RELAY BASED IDENTIFICATION AND CONTROL OF SISO PROCESS – AN EXPERIMENTAL APPROACH VIA CONICAL TANK

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Abstract: Conventional method of model identification for any process involves different trial and error sequence. Relay based model identifications helps to overcome this problem and aids to obtain optimal model. This paper makes contribution of attaining FOPDT models using ideal relay, symmetric relay and asymmetric relay and compared with closed loop model identification of Ziegler Nichols methodology. For all the designed models control design has been carried out using skogestad’s PI tuning technique. It is observed that model using relay is more efficient than conventional techniques and it has been validated by using different performance indices.

Keywords: FOPDT, Sustained oscillations, Skogestad’s control, level process, Symmetric and Asymmetric Relay.

1. Introduction
Astrom and Hagglund initially suggested the practice of an ideal relay to produce sustained oscillations in the closed loop process. Using those oscillations ultimate gain and ultimate frequency (phase crossover frequency of relay) can be calculated [1]. Luyben worked on the identification of FOPDT and SOPDT models by using autotuned relay method only when steady state gain is known as priori [2]. Mahji et al. contributed his work on relay tuning to calculate process parameters of transfer function model [3] and also proposed method to identify FOPDT, SOPDT and IPDT models using offline and online tuning using relay [4]. Shen et al. proposed parameter identification of system using input biased relay in which steady state gain can also be obtained using single relay test [5]. Conical tank level process has been considered as setup to carry out the experimentation shown in Figure 1.

Figure 1. Experimental setup: Conical tank level process

This Paper comprises of model identification using simple feedback test as a conventional approach by varying proportional gain to attain sustained oscillations at verge of instability. Using those oscillations model parameters can be obtained. Further introducing ideal and biased relay feedback tests model...
parameters can be estimated, which was mentioned in the section. To compare optimality in the model design all the obtained models are subjected to same control design method. Skogestad’s PI tuning has been performed to develop closed loop response of the process, mentioned in section. Section 4 depicts the results analysis and model validation using performance indices.

2. Model Identification

A. Conventional Model Identification

Closed loop model identification using conventional method by Ziegler Nichols states that, one has to increase the proportional gain of the process reaches verge of instability (where sustained oscillations would be obtained). Based on those oscillations by calculating amplitude and time period FOPDT model can be identified. In this paper closed loop response has been performed for different values of proportional gain ($K_p$). Verge of instability is observed at $K_p = 28$ at 275 (sampling time) which was shown in the Figure 2 and controller output for variable $K_p$ is shown in Figure 3.

First order plus dead time (FOPDT) model is given as follows

$$G_p(s) = \frac{K}{\tau s + 1} e^{-\zeta \tau}$$

Where $K$ is process gain, $\zeta$ is time delay and $\tau$ is time constant of the process estimated.

![Figure 2. System response to identify model](image1)

![Figure 3. Controller output identify model](image2)
At the verge of instability from Figure 2, ultimate gain and ultimate frequency can be calculated. Process gain can be calculated by using closed loop steady state gain, where loop gain and loop phase must be equated to ‘1’ and ‘-π’ respectively. Therefore from amplitude and phase angle of the closed loop response time delay \( \tau_c \) and time constant \( \tau \) can be estimated as follows.

\[
\angle G_{c}(j\omega) = -n \tan^{-1}(\omega \tau) = -\pi \tag{2}
\]

\[|G_{c}(j\omega)| = 1 \tag{3}\]

**B. Model Identification Using Symmetric Relay Feedback**

Symmetric relay response can be obtained by using equivalent switch on-off points, in this paper switch on-off point has been selected as shown in the Table 1.

The response of symmetric relay with respect to the assumed settings as shown in the Figure 4.

**Symmetric Relay parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switch on point</td>
<td>2</td>
</tr>
<tr>
<td>Switch off point</td>
<td>-2</td>
</tr>
<tr>
<td>Relay output when on</td>
<td>60</td>
</tr>
<tr>
<td>Relay output when off</td>
<td>40</td>
</tr>
</tbody>
</table>

Relay response is obtained based on the parameters mentioned in the block. Switch on and switch off points imply the error with respect to process variable and manipulated action is assigned in the relay block based on that error.

**C. Model Identification Using Asymmetric Relay Feedback**

The procedure of attaining model is same as that of symmetric relay feedback approach but the relay parameters were varied with respective process error. The error is considered as asymmetric and upper and lower limits of manipulated variables are assigned based on asymmetric switch on and switch off points. Table 2 depicts relay parameter specifications and Figure 5 shows the response of asymmetric relay.

**Asymmetric Relay parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switch on point</td>
<td>10</td>
</tr>
<tr>
<td>Switch off point</td>
<td>0</td>
</tr>
<tr>
<td>Relay output when on</td>
<td>60</td>
</tr>
<tr>
<td>Relay output when off</td>
<td>30</td>
</tr>
</tbody>
</table>
Figure 5. Asymmetric relay response

Assume $K=1$ or one can consider steady state gain to obtain process gain and calculation for $T$ and $\theta$ as follows:

\[
\omega_n = \frac{2\pi}{P_s} = 0.26 \tag{9}
\]

Describing function is given by:

\[
N(A) = \frac{4h}{\pi a} = 2.38 \tag{8}
\]

Critical frequency

Table 3. Comparison of Different FOPDT Models

<table>
<thead>
<tr>
<th></th>
<th>Closed loop conventional method</th>
<th>Symmetric Relay</th>
<th>Asymmetric Relay</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOPDT models</td>
<td>$G(s) = \frac{0.25}{16.9s + 1} e^{0.77s}$</td>
<td>$G(s) = \frac{1}{14.6s + 1} e^{-5.2s}$</td>
<td>$G(s) = \frac{1}{7.7s + 1} e^{-7.7s}$</td>
</tr>
</tbody>
</table>

3. Control Design

Skogestad’s has proposed an analytical PID tuning rules for all types of processes for good closed loop behavior and optimal disturbance rejection [6]. The motivation of this paper is to deliver ideal model using relay feedback test and to compare the efficiency of the model with conventional model identification methods. Henceforth same control design has been implemented on all the obtained models and performance has been compared. Skogestad’s PI tuning for FOPDT models as follows:

Table 4. Skogestad’s PI tuning parameters for all FOPDT processes

<table>
<thead>
<tr>
<th>Methods</th>
<th>FOPDT models</th>
<th>$K_p = \left( \frac{1}{K} \right) \frac{\tau}{\tau + \tau_i}$</th>
<th>$\tau_i = \min(\tau, 4(\tau_i + \tau))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed loop conventional method</td>
<td>$G(s) = \frac{0.25}{16.9s + 1} e^{0.77s}$</td>
<td>38.1</td>
<td>6.16</td>
</tr>
<tr>
<td>Symmetric Relay</td>
<td>$G(s) = \frac{1}{14.6s + 1} e^{-5.2s}$</td>
<td>2.35</td>
<td>14.6</td>
</tr>
<tr>
<td>Asymmetric Relay</td>
<td>$G(s) = \frac{1}{7.7s + 1} e^{-7.7s}$</td>
<td>0.88</td>
<td>7.7</td>
</tr>
</tbody>
</table>
Where $\tau_c$ closed loop time constant, $K$ is system gain, $\tau$ system time constant and $t_d$ is time delay $\tau_c$. $\tau_c$ is tuning parameter, it can be chosen based on tuning user requirement, i.e., for speedup of response and good disturbance rejection (favored by a small value of $\tau_c$) and for Stability, robustness and small input variation (favored by a large value of $\tau_c$). In this paper $\tau_c$ is equated to delay time $t_d$.

4. Result Analysis

It is observed that relay feedback method of model identification gives better reference tracking and load rejection compared with conventional model. Figure 6. shows the closed loop PI response of conventional model. Due to high PI values to the system, the response is oscillating over the setpoint and also settling time is large compared with relay modeling approach.

**Figure 6. Closed loop PI response for conventional model**

Figure 7. depicts the closed loop PI response for symmetric relay feedback model. Setting time is less compared with conventional and asymmetric relay.

**Figure 7. Closed loop PI response for symmetric relay feedback model**

Figure 8. represents closed loop PI response for asymmetric relay feedback model, where servo operation is efficient compared with conventional approach but lags behind the performance compared with symmetric relay based model with respect to settling time. It is also observed that the actuator energy utilization in asymmetric relay model is less compared to symmetric and conventional methods.
4.1 Performance Indices

Controller is said to be optimal if it satisfies the minimization of the error of the process. There are different criteria which reflects the performance of the controller. Each of the indices (function of error) has been calculated over a period of time and also plays an important role in estimating future moves of controller. In this paper ISE (integral square error), IAE (integral absolute error), ITAE (Integral time absolute error) and ITAE (Integral time square error) is considered to analyze the performance.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Integral absolute error (IAE)</th>
<th>Integral square error (ISE)</th>
<th>Integral time absolute error (ITAE)</th>
<th>Integral time square error (ITSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed loop</td>
<td>1195</td>
<td>1.22e+04</td>
<td>1.63e+04</td>
<td>7.50e+05</td>
</tr>
<tr>
<td>conventional method</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric Relay</td>
<td>603.3</td>
<td>8560</td>
<td>6.95e+04</td>
<td>2.96e+05</td>
</tr>
<tr>
<td>Asymmetric Relay</td>
<td>1040</td>
<td>1.58e+04</td>
<td>1.21e+05</td>
<td>5.68e+05</td>
</tr>
</tbody>
</table>

5. Conclusion

Relay plays important role in mathematical modeling of physical process. Identification of the model can be precisely selected using relay limits. Based on threshold limits of an error of process the relay outputs be varied. Operating region of the model is based on the relay output limits. In this paper different models of nonlinear level control process has been identified and control design for all the models has been carried out using skogestad’s PI tuning. Result analysis depicts the efficiency of using relay (symmetric and asymmetric) with respect to reference tracking and energy consumption on manipulated variable. Performance indices also performed for all the model to compare their efficient of handling error.

6. Future work

The opportunity of extending this paper, different literatures has been studied towards model identification of MIMO (Multi Input Multi Output) process [7][8]. Binary distillation column is an experimental setup which is considered to implement model identification through relay (under progress).

References


