International Journal of Pure and Applied Mathematics
Volume 118 No. 20 2018, 2007-2013
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue

ijpam.eu

DESIGN OF MODEL REFERENCE ADAPTIVE CONTROLLER FOR CYLINDER TANK SYSTEM

[1]Nandhinipriyanka G, [2]Ishwarya S, [3]Janakiraman S, [4]Thana Sekar C, [5]Vaishali P
[1]Assistant Professor, Department of EIE, Karpagam college of Engineering.
[2,3,4,5] UG Scholar, Department of EIE, Karpagam college of Engineering.
[1]nandhu521994@gmail.com

Abstract: In process industries, liquid level control is mandatory. The conventional PID controller (Proportional-Integral-Derivative controller) is mostly used in process industries for level control. In this paper, the mathematical model of cylindrical tank interacting and non-interacting system is obtained. To control the liquid level in the tank, Model Reference Adaptive Controller(MRAC) is employed. The design, implementation and performance evaluation are demonstrated via Matlab/Simulink. For performance analysis, the PID controller is designed and is compared with the proposed MRAC. The simulation results shows that MRAC gives better transient performance.

Index Terms: Model Reference Adaptive Controller, cylindrical tank system, Z-N tuned PID.

1. Introduction

System identification is the process of constructing model from experimental data. Based on set of measured stimulus and response samples, system identification involves building a mathematical model of dynamic system. It is a process of acquiring, formatting, processing and identifying mathematical models based on raw data from the real-world system. System Identification of an Interacting Series Process for Real-Time Model Predictive Control is done [1]. Many challenging problems are occurring due to the non-linear behavior in most of the chemical industries. Therefore, traditional control techniques are needed for most of the chemical process industries. Conical tanks exhibit non-linear behavior [2] which is suitable for food process industries, concrete mixing industries, hydrometallurgical industries and waste water treatment industries. Due to the shape, it contributes better drainage of solid mixtures, slurries and viscous liquids. Because of non-linearity, the controller design becomes a challenging task to achieve satisfactory performance using conical tanks. The non-linearity arises due to its shape as at the end, it is broad and at the lower end it becomes narrow. The primary task of a controller is to maintain the process at the desired set point. A Mathematical model is developed for Two Tank Conical Interacting system using the Mass Balance Equation. The controller is tuned using Skogestad method named after the originator is based on the direct method. Here by tuning of PI controllers, an optimization can be achieved[3]. Identifying a linear time invariant (LTI) with lumped parameters state space model of the gaseous pilot plant which has a typical structure of interacting series process and the model has been developed around an operating point. [4] have done work on Model based control of a four tank system. Model based controller is designed for spherical tank system in real time [5]. For estimating FOPDT model parameters, tangent and point of inflection methods are proposed. The major disadvantage of all these methods is the difficulty in locating the point of inflection in practice. The extension to cover general PID control systems was accomplished by introducing an identification scheme. Due to the employing of the Taylor expansion, the convergence is limited by the nearest branch point where complex conjugate poles coalesce and split into real poles. The resulting proportional gain is usually small or the derivative gain is very large, with these quantitative relationships, desired dominant poles can be easily determine. The output of a model reference adaptive system (MRAS) is capable of well tracking the reference output, it is normally considered that the model of the system follows the reference model[6].The Model Reference Adaptive Systems (MRAS) derived for deterministic continuous-time signals is an important adaptive control where desired performance expressed in terms of reference model thus responding to the command signal, other than the normal feedback loop there is another to change the controller parameters with respect to the error. The parameter adjustment mechanism can be gradient method or by using stability theory. The characteristics or system parameters are usually not always the same. For these reasons, tuning of the traditional PID controller parameters to control this system for the required performance faces a strong challenge, so it is better to adapt the tuning parameters by an adaptation rule. Many adaptive control techniques used to provide automatic tuning, in such applications the adaptation loop is simply switched for a period of time when tuning is required. Perturbation signals are normally added to improve parameter estimation. The adaptive controller is run until the performance is satisfactory, then the adaptation loop is disconnected and the system is left running with fixed controller parameters[7].

2. System Description

For collecting the input-output data in the present work, the real time interacting and non-interacting fabricated system was used. The set-up consists of supply tank, pump for water circulation, rotameter for flow measurement, transparent tanks with graduated scales. The tanks can be connected either in interacting or in non-interacting mode. The components are assembled on frame to form tabletop mounting. The set-up is designed such that when connected in interacting or non-interacting mode, dynamic response of single and multi- capacity processes is studied. Here, interacting mode is used for analysis and control of liquid level in the tank. The experimental set up is shown in Fig.1. The specifications are tabulated in Table 1.

Figure 1. Experimental Set up

Table 1. Specifications of the set up

Components	Details
Rotameter	10-100 LPH
Process tank	Acrylic, Cylindrical, Inside Diameter 92mm With graduated scale in mm.
Supply tank	SS304
Pump	Fractional horse power, type submersible
Overall dimensions	550W*475D*520H mm

A. *Mathematical Modeling of Two-Tank Interacting system*

Consider the process consisting of two interacting liquid tanks as shown in Fig.2. The volumetric flow into tank1 is q_{in}(cm3/min), the volumetric flow rate from tank1 to tank2 is q1(cm3/min), and the volumetric flow rate from tank2 is q_0(cm3/min). The height of the liquid level is h1(mm) in tank1 and h2 in tank2(mm). Both tanks have the same cross sectional area denotes the area of tank1 is A1(cm2) and area of tank2 is A2(cm2), q_{l1} is the inflow of tank1 as load disturbance(cm3/min) and q_{l2} is the inflow of tank2 as load disturbance(cm3/min).The differential equations for tank1 and tank 2 are given in equations (1) and (2).

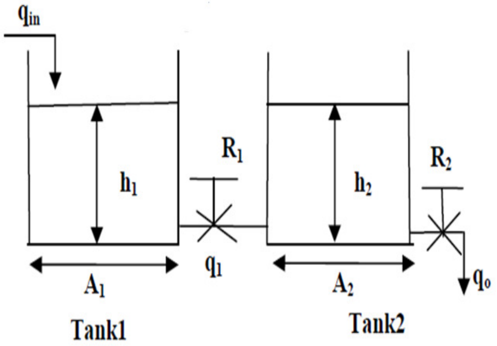

Figure 2. Interacting system

For Tank1, the mass balance equation can be written as,

$$A_1 . \frac{dh_1}{dt} = q_{in} - q_1 \tag{1}$$

Assume linear Resistance to flow,

$$q_1 = \frac{h_1 - h_2}{R_1} \tag{2}$$

$$A_1 \frac{dh_1}{dt} = q_{in} - \frac{h_1 - h_2}{R_1} \tag{3}$$

$$A_1 R_1 \frac{dh_1}{dt} = q_{in} R_1 - h_1 + h_2 \tag{4}$$

By taking Laplace transform

$$A_1 R_1 s h_1(s) = q_{in}(s) R_1 - h_1(s) + h_2(s) \tag{5}$$

$$h_1(s) = \frac{R_1 q_{in}(s) + h_2(s)}{1 + A_1 R_1 s} \tag{6}$$

$$h_1(s) = \frac{R_1 q_{in}(s) + h_2(s)}{1 + \tau_1 s} \tag{7}$$

where,$\tau_1 = A_1 R_1$. Similarly for tank 2, the mass balance equation is

$$A_2 . \frac{dh_2}{dt} = q_1 - q_0 \tag{8}$$

Assume linear Resistance to flow,

$$A_2 \frac{dh_2}{dt} = \left(\frac{h_1 - h_2}{R_1}\right) - \left(\frac{h_2}{R_2}\right) \tag{9}$$

where, $q_0 = \frac{h_2}{R_2}$

$$A_2 R_1 R_2 \frac{dh_2}{dt} = R_2 h_1 - R_2 h_2 - h_2 R_1 \tag{10}$$

On dividing by R_1 and taking Laplace transform,

$$A_2R_2sh_2(s)+\frac{R_2}{R_1}h_2(s)+h_2(s)=\frac{R_2}{R_1}h_1(s) \quad (11)$$

$$h_2(s)\left(\tau_2s+\frac{R_2}{R_1}+1\right)=\frac{R_2}{R_1}h_1(s) \quad (12)$$

where$\tau_2=R_2A_2$

$$h_2(s)\left(\tau_2s+\frac{R_2}{R_1}+1\right)=\frac{R_2}{R_1}\left(\frac{R_1q_{in}(s)+h_2(s)}{1+\tau_1s}\right) \quad (13)$$

$$h_2(s)\left(\tau_2s+\tau_2\tau_1s^2+\frac{R_2}{R_1}\tau_1s+1+\tau_1s\right)=R_2q_{in}(s) \quad (14)$$

therefore, the transfer function of the two tank interacting system is expressed as

$$\frac{h_2(s)}{q_{in}(s)}=\frac{R_2}{\tau_2\tau_1s^2+(\tau_1+\tau_2+A_1R_2)s+1} \quad (15)$$

Table 2. Model parameters of cylindrical tank system

Symbol	Description	Value
R1	Resistance	1.5ohm
R2	Resistance	1.7ohm
D	Diameter of tank	92cm
Qin	Initial flow	20lph
τ1	Time constant	0.9
τ2	Time constant	1.02

By substituting the parameters values in equation (15) as listed in Table 2, the transfer function for the interacting system obtained is

$$G(s)=\frac{1.7}{0.918s^2+1.935s+1} \quad (16)$$

B. Mathematical Modeling of Two-Tank Non-Interacting system

Two tanks are arranged in series so that the outflow from the first tank is the inlet flow to the second tank. Here the outflow from the first tank depends only on the height of the first tank and the variation in the height of the second tank does not affect the response of first tank. This type of system is said to be non-interacting system and is shown in Fig.3.

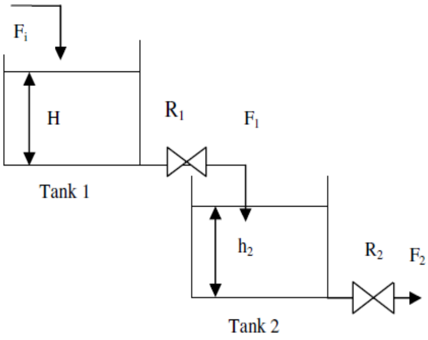

Figure 3. Non interacting system

From tank1 the mass-balance equation,

$$q_{in}-q_1=A_1\frac{dh_1}{dt} \quad (17)$$

Substitute $q_1=\frac{h_1}{R_1}$ and by taking Laplace transform on both sides,

$$q_{in}(s)-\frac{h_1(s)}{R_1}=A_1h_1(s)s \quad (18)$$

$$q_{in}(s)=h_1(s)\left(\frac{R_1A_1s+1}{R_1}\right) \quad (19)$$

$$\frac{h_1(s)}{q_{in}(s)}=\frac{R_1}{\tau_1s+1}q_{in}(s) \quad (20)$$

where$\tau_1=A_1R_1$

From Tank 2 the mass balance equation,

$$q_1-q_2=A_2\frac{dh_2}{dt} \quad (21)$$

$$\frac{h_1}{R_1}-\frac{h_2}{R_2}=A_2\frac{dh_2}{dt} \quad (22)$$

Taking laplace transform on both sides

$$\frac{h_1(s)}{R_1}-\frac{h_2(s)}{R_2}=A_2h_2(s)s \quad (23)$$

$$\frac{h_1(s)}{R_1}=A_2h_2(s)s+\frac{h_2(s)}{R_2} \quad (24)$$

$$\frac{h_1(s)}{R_1}=h_2(s)\left(\frac{\tau_2s+1}{R_2}\right) \quad (25)$$

$$\frac{q_{in}(s)}{\tau_1s+1}=\left(\frac{\tau_2s+1}{R_2}\right)h_2(s) \quad (26)$$

Therefore by substituting the values, the transfer function of the two tank interacting system is expressed as

$$\frac{h_2(s)}{q_{in}(s)}=\frac{R_2}{(\tau_1s+1)(\tau_2s+1)} \quad (27)$$

By substituting the parameters values in equation (27) as listed in Table 2, the transfer function for the interacting system obtained is

$$G(s)=\frac{4.6}{3.0349s^2+3.044s+1} \quad (28)$$

3. Controller Design Aspect

A. Model Reference Adaptive Controller

Model Reference Adaptive Control strategy is used to design the adaptive controller that works on the principle of adjusting the controller parameters so that the output of the actual plant tracks the output of a reference model having the same reference input.

Adjustment Mechanism is a component used to alter the parameters of the controller so that actual plant could track the reference model. Mathematical approaches like MIT rule, Lyapunov theory and theory of augmented error can be used to develop the adjusting mechanism. The MRAC approach can be used for auto-tuning of PID controller. The basic block diagram of MRAC system is shown in the “Fig. 3”. As shown in the figure, ym(t) is the output of the reference model and y(t) is the output of the actual plant and difference between them is denoted by e(t).

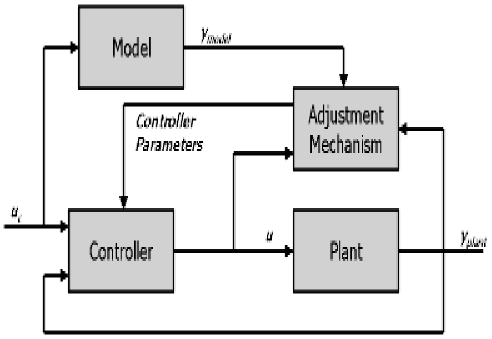

Figure 4. Model Reference Adaptive Control

Consider a plant,

$$\frac{dy}{dt} = -ay + bu \tag{17}$$

Model,

$$\frac{dy}{dt} = -a_m y_m + b_m u_c \tag{18}$$

Control equation

$$U = \theta_1 u_c(t) - \theta_2 y(t) \tag{19}$$

$$\frac{dy}{dt} = -ay + b(\theta_2 b) + b\theta_1 u_c(t) \tag{20}$$

If $y = y_m$ that is perfect model following, then

$$\theta_2 = \frac{a_m - a}{b} \tag{21}$$

$$\theta_1 = \frac{b_m}{b} \tag{22}$$

We know that,

$$\frac{d\theta_1}{dt} = -\gamma e \frac{\partial e}{\partial \theta_1} \tag{23}$$

$$\frac{d\theta_2}{dt} = -\gamma e \frac{\partial e}{\partial \theta_2} \tag{24}$$

$$e = y(s) - y_m(s) \tag{25}$$

By taking Laplace transform

$$sy(s) = -y(s)(a + \theta_2 b) + b\theta_1 u_c(s) \tag{26}$$

$$y(s) = \frac{b\theta_1}{s+a+b\theta_2} u_c(s) \tag{27}$$

we know that,

$$u_c(s) = \frac{s+a+b\theta_2}{b\theta_1} y(s) \tag{28}$$

Therefore,

$$\frac{\partial e}{\partial \theta_2} = \frac{-b}{s+a_m} y(s) \tag{29}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s+a_m} u_c(s) \tag{30}$$

Apply MIT rule, such that

$$\theta_1 = -\gamma \int e \frac{b}{s+a_m} u_c(s) \tag{31}$$

$$\theta_2 = \gamma \int e \frac{b}{s+a_m} y(s) \tag{32}$$

4. Simulation Results and Analysis

The interacting cylindrical tank system is studied both under normal operating conditions and in the presence of disturbance. For tank system with MRAC controller, three different adaptation gain values are chosen and the results are compared.

A. Open Loop Response

Open loop response of a system is giving a input directly to the system and analyzing the properties of the system. In the open loop control system the output is neither measured nor feedback for comparison with the input signal. The open loop response test is used to determine the process time constant, process gain and process dead time.

The open loop response of the two tank interacting and non-interacting system is shown in Fig. 4 when given unit step signal of final value 1 and the step time of 1.

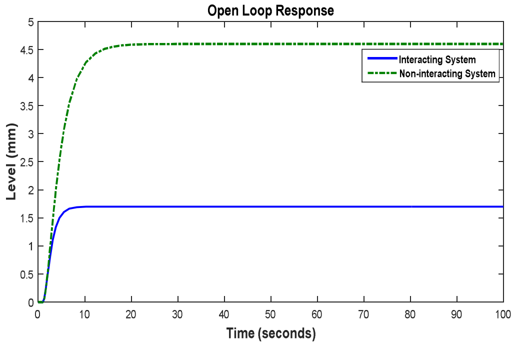

Figure 4. Open Loop Response

B.Closed Loop Response

In a closed loop system, a controller is used to compare the response of a system with the required condition and convert the error into a control action. The controller is designed in a way such that it reduces the error and make the system to achieve the desired response.

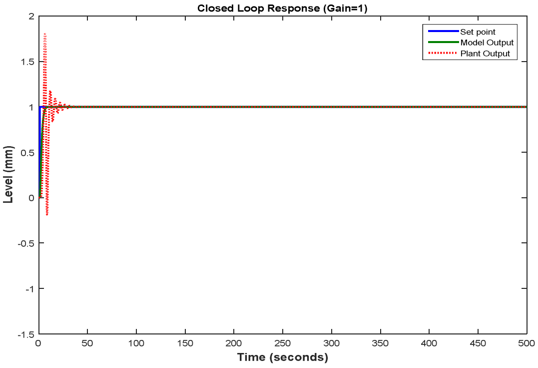

Figure 5. Closed loop response of interacting tank system with MRAC (Gain=1)

The closed loop response for level control in two tank interacting process is shown in Fig.5. The model reference adaptive controller takes the output response of the model as the reference signal and it adapts itself such that it can track the model instead of tracking the set point. The model is chosen in such a way that it can track the set-point with minimum rise time and no overshoot.

In MRAC, the adaptation gain is the parameter that affects the performance of the system directly. The adaptation gain should neither be too low nor too high. From Fig.5, it is seen that the plant have high overshoot and it undergoes oscillations when adaptation gain is one.

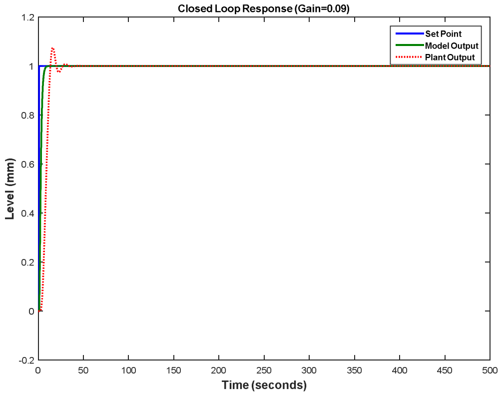

Figure 6. Closed loop response of interacting tank system with MRAC (Gain=0.09)

It is observed that the plant have minimum overshoot of about 0.05mm and it the oscillations are reduced for the adaptive gain of 0.09 as shown in Fig.6. It is also seen that the plant can track the model perfectly. Hence in order to reduce the overshoot and oscillations, the adaptation gain is reduced.

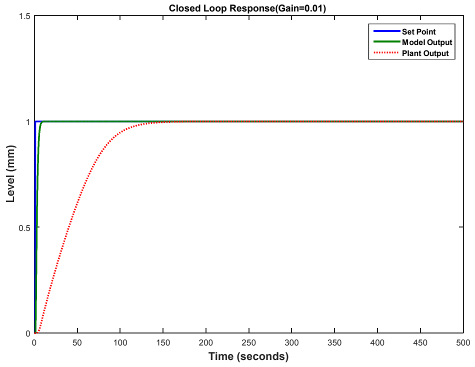

Figure 7. Closed loop response of interacting tank system with MRAC (Gain=0.01)

On reducing the adaptation gain further to 0.01, the plant model eliminates the overshoot but it leads to increase in settling time as shown in Fig.7.

The adaptive gain is the parameter that determines the performance characteristics of the system when model reference adaptive controller is employed. Thus, for interacting system when adaptive gain is 0.09 better response is achieved.

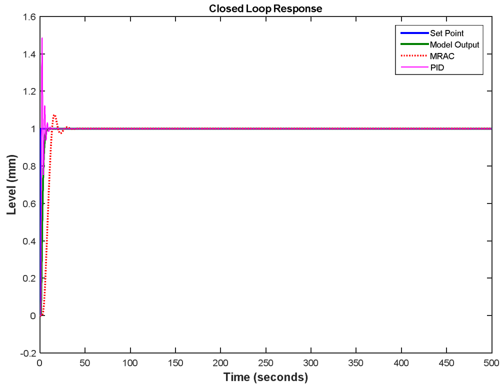

Figure 8. Closed loop response of interacting tank system with MRAC and PID controller

For performance analysis, the convention PID controller is used here. The comparative analysis of MRAC and PID controller is shown in Fig.8. The advantage in using PID is that the settling time is small whereas by using MRAC the settling time is large. From the figure it is observed that with MRAC minimum overshoot is achieved and the oscillations are highly reduced whereas with PID controller there is a overshoot of about 0.5mm and the oscillations are also high.

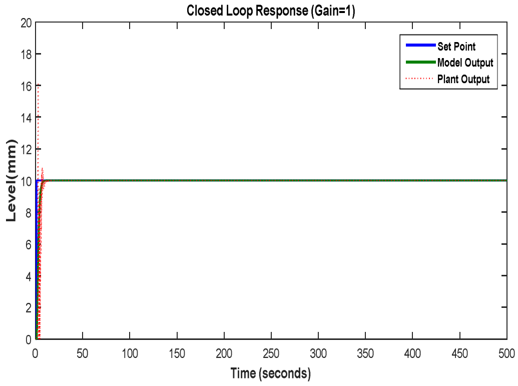

Figure 9. Closed loop response of interacting tank system with MRAC (Gain=1)

The closed loop response for level control in two tank non-interacting process is shown in Fig.9.It is also seen that the plant can track the model perfectly. Hence in order to reduce the overshoot and oscillations, the adaptation gain is increased.

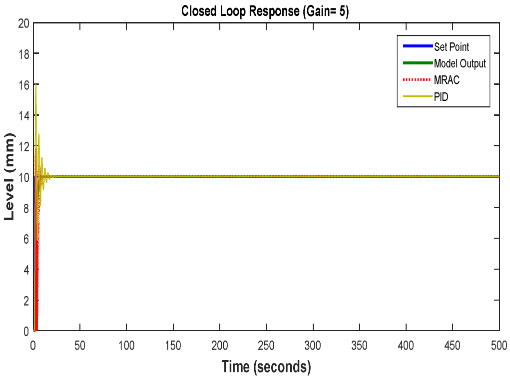

Figure 10. Closed loop response of non-interacting tank system with MRAC and PID controller

It is observed from the above Fig.10, that MRAC controller has minimum overshoot, and the settling time is about 12sec whereas the PID controller has an overshoot of 6mm and it takes 26sec to settle. There is oscillation in the MRAC controller as the controller incorporates the changes continuously and it tries to minimize the error as much earlier as possible.

5. Conclusion

The mathematical model of the cylindrical tank system is derived and the transfer function is obtained so as to control the level of the liquid in the tank. Open loop response of the interacting and non-interacting system is obtained and is found that the system is stable but to improve the performance and to track the reference trajectory a Model Reference Adaptive Controller has been designed in this paper. Through simulation results it is shown that the tuning parameter is the adaptation gain that affects the system response. The simulated results show that MRAC achieves a far better consistent set-point tracking performance as compared to PID.

References

[1] Nithya,S., Abhay Singh Gour, N. Sivakumaran, T. K. Radhakrishnan, T. Balasubramanian, and N. Anantharaman. "Design of intelligent controllers for nonlinear processes." Asian Journal of Applied Sciences 1 (2008): 33-45.

[2] Wibowo, Tri Chandra, NordinSaad, and Mohd Noh Karsiti. "Tri Chandra S. Wibowo, NordinSaad, and Mohd Noh Karsiti,"A Heuristic Approach for Tuning Model Predictive Controller", Journal ELEKTRIKA 11, no. 1 (2009): 8-14.

[3] Gatzke, Edward P., Edward S. Meadows, Chung Wang, and Francis J. Doyle Iii. "Model based control of a four-tank system." Computers & Chemical Engineering 24, no. 2-7 (2000): 1503-1509.

[4] Nithya, S., N. Sivakumaran, T. Balasubramanian, and N. Anantharaman. "Model based controller design for a spherical tank process in real time." IJSSST 9, no. 4 (2008): 25-31.

[5] Weyer, Erik. "System identification of an open water channel." Control engineering practice 9, no. 12 (2001): 1289-1299.

[6] Jain, Priyank, and M. J. Nigam. "Design of a model reference adaptive controller using modified MIT rule for a second order system." Advance in Electronic and Electric Engineering 3, no. 4 (2013): 477-484.

[7] Casagrande, Daniele, Alessandro Astolfi, and Thomas Parisini. "Switching-driving Lyapunov function and the stabilization of the ball-and-plate system." IEEE Transactions on Automatic Control 54, no. 8 (2009): 1881-1886.

[8] Lee, Jung-Hoon. "A New PID Controller with Lyapunov Stability for Regulation Servo Systems." Journal of IKEEE 13, no. 1 (2009): 11-18.

[9] Upadhyay, Rahul, and Rajesh Singla. "Analysis of CSTR temperature control with adaptive and PID controller (a comparative study)." International Journal of Engineering and Technology 2, Vol 5 pp 453 (2010).

[10] Kochummen, Sherin A., N. E. Jaffar, and A. Nasar. "Model Reference Adaptive Controller designs of steam turbine speed based on MIT Rule." In Control Communication & Computing India (ICCC), 2015 International Conference on, pp. 7-11.IEEE, 2015.

[11] T. Padmapriya and V. Saminadan, "Improving Throughput for Downlink Multi user MIMO-LTE Advanced Networks using SINR approximation and Hierarchical CSI feedback", International Journal of Mobile Design Network and Innovation- Inderscience Publisher, ISSN : 1744-2850 vol. 6, no.1, pp. 14-23, May 2015.

[12] S.V.Manikanthan and K.Baskaran "Low Cost VLSI Design Implementation of Sorting Network for ACSFD in Wireless Sensor Network", CiiT International Journal of Programmable Device Circuits and Systems,Print: ISSN 0974 – 973X & Online: ISSN 0974 – 9624, Issue :November 2011, PDCS112011008.

2014