 4, elements are present in the highest hexagonal fuzzy number in all the hexagonal fuzzy matrix relation, (i.e), $(10, 10, 10, 10, 10)$.

Now, we calculate $\hat{C} - \hat{R}_o = \hat{R}_3 = \hat{R}_s = \hat{R}_o$

Using the ranking function the above $\hat{R}_3$ is $\hat{R}_3 = \hat{R}_4$ is calculated by $\hat{R}_4 = (\hat{C} - \hat{R}_s) \ast \hat{R}_o$

$\hat{C} - \hat{R}_s = \hat{R}_4 =$
Using the ranking function the above \( \hat{R}_4 \) is
\[
\hat{R}_4 =
\]

**Case(ii)**

Using hexagonal fuzzy matrix relation \( \hat{R}_s \), \( \hat{R}_o \) and \( \hat{R}_c \) can be calculated four different indication relations as below with the aid of 2.6.2 arithmetic operations of HFNs.

The occurrence indication relation: \( \hat{R}_1 \) calculated by \( \hat{R}_1 = \hat{R}_s \ast \hat{R}_o \)

By using 2.6.2 arithmetic operations of hexagonal fuzzy number we get,
\[
\hat{R}_1 =
\]

Using the ranking function the above \( \hat{R}_1 \) is
\[
\hat{R}_1 =
\]

The conformability indication relation: \( \hat{R}_2 \) is calculated by \( \hat{R}_2 = \hat{R}_s \ast \hat{R}_c \)

Using the ranking function the above \( \hat{R}_2 \) is
\[
\hat{R}_2 =
\]

The non-occurrence indication relation: \( \hat{R}_3 \) is calculated by \( \hat{R}_3 = \hat{R}_s \ast \left( \hat{C} - \hat{R}_o \right) \)

Where \( \hat{C} \) is the constant hexagonal fuzzy matrix of order 4, elements are present.
in the highest hexagonal fuzzy number in all the hexagonal fuzzy matrix relation, 
(i.e), (10, 10, 10, 10, 10).

Now, we calculate

\[ \hat{R}_3 = \]

Using the ranking function the above \( \hat{R}_3 \) is

\[ \hat{R}_3 = \]

The non-symptom indication relation: \( \hat{R}_4 \) is calculated by \( \hat{R}_4 = (\hat{C} - \hat{R}_s) \ast \hat{R}_o \)

\[ \hat{R}_4 = \]

Using the ranking function the above \( \hat{R}_4 \) is

\[ \hat{R}_4 = \]

6 Conclusion

Among the two cases, we draw the following conclusions In the same example, to compare the two cases by using the 2.6.1 and 2.6.2, the results of \( \hat{R}_1 \) and \( \hat{R}_2 \) indicate
that both disease $d_3$ and $d_4$ are suitable diagnosis to affect in diabetes of patients $p_3$ and $p_4$ respectively, whereas in case(i) $\hat{R}_3$ and $\hat{R}_4$ indicates $p_2$ and $p_4$ not having the disease $d_3$ and case(ii), compare with the values of $\hat{R}_3$ and $\hat{R}_4$ is highest extreme value of matrix i.e $p_2$ not having the disease of $d_1$. In this paper, A comparison have been made by taking two different fuzzy operations in the same illustration. This paper is more useful to deduct the conformation of diabetes for proper diagnostic system using Hexagonal Fuzzy Matrix relation in medical science.

References


