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The FM/FM/1 queue with single working vacation and set-up times

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Abstract

Using Fuzzy analysis is technique, a $FM/FM/1$ queue with single working vacation and set-up times is discussed in this paper. Then non-linear programming technique for the mean queue length, mean waiting time of a customer, server is in a set-up period and server is in a close-down period. Finally, numerically results are discussed.

AMS Subject Classification: 68M20, 90B22

Key Words and Phrases: Fuzzy single working vacation; mean queue length; mean waiting time; set-up period and close-down period

1 Introduction

During last two decades, vacation queue have been investigated extensively because of their application in computer systems, communication networks, production managing and so forth. In various vacation queue models, the server completely stops serving customers during a vacation, but many perform other supplementary jobs, proposing of various vacation polices provides more flexibility for optimal design and operation control of the system. The details can be seen in the monographs of Takagi (1991) and Tian and Zhang (2006), the surveys of Doshi (1996) and Teghem(1986).

Working vacation is a kind of semi-vacation policy and is also a new vacation policy that was first introduced by Servi and Finn (2002). Part of the service ability keeps the system operating in a lower speed rather then completely stopping the service during a working vacation. If service speed degenerates into zero in a working vacation, the vacation queue becomes a classical vacation queue model. Therefore the working vacation queue is the generalization of the classical vacation queue. Moreover, Kim et al (2003), Wu and Takagi (2006) generalized the work of the Servi and Finn (2002) a $M/G/1$ queue with multiple working vacation.

G.Kannadasan and N.Sathiyamoorthi (2017) investigate the $FM/FM/1$ queue with single Working vacation. They obtain some system characteristic such as the number of customer in the system in study-state, the virtual time of a customer in the system, the server is in idle period, the server is in regular busy period. G.Kannadasan, et.al (2017) also gave Analysis for the $FM/FM/1$ queue with multiple working vacation with N-Policy, using non-linear programming method. They obtain some performance measure of interest such as membership of mean queue length, mean waiting time, with N=2. K.Julia Rose Mery And T.Gokilavani (2011) investigate the performance measure of a $M^X/M/1$ multiple working vacation (MWV) queuing model in a fuzzy environment. Mary George and Jayalakshmi (2013) studied on the analysis of $G/M(n)/1/k$ queuing system with multiple exponential vacations and vacations of fuzzy length. R.Ramesh et.al (2013) constructs the membership function of the system characteristics of a batch-arrival queuing system with multiple servers, in which the batch-arrival rate and customer service rate are all fuzzy numbers.

In this paper, we investigate a $FM/FM/1$ queue with single working vacation and set-up times, if set up time equal to zero, our model becomes a $FM/FM/1$ queue with single working vacation : furthermore, if working vacation time equal zero at the same time, this model becomes as $FM/FM/1$ queue. In section 2, we describe the queue model, section 3 and 4 respectively discuss the fuzzy model with mean queue length, mean waiting time, server is in a closed-down period, the server is in a set-up period are studied in fuzzy environment. In section 5 includes numerical study about the performance measures. Finally, conclusions are gained.

2 The crisp model

We introduce the policy of single working vacation and set-up times into a classical $M/M/1$ queue with arrival rate μ_ν and service rate μ_b. The server begins a working vacation of random length V at the instants when the queue becomes empty, and vacation duration, V follows an exponential distribution with parameter θ. During a working vacation, arriving customers are served at a rate of $\mu_\nu(\mu_\nu < \mu_b)$ according to arrival order. When a vacation ends, if there are customers in the queue, the server changes service rate from $\mu_\nu \ to \ \mu_b$, and a regular busy period starts; otherwise, the server begins a closed-down period. During a closed-down period, an arriving customer cannot be served immediately and experiences a period of set-up time, set-up duration S follows an exponential distribution with parameter α and a regular busy period starts after a set-up period. Working vacation V is a operating period in a lower speed. When the number of customers in the system is relatively few, we set a lower speed operating period and a closed-down period to economies operating cost together with serving customers; these two periods have essential differences because customers can be served in the former period and not do so in the

latter period. Therefore, this working vacation and set-up time policy has practical significance in optimal design of the system.
We assume that inter-arrival times, service times, working vacation times and set-up times are mutually independent. In addition, the service order is First In First Out (FIFO).

$$J(t) = \begin{cases} 0, & \text{the system is in a working vacation period at time } t, \\ 1, & \text{the system is in a set-up period or closed-down period at time } t, \\ 2, & \text{the system is in a regular busy period at time } t. \end{cases}$$

Then $Q(t), J(t)$ is a Markov process with the state space:

$$\Omega = \{(0,0),(0,1)\} \cup (k,j) : k \geq 1, j = 0,1,2.$$

where state $(k,0), k \geq 0$ indicates that the system is in a working vacation period and there are k customers in the queue; state $(k,1), k \geq 1$ indicates that the system is in a set-up period and there are k customers in the queue; state $(0,1)$ indicates that the system is in a closed-down period; state $(k,2), k \geq 1$ indicates that the system is in a regular busy period.

3 The model in fuzzy environment

In this section, the arrival rate λ, service rate μ_b, arriving customers are served at a rate of μ_ν, working vacation θ are assumed to be fuzzy numbers respectively. Now
$\bar{\lambda} = \{w, \mu_{\bar{\lambda}}(w); w \in S(\bar{\lambda})\}$, $\bar{\beta} = \{x, \mu_{\bar{b}}(x); x \in S(\bar{\beta})\}$, $\bar{\nu} = \{y, \mu_{\bar{\nu}}(y); y \in S(\bar{\nu})\}$
and

$$\bar{\theta} = \{z, \mu_{\bar{\theta}}(z); z \in S(\bar{\theta})\}.$$

where $S(\bar{\lambda})$, $S(\bar{\beta})$, $S(\bar{\nu})$ and $S(\bar{\theta})$ are the universal sets of the arrival rate, service rate , arriving customers are served at a rate of μ_ν, and working vacation respectively. It defines $f(w,x,y,z)$ as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function $f(\bar{\lambda},\bar{\beta},\bar{\nu},\bar{\theta})$.
Applying Zadeh's extension principle (1978) the membership function of the performance measure $f(\bar{\lambda},\bar{\beta},\bar{\nu},\bar{\theta})$ can be defined as:

$$\mu_{\bar{f}(\bar{\lambda},\bar{\beta},\bar{\nu},\bar{\theta})}(H) = \sup_{\substack{w\in S(\bar{\lambda}) \\ x\in S(\bar{\beta}) \\ y\in S(\bar{\nu}) \\ z\in S(\bar{\theta})}} \{\mu_{\bar{\lambda}}(w), \mu_{\bar{\beta}}(x), \mu_{\bar{\nu}}(y), \mu_{\bar{\theta}}(z)/H = f(w,x,y,z)\} \tag{1}$$

If the α- cuts of $f(\bar{\lambda},\bar{\beta},\bar{\nu},\bar{\theta})$ degenerate to some fixed value then, the system performance is a crisp number. Otherwise it is a fuzzy number.
The mean queue length

$$E(Q) = \frac{\lambda}{\mu_b - \lambda} + \left[\left[\theta^3\left(\frac{\alpha}{\alpha+\lambda}\right)^3\left(\frac{\mu_b-\lambda}{\mu_b}\right)^2\left(\frac{(2\mu_v - L)^4}{(2\mu_v)^2\mu_b(\lambda+\alpha)}\right)\right]^{-1}$$

$$\times\left[\left(\frac{\theta}{\mu_b}\right)+\left(\frac{L\times(\mu_b-\lambda)-\mu_b\lambda+\lambda^2}{2\mu_v-\mu_b^2}\right)+\left(\frac{\theta.2\mu_v-L}{\lambda 2\mu_v}\frac{\mu_b-\lambda+\alpha}{(\lambda+\alpha)\mu_b}\right)\right]\left(\frac{\alpha}{\lambda+\alpha}\right)$$
$$+\frac{L}{2\mu_v-L}.\left[\frac{L-\lambda}{2\mu_v-\mu_b}+\frac{\theta.L}{2\mu_v\mu_b(2\mu_v-L)}\right]\left(\frac{\alpha}{\lambda+\alpha}\right)+\left(\frac{2\lambda-\lambda^2}{\alpha}\right)\times$$
$$\left(\frac{\theta(2\mu_v-L)}{2\mu_v-(\lambda+\alpha)}\right)\Bigg].$$

where $L=\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_\nu}$.

The mean waiting time

$$E(W)=\frac{1}{\mu_b-\lambda}+\frac{1}{\lambda}\left[\left[\theta^3\left(\frac{\alpha}{\alpha+\lambda}\right)^3\left(\frac{\mu_b-\lambda}{\mu_b}\right)^2\left(\frac{(2\mu_v-L)^4}{(2\mu_v)^2\mu_b(\lambda+\alpha)}\right)\right]^{-1}\right.$$
$$\times\left[\left(\frac{\theta}{\mu_b}\right)+\left(\frac{L\times(\mu_b-\lambda)-\mu_b\lambda+\lambda^2}{2\mu_v-\mu_b^2}\right)+\left(\frac{\theta.2\mu_v-L}{\lambda 2\mu_v}\frac{\mu_b-\lambda+\alpha}{(\lambda+\alpha)\mu_b}\right)\right]\left(\frac{\alpha}{\lambda+\alpha}\right)$$
$$+\frac{L}{2\mu_v-L}.\left[\frac{L-\lambda}{2\mu_v-\mu_b}+\frac{\theta.L}{2\mu_v\mu_b(2\mu_v-L)}\right]\left(\frac{\alpha}{\lambda+\alpha}\right)+\left(\frac{2\lambda-\lambda^2}{\alpha}\right)\times$$
$$\left.\left(\frac{\theta(2\mu_v-L)}{2\mu_v-(\lambda+\alpha)}\right)\right].$$

The server is in a close-down period

$$P_0=\left[\frac{1}{2\mu_v^2-2\mu_v.L}+\frac{\theta}{\lambda}+\frac{\theta}{\alpha\mu_b(\mu_b-\lambda)}+\frac{\theta\mu_b(2\mu_v+L^2-L)(\mu_b-L)}{4\mu_v^2(2\mu_b\mu_v-L)^2(\mu_b-\lambda)}\right]\times\frac{\theta}{\lambda}.$$

The server is in a set-up period

$$P_1=\left[\frac{1}{2\mu_v^2-2\mu_v.L}+\frac{\theta}{\lambda}+\frac{\theta}{\alpha\mu_b(\mu_b-\lambda)}+\frac{\theta\mu_b(2\mu_v+L^2-L)(\mu_b-L)}{4\mu_v^2(2\mu_b\mu_v-L)^2(\mu_b-\lambda)}\right]\times\left[\frac{\lambda\theta}{\alpha\mu_b-\alpha\lambda}\right.$$
$$\left.+\frac{\theta\mu_b(2\mu_v+L^2-L)(\mu_b-L)}{4\mu_v^2(2\mu_b\mu_v-L)^2(\mu_b-\lambda)}\right].$$

Under the study state condition $\rho=\frac{\lambda}{\mu_b}$. We obtain the membership function of some performance measure's namely the mean queue length, the mean waiting time, the server is in a close-down period and the server is in a set-up period.
For the system in terms of the membership functions are:

$$\mu_{\overline{E(L)}}(A)=\sup_{\substack{w\in S(\bar\lambda)\\ x\in S(\bar\beta)\\ y\in S(\bar\nu)\\ z\in S(\bar\theta)}}\left\{\mu_{\bar\lambda}(w),\mu_{\bar\beta}(x),\mu_{\bar\nu}(y),\mu_{\bar\theta}(z)/A\right\},\tag{2}$$

Where

$$A=\frac{w}{x-w}+\left[\left[z^3\left(\frac{\alpha}{\alpha+w}\right)^3\left(\frac{x-w}{x}\right)^2\left(\frac{(2y-L)^4}{(2y)^2x(w+\alpha)}\right)\right]^{-1}\right.$$
$$\times\left[\left(\frac{z}{x}\right)+\left(\frac{L\times(x-w)-xw+w^2}{2y-x^2}\right)+\left(\frac{2yz-L}{2wy}.\frac{x+\alpha}{(w+\alpha)x}\right)\right]\left(\frac{\alpha}{w+\alpha}\right)$$

$$+\frac{L}{2y-L}.\left[\frac{L-w}{2y-x}+\frac{z.L}{2xy(2y-L)}\right]\left(\frac{\alpha}{w+\alpha}\right)+\left(\frac{2w-w^2}{\alpha}\right)\times\left(\frac{2yz-zL}{2y-(w+\alpha)}\right)\Bigg].$$

$$\mu_{\overline{E(W)}}(B)=\sup_{\substack{w\in S(\bar\lambda)\\x\in S(\bar\beta)\\y\in S(\bar\nu)\\z\in S(\bar\theta)}}\left\{\mu_{\bar\lambda}(w),\mu_{\bar\beta}(x),\mu_{\bar\nu}(y),\mu_{\bar\theta}(z)/B\right\},\tag{3}$$

Where

$$B=\frac{1}{x-w}+\frac{1}{w}\left[\left[z^3\left(\frac{\alpha}{\alpha+w}\right)^3\left(\frac{x-w}{x}\right)^2\left(\frac{(2y-L)^4}{(2y)^2x(w+\alpha)}\right)\right]^{-1}\right.$$
$$\times\left[\left(\frac{z}{x}\right)+\left(\frac{L\times(x-w)-xw+w^2}{2y-x^2}\right)+\left(\frac{2yz-L}{2wy}.\frac{x+\alpha}{(w+\alpha)x}\right)\right]\left(\frac{\alpha}{w+\alpha}\right)$$
$$+\frac{L}{2y-L}.\left[\frac{L-w}{2y-x}+\frac{z.L}{2xy(2y-L)}\right]\left(\frac{\alpha}{w+\alpha}\right)+\left(\frac{2w-w^2}{\alpha}\right)\times\left(\frac{2yz-zL}{2y-(w+\alpha)}\right)\Bigg].$$

$$\mu_{\overline{P_0}}(C)=\sup_{\substack{w\in S(\bar\lambda)\\x\in S(\bar\beta)\\y\in S(\bar\nu)\\z\in S(\bar\theta)}}\left\{\mu_{\bar\lambda}(w),\mu_{\bar\beta}(x),\mu_{\bar\nu}(y),\mu_{\bar\theta}(z)/C\right\},\tag{4}$$

Where

$$C=\left[\frac{1}{2y^2-2yL}+\frac{z}{w}+\frac{z}{\alpha x(x-w)}+\frac{zx(2y+L^2-L)(x-L)}{4y^2(2xy-L)^2(x-w)}\right]\times\frac{z}{w}.$$

$$\mu_{\overline{P_1}}(D)=\sup_{\substack{w\in S(\bar\lambda)\\x\in S(\bar\beta)\\y\in S(\bar\nu)\\z\in S(\bar\theta)}}\left\{\mu_{\bar\lambda}(w),\mu_{\bar\beta}(x),\mu_{\bar\nu}(y),\mu_{\bar\theta}(z)/D\right\},\tag{5}$$

Where

$$D=\left[\frac{1}{2y^2-2yL}+\frac{z}{w}+\frac{z}{\alpha x(x-w)}+\frac{zx(2y+L^2-L)(x-L)}{4y^2(2xy-L)^2(x-w)}\right]\times\left(\frac{z}{w}\right)$$
$$\times\left[\frac{wz}{\alpha(x-w)}+\frac{zx(2y+L^2-L)(x-L)}{4y^2(2xy-L)^2(x-w)}\right].$$

Using the fuzzy analysis technique explain, we can find the membership of $\overline{E(L)}$, $\overline{E(W)}$, $\overline{P_0}$ and $\overline{P_1}$ as a function of the parameter α, thus the α-cut approach can be used to develop the membership function of $\overline{E(L)}$, $\overline{E(W)}$, $\overline{P_0}$ and $\overline{P_1}$.

4 Performance of measure

The mean queue length

Based on Zadeh's extension Principle $\mu_{E(L)}(A)$ is the superimum of minimum over $\mu_{\bar\lambda}(w)$, $\mu_{\bar\beta}(x)$, $\mu_{\bar\nu}(y)$, $\mu_{\bar\theta}(z)\}$ to satisfying $\mu_{E(L)}(A)=\alpha$, $0<\alpha\leq 1$.
The following four cases arise:
Case$(i)\mu_{\bar\lambda}(w)=\alpha,\mu_{\bar\beta}(x)\geq\alpha,\mu_{\bar\nu}(y)\geq\alpha,\mu_{\bar\theta}(z)\geq\alpha,$

Case$(ii)\mu_{\bar{\lambda}}(w) \geq \alpha, \mu_{\bar{\beta}}(x) = \alpha, \mu_{\bar{\nu}}(y) \geq \alpha, \mu_{\bar{\theta}}(z) \geq \alpha,$
Case$(iii)\mu_{\bar{\lambda}}(w) \geq \alpha, \mu_{\bar{\beta}}(x) \geq \alpha, \mu_{\bar{\nu}}(y) = \alpha, \mu_{\bar{\theta}}(z) \geq \alpha,$
Case$(iv)\mu_{\bar{\lambda}}(w) \geq \alpha, \mu_{\bar{\beta}}(x) \geq \alpha, \mu_{\bar{\nu}}(y) \geq \alpha, \mu_{\bar{\theta}}(z) = \alpha.$

For case (i) the lower and upper bound of α- cuts of $\overline{E(L)}$ can be obtained through the corresponding parametric non-linear programs:
$[\overline{E(L)}]_\alpha^{L_1} = \min_\Omega \{[A]\}$, & $[\overline{E(L)}]_\alpha^{U_1} = \max_\Omega \{[A]\}$.

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{E(L)}$ for the case $(ii), (iii)$ & (iv). By considering all the cases simultaneously the lower and upper bounds of the α-cuts of $\overline{E(L)}$ can be written as:
$[\overline{E(L)}]_\alpha^{L} = \min_\Omega \{[A]\}$ & $[\overline{E(L)}]_\alpha^{U} = \max_\Omega \{[A]\}$
such that $w_\alpha^L \leq w \leq w_\alpha^U,\ x_\alpha^L \leq x \leq x_\alpha^U,\ y_\alpha^L \leq y \leq y_\alpha^U,\ z_\alpha^L \leq z \leq z_\alpha^U$.
If both $(\overline{E(L)})_\alpha^L$and $(\overline{E(L)})_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(A) = [(E(L))_\alpha^L]^{-1}$ and $R(A) = [(E(L))_\alpha^U]^{-1}$ can be derived from, which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as:

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A), & E(L)_{\alpha=0}^L \leq A \leq E(L)_{\alpha=0}^U \\ 1, & E(L)_{\alpha=1}^L \leq A \leq E(L)_{\alpha=1}^U \\ R(A), & E(L)_{\alpha=1}^L \leq A \leq E(L)_{\alpha=0}^U \end{cases} \tag{6}$$

In the same way as we said before we get the following results.

The mean waiting time

$$\mu_{\overline{E(W)}}(B) = \begin{cases} L(B), & E(W)_{\alpha=0}^L \leq B \leq E(W)_{\alpha=0}^U \\ 1, & E(W)_{\alpha=1}^L \leq B \leq E(W)_{\alpha=1}^U \\ R(B), & E(W)_{\alpha=1}^L \leq B \leq E(W)_{\alpha=0}^U \end{cases} \tag{7}$$

The server is in a close-down period

$$\mu_{\overline{P_0}}(C) = \begin{cases} L(C), & (P_0)_{\alpha=0}^L \leq C \leq (P_0)_{\alpha=0}^U \\ 1, & (P_0)_{\alpha=1}^L \leq C \leq (P_0)_{\alpha=1}^U \\ R(C), & (P_0)_{\alpha=1}^L \leq C \leq (P_0)_{\alpha=0}^U \end{cases} \tag{8}$$

The server is in a set-up period

$$\mu_{\overline{P_1}}(D) = \begin{cases} L(D), & (P_1)_{\alpha=0}^L \leq D \leq (P_1)_{\alpha=0}^U \\ 1, & (P_1)_{\alpha=1}^L \leq D \leq (P_1)_{\alpha=1}^U \\ R(D), & (P_1)_{\alpha=1}^L \leq D \leq (P_1)_{\alpha=0}^U \end{cases} \tag{9}$$

5 Numerical study

The mean queue length

Suppose the arrival rate $\overline{\lambda}$, the service rate $\overline{\mu_b}$, arriving customers are served at a rate $\overline{\mu_\nu}$ and vacation rate $\overline{\theta}$ are assumed to be trapezoidal fuzzy numbers described

by:
$\overline{\lambda} = [11, 12, 13, 14]$, $\overline{\mu} = [41, 42, 43, 44]$, $\overline{\nu} = [61, 62, 63, 64]$ and $\overline{\theta} = [71, 72, 73, 74]$ per hours respectively. Then:

$$\lambda(\alpha) = \min\{w \in s(\bar{\lambda}), \begin{cases} w - 11, & 11 \leq w \leq 12 \\ 11 \quad , & 12 \leq w \leq 13 \\ 14 - w, & 13 \leq w \leq 14 \end{cases} \geq \alpha\},$$

$$\max\{w \in s(\bar{\lambda}), \begin{cases} w - 11, & 11 \leq w \leq 12 \\ 11 \quad , & 12 \leq w \leq 13 \\ 14 - w, & 13 \leq w \leq 14 \end{cases} \geq \alpha\}.$$

That is., $\lambda(\alpha) = [11 + \alpha, 14 - \alpha], \mu(\alpha) = [41 + \alpha, 44 - \alpha]$,
$\nu(\alpha) = [61 + \alpha, 64 - \alpha]$ and $\theta(\alpha) = [71 + \alpha, 74 - \alpha]$.
It is clear that, when $w = w^U_\alpha,\ x = x^U_\alpha,\ y = y^U_\alpha\ \ \&\ \ z = z^U_\alpha$, A attains its maximum value and, when $w = w^L_\alpha, x = x^L_\alpha, y = y^L_\alpha\ \ \&\ \ z = z^L_\alpha$, A attains its minimum value.
From the generated for the given input value of $\overline{\lambda},\ \overline{\mu},\ \overline{\theta}\ \ \&\ \ \overline{\beta}$.
i) For fixed values of $w, x\ \ \&\ \ y,\ A$ decreases as z increase.
ii) For fixed values of $x, y\ \ \&\ \ s,\ A$ decreases as w increase.
iii) For fixed values of $y, z\ \ \&\ \ w,\ A$ decreases as x increase.
iv) For fixed values of $z, w\ \ \&\ \ x,\ A$ decreases as y increase.
The smallest value of occurs, when w-takes its lower bound.
That is, $w = 11+\alpha$ and x, y and z, take their upper bounds given by $x = 44-\alpha, and$ $y = 64 - \alpha,$ and $z = 74 - \alpha$ respectively. And maximum value of $E(L)$ occurs when $w = 14 - \alpha,\ x = 41 + \alpha,\ y = 61 + \alpha,\ z = 71 + \alpha$. If both $E(L)^L_\alpha\ \ \&\ \ E(L)^\alpha_U$ are invertible with respect to $'\alpha'$ then, the left shape function $L(A) = [E(L)_\alpha]^{-1}$ and right shape function $R(A) = [E(L)^L_\alpha]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as:

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A), A_1 \leq A \leq A_2 \\ 1, \quad\quad A_2 \leq A \leq B_3 \\ R(A), A_3 \leq A \leq A_4\ . \end{cases} \tag{10}$$

The values of $A_1, A_2, A_3\ \ \&\ \ A_4$ as obtained from (10) are:

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A), & 0.3581 \leq A \leq 0.6251 \\ 1, & 0.6251 \leq A \leq 17.2943 \\ R(A), & 17.2943 \leq A \leq 23.6014\ . \end{cases} \tag{11}$$

In the same way as we said before we get the following results.

The mean waiting time

$$\mu_{\overline{E(W)}}(B) = \begin{cases} L(B), & 0.5854 \leq B \leq 0.7017 \\ 1, & 0.7017 \leq B \leq 1.2331 \\ R(B), & 1.2331 \leq B \leq 1.6276\ . \end{cases} \tag{12}$$

The server is in a close-down Period

$$\mu_{\overline{P_0}}(C) = \begin{cases} L(C), & 0.4117 \leq C \leq 0.6370 \\ 1, & 0.6370 \leq C \leq 0.7519 \\ R(C), & 0.7519 \leq C \leq 0.8932\ . \end{cases} \tag{13}$$

The server is in a set-up Period

$$\mu_{\overline{P_1}}(D) = \begin{cases} L(D), & 0.2519 \leq D \leq 0.2974 \\ 1, & 0.2974 \leq D \leq 0.4120 \\ R(D), & 0.4120 \leq D \leq 0.5709\ . \end{cases} \tag{14}$$

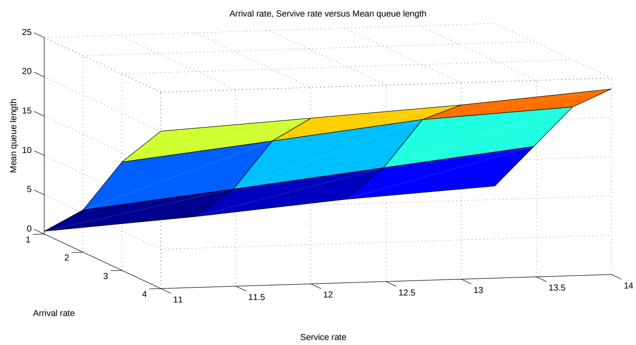

Figure 1: arrival rate, service rate versus the mean queue length

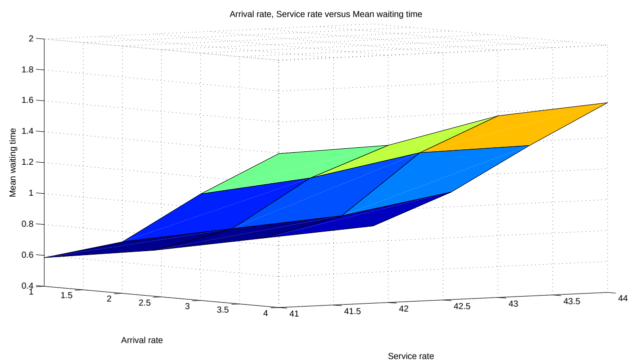

Figure 2: arrival rate, service rate versus the mean waiting time

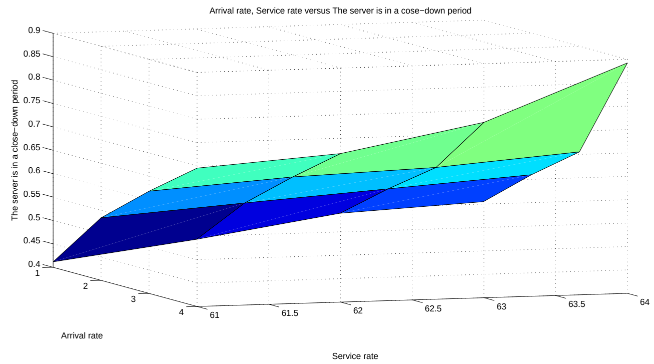

Figure 3: arrival rate, service rate versus the server is in a close-down period

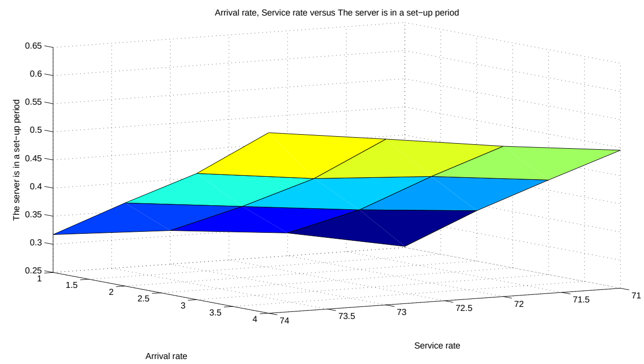

Figure 4: arrival rate, service rate versus the server is in a set-up period

Further by fixed the vacation rate by a crisp value $\bar{\theta} = 71.4$ and $\bar{\nu} = 62.7$ taking arrival rate $\overline{\lambda} = [11, 12, 13, 14]$, service rate both trapezoidal fuzzy numbers the values of the mean queue length are generated and are plotted in the figure 1. It can be observed that as $\overline{\lambda}$ increases the mean queue length increases for the fixed value of the service rate, whereas for fixed value of arrival rate, the mean queue length decreases as service rate increases. Similar conclusion can be obtained for the case $\bar{\theta} = 73.6$ and $\bar{\nu} = 63.8$. Again for fixed values of taking $\overline{\lambda} = [11, 12, 13, 14]$, the graphs of the mean waiting time are drawn in figure 2. The figure shows that as arrival rate increases the mean waiting time also increases, while the mean waiting time decreases as the service rate increases in both the case.

It is also observed from the data generated that the membership value of the mean queue length is 23 and the membership value of the mean waiting time 1.65 when the ranges of arrival rate, service rate, and the vacation rate lie in the intervals$(12, 13.4), (42, 43.4)$,
& $(61.2, 63.7)$ respectively.

6 Conclusion

In this paper, we propose a new fuzzy queue model with single working vacation and set-up times. We are discuses about the performance measure and numerical examples demonstrate the results.

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