A Performance Valuation for NHPP Software Reliability Model depend on Weibull-Type Distribution

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Abstract

Background/Objectives: In this task, the reliability software reliability model, depend on Weibull and extended Weibull distribution, Weibull extension distribution that controls the distribution of several life style using the administration of software creation difficult, was planned.

Methods/Statistical analysis: The software disappointment reliability characteristic point was founded on infinite failure non-homogeneous Poisson process property and the parameter estimation by means of maximum likelihood approximation can be complemented. As distinct cases, the model valuation founded from the mean square error and coefficient of determination aimed at the typical model was accompanied. Using this investigation, the software schemer can be able to help when used as basic information of the software produce.

Findings: From this paper, the reliability software reliability model, depend on Weibull and extended Weibull distribution, Weibull extension distribution that controls the distribution of several life style since the administration of software produce difficult, can be planned. The ensuing conclusions were obtained. Ultimately, in terms of the non-conformity for the among of the predicted estimation values
with the real observations and predictive power of the difference the predicted estimation values, the case of Weibull extension distribution model than extended Weibull distribution model and Weibull model is the helpfulness model.

**Improvements/Applications:** Finally, from contrast about the mean value function, Weibull extension distribution model can be realized that the closer to the true value than the extended Weibull distribution model and Weibull distribution model.

**Key Words:** Extended Weibull Distribution, Weibull Extension Distribution, NHPP, Mission Time, Mean Square Error, Coefficient of Determination

1 Introduction

Up to the present period, frequent software disappointment reliability models have been acknowledged. These contents for Non-homogenous Poisson Process\(^1,2\) (NHPP) since the responsibility discovery method, if a responsibility rises, straight can be separated throughout the correcting procedure and has the conjecture that new errors have not encountered. Goel and Okumoto were accompanied an exponential software reliability model\(^2\). Using characteristic model,, the full number of faults can be displayed S-molded or exponential-considered features from mean value function. Subsequently, in this article, the reliability software disappointment dependability model, happening Weibull and extended Weibull distribution, Weibull extension distribution that controls the distribution of numerous life style from the administration of software product testing, was argued.

2 NHPP feature using Infinite Situation

Form the non-homogeneous Poisson process (NHPP) model\(^3\) by considering infinite property, the corresponding mean value function \(m(t)\) and intensity function \(\lambda(t)\) are respectively.

\[
m(t) = \int_0^t \lambda(\omega)d\omega, \quad \frac{dm(t)}{dt} = \lambda(t)
\]
Consequently, $N(t)$ shadows Probability mass Function (PMF) of Poisson situation by means of a factor.

$$p(N(t) = n) = \frac{[m(t)]^n}{n!}e^{-m(t)}, n = 0, 1, \ldots, \infty$$  \hspace{1cm} (2)

These period-sequence models can be found by NHPP remained famous using a probability of the failure-period. The corresponding mean value function was recognized to following circumstance:

$$m(t) = -1n(1 - F(t))$$  \hspace{1cm} (3)

Specifically, the intensity function container be founded the hazard function ($h(t)$).

$$\lambda(t) = m'(t) = f(t)/(1 - F(t)) = h(t)$$  \hspace{1cm} (4)

Using calculation (4) $f(t)$ monitors PMF (Probability Mass Function) and is CDF (Cumulative Distribution Function). Use $\{t_i, i = 1, 2, \ldots\}$ can be founded order statistic of the periods amongst succeeding software disappointment. Therefore, the failure last period $x_n$ can be founded $n^{th}$ failure period.

$$x_n = \sum_{i=1}^{n} t_i (i = 1, 2, \ldots, n; \ 0 \leq x_1 \leq x_2 \leq \ldots \leq x_n)$$  \hspace{1cm} (5)

A mathematical formula of the likelihood function of $x_1, x_2, \ldots, x_n$ is clear as next follows:

$$fX_1, X_2, \ldots, X_n(x_1, x_2, \ldots, x_n) = L(\Theta | x) = e^{-m(x_n)}\prod_{i=1}^{n} \lambda(x_i)$$  \hspace{1cm} (6)

Using these conditions, the reliability for constrained process $\hat{R}(\xi | x_n)$ can be founded following measures using the work time (or mission time) $\zeta$.

$$\hat{R}(\xi | x_n) = e^{-\int_{x_n}^{x_n+\xi} \lambda(t)dt} = exp[- \{m(\xi + x_n) - m(x_n)\}]$$  \hspace{1cm} (7)

3 Software reliability NHPP model using Weibull-Type distribution

3.1 Weibull NHPP (Crow-AMSAA) model

One actual and furthermore normally used model of the NHPP is the Weibull distribution model (power law procedure model). A model was foremost planned by Duane was established by Crow.
as a NHPP property. This prototypical point has the aptitude to be useful for the forecast property of software failure time. The intensity function and mean value function were known mathematical formulas of next property.

\[ \lambda_{W_{\alpha}}(t) = ab^{t-b-1}, a, b, t > 0, m_{W_{\alpha}}(t) = \int_0^t ab^{t-b-1}dt = a t^b \]  

In Equation (8), a indicates scale parameter and b follows shape parameter. From the Equation (8), the likelihood function by considering infinite failure NHPP can be founded as next mathematical formula.

\[ L_{NHPP}(\Lambda|x) = \prod_{i=1}^{n} ab x_i^{b-1} \exp[-ax_i^b] \]  

So that, \( x = (x_1 \leq x_2 \leq x_3, ..., \leq x_n) \), \( \Theta \) indicates parameterspace.

It is relaxed effort out for estimation of the parameters aimed at the log likelihood function was calculated as subsequent design using the calculation (9).

\[ \ln L_{NHPP}(\Theta|x) = n \ln a + n \ln b + \sum_{i=1}^{n} (b - 1)ln x_i - a x_n^b \]  

The MLE estimator \( \hat{a}_{MLE} \) and \( \hat{b}_{MLE} \) must be pleased the following condition.

\[ \frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial a} = \frac{n}{a} - x_n^b = 0 \text{ (solving for } a, \hat{a}_{MLE} = \frac{n}{x_n^b} \)  

\[ \frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial b} = -a x_n^b \ln x_n + \frac{n}{b} + \sum_{i=1}^{n} \ln x_i = 0 \]  

(solving for \( b \), \( \hat{b}_{MLE} = \frac{n \ln x_n - \sum_{i=1}^{n} \ln x_i}{n} \))

3.2 Extended Weibull NHPP model

The extended Weibull model reduces to the Weibull form. The extended Weibull is a distribution that is extensively used in the arena of software reliability and communal sciences. The corresponding intensity function \( \lambda(t) \) and mean value function \( m(t) \) by considering extended Weibull model are respectively.

\[ \lambda_{Edw}(t) = a t^{b-1}, a, b, t > 0, m_{Edw}(t) = \int_0^t \lambda(\omega)d\omega = a t^b \]  

In Equation (13), a represents scale parameter and b means shape parameter. Using the Equation (13), Form of the likelihood
function using infinite failure NHPP can be founded in place of next property.

$$L_{NHPP}(\Theta|x) = \prod_{i=1}^{n} a x_i^{b-1} \exp[-\frac{a}{b} x_i^b]$$ (14)

So that, \(x = (x_1 \leq x_2 \leq x_3, \ldots \leq x_n)\), \(\Theta = \{a, b\}\) represents parameter space.

In order to the parameter approximation, the log likelihood function was originated as following formula procedure using the Equation (14).

$$\ln L_{NHPP}(\Theta|x) = n \ln a + (b - 1) \sum_{i=1}^{n} x_i - \frac{a}{b} x_n^b$$ (15)

The MLE estimator \(\hat{a}_{MLE}\) and \(\hat{b}_{MLE}\) must be pleased the following condition.

$$\frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial a} = \frac{n}{a} - \frac{1}{b} x_n^b = 0 (solving \ for \ a, \ \hat{a}_{MLE} = \frac{nb}{x_n^b})$$ (16)

Also, discovery the first partial derivative with respect to \(b\) and usual the calculation equal to zero yields.

$$\frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial b} = \sum_{i=1}^{n} x_i + \frac{a}{b^2} x_n^b - \frac{a}{b} x_n^b \ln x_n = 0$$ (17)

### 3.3 Weibull extension distribution NHPP model

The Weibull extension distribution\(^{10}\) is a distribution useful to the reliability model and has the resulting probability mass function and distribution function as a request distribution of the Weibull distribution.

$$f_{Wed}(t) = \lambda b(at)^{b-1} \exp[(at)^b + \lambda^b_1(1 - e^{(at)^b})]$$ (18)

$$f_{Wed}(t) = 1 - \exp[-\lambda^b_1(1 - e^{(at)^b}) - 1]$$ (19)

In equation (18) and (19), so that, \(\lambda, a, b > 0, t > 0, a\), \(\lambda\) denotes scale parameter and \(b\) is shape parameter. Using equation (3) and (4), the intensity function \(\lambda(t)\), hazard function \(h(t)\) and the mean value function \(m(t)\) are as following pattern respectively.

$$\lambda_{Wed}(t) = f(t)/(1-F(t)) = \lambda b(at)^{b-1} \exp[(at)^b] = h(t)$$ (20)
\[ m_{Wad}(t) = -\ln(1-F(t)) = \lambda \left( e^{(at)b} - 1 \right) \] (21)

Using the equation (20) and (21), the log-likelihood function using infinite failure NHPP can be pleased as next relationship.

\[
\ln L_{NHPP}(\Theta|x) = n \ln \lambda + n \ln b + (b-1) \sum_{i=1}^{n} \ln(ax_i) + \sum_{i=1}^{n} (ax_i)^b - \frac{\lambda}{a}(e^{(ax_n)b} - 1)
\] (22)

So that. \( x = (x - 1 \leq x_2 \leq x_3 \ldots \leq x_n), \Theta = \{a, b, \lambda\} \) , is parameter space. In order to simplify convergence for point approximation, want to progress the equation by fixing the scale parameter (\( \hat{b}_{MLE} \)) to 1.0. Thus, discovery the first partial derivative with admiration to \( \lambda, b \), usual the equation equal to zero harvests respectively. So that, \( \hat{a}_{MLE} \) and \( \hat{\lambda}_{MLE} \) must be pleased the following relationship.

\[
\frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial \lambda} = \frac{n}{\lambda} - \frac{1}{a} (e^{(ax_n)b} - 1) = 0
\] (23)

\[
\frac{\partial \ln L_{NHPP}(\Theta|x)}{\partial a} = (b-1) \frac{n}{a} + ba \sum_{i=1}^{n} x_i^b + \frac{\lambda}{a^2} \left[ e^{ax_n} - 1 - x_n^b e^{(ax_n)b} \right]
\] (24)

4 Illustration

In this section, using the failure time structure, the property for software reliability model considering the Weibull, extended Weibull and Weibull extension distribution that controls the several life style distribution were studied. Table 1 is creation of the software failure time. The Laplace trend test in the chief, would be headed to control the worth of the statistics. Some significances of this test from Figure 1 display that approximate value of Laplace features need the environment from 2 to -2. Since a possessions of the reliability growing display, it attains designate the unsafe price remains not happened. So, the calculation of the reliability using this data might be possible.

The approximation price of the parameters for the projected model was recycled the maximum likelihood method. In this situation, the exact change forms (Failure time (time) \times 0.1) for
Table 1: Software Failure Time Information for Proposed Model

<table>
<thead>
<tr>
<th>Failure number</th>
<th>Failure time (hours)</th>
<th>Failure Number</th>
<th>Failure time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.479</td>
<td>16</td>
<td>10.771</td>
</tr>
<tr>
<td>2</td>
<td>0.745</td>
<td>17</td>
<td>10.906</td>
</tr>
<tr>
<td>3</td>
<td>1.022</td>
<td>18</td>
<td>11.183</td>
</tr>
<tr>
<td>4</td>
<td>1.576</td>
<td>19</td>
<td>11.779</td>
</tr>
<tr>
<td>5</td>
<td>2.61</td>
<td>20</td>
<td>12.536</td>
</tr>
<tr>
<td>6</td>
<td>3.559</td>
<td>21</td>
<td>12.973</td>
</tr>
<tr>
<td>7</td>
<td>4.252</td>
<td>22</td>
<td>15.203</td>
</tr>
<tr>
<td>8</td>
<td>4.849</td>
<td>23</td>
<td>15.64</td>
</tr>
<tr>
<td>9</td>
<td>4.966</td>
<td>24</td>
<td>15.98</td>
</tr>
<tr>
<td>10</td>
<td>5.136</td>
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<tr>
<td>11</td>
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<tr>
<td>12</td>
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<td>27</td>
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<tr>
<td>13</td>
<td>6.996</td>
<td>28</td>
<td>17.60</td>
</tr>
<tr>
<td>14</td>
<td>8.170</td>
<td>29</td>
<td>18.122</td>
</tr>
<tr>
<td>15</td>
<td>8.863</td>
<td>30</td>
<td>18.735</td>
</tr>
</tbody>
</table>

Figure 1. Test of Laplace Trend

Table 2: Estimated Value of MLE, MSE, $R^2$ for Proposed Model

<table>
<thead>
<tr>
<th>Model</th>
<th>Maximum likelihood estimation</th>
<th>Model Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\alpha}_{MLE} = 22.6889$</td>
<td>$\hat{\beta}_{MLE} = 0.4449$</td>
</tr>
<tr>
<td>Extended Weibull</td>
<td>$\hat{\alpha}_{MLE} = 16.9888$</td>
<td>$\hat{\beta}_{MLE} = 1.0908$</td>
</tr>
<tr>
<td>Weibull Extension</td>
<td>$\hat{\alpha}_{MLE} = 0.0083$</td>
<td>$\hat{\lambda}_{MLE} = 15.0109$</td>
</tr>
</tbody>
</table>

shorten the parameter calculation was used. A consequence of the parameter estimate can be attained in the Table 2. In this section, outcome of parameter estimation was recorded in Table 2. These
controls to estimate the root, solving mathematically, because the initial values were given 0.001 and 5.000 and acceptance rate of interval value \(10^{-5}\) were specified, were accomplished repetition of 100 times using C-language checking acceptable convergent.

The mathematical formula of Mean Square Error \(^3,12\) (MSE) indicates difference of the real value and estimated value. It was obtained as next measure.

\[
MSE = \frac{\sum_{i=1}^{n} [m(x_i) - \hat{m}(x_i)]^2}{n-k} \quad (25)
\]

Also, \(R^2\) indicates the predictive measure of the difference among the forecasting values\(^3,11\)

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} [m(x_i) - \hat{m}(x_i)]^2}{\sum_{i=1}^{n} m(x_i) - \sum_{j=1}^{n} m(x_j)/n} \quad (26)
\]

Memo. \(m(x_i)\) means the full cumulated number of the faults noticed in \((0, x_i]\) and \(\hat{m}(x_i)\) estimating full cumulated number of the faults noticed in \((0, x_i]\), specifies the amount of realizing values and \(k\) is the amount of the parameter.

For the software failure reliability model valuation in Table 2, MSE displays that the circumstance of Weibull extension distribution model than extended Weibull distribution model and Weibull distribution model consumes a small estimation value. Thus, the case of Weibull extension distribution model is significantly better than extended Weibull distribution model and Weibull distribution model. Similarly, \(R^2\) displays that the case of Weibull extension distribution model than extended Weibull distribution model and Weibull distribution model and Weibull distribution has a high estimation value. So, the case of Weibull extension distribution model than extended Weibull distribution model and Weibull model is the helpfulness model.
The consequence of hazard functions (or intensity functions) remained itemized from Figure 2. Since this figure, the case of Weibull distribution shows decreases sharply with failure time and the shapes of hazard function of Weibull extension and extend Weibull distribution have the property of the non-decreasing form.
The outcome of mean value meanings was enumerated from Figure 3. Since this figure, the shapes of mean value function show the tendency of the non-decreasing procedure. Lastly, from contrast about the mean value function, Weibull extension distribution model can be comprehended that the closer to the true value than the extended Weibull distribution model and Weibull distribution model. As compared to the true value, Weibull distribution model was done overestimation and the Weibull extension distribution model. The circumstance of the extended Weibull distribution model, the shapes of mean value function, in the first half, was underestimated and in the second half was overestimated.

In Figure 4, in terms of decision of reliability, the circumstance of the reliability for assumed effort time (mission time), displays the proposed model have possessions of the non-reduced shape. Specifically, the reliability has been delicate to the work time. In positions of reliability, Weibull distribution model than extended Weibull distribution and Weibull extension distribution model shows the high trend. So that, in terms of reliability, Weibull distribution model than extended Weibull distribution and Weibull extension distribution model can be decided more dependable model in this arena.
5 Conclusions

At this conclusion, the software failure reliability model, proceeding Weibull and extended Weibull distribution, Weibull extension distribution that controls the distribution of several life styles after the administration about software produce testing, was planned. The subsequent results were obtained. Eventually, for the nonconformity for the amongst of the foretold estimation standards with the real comments and a predictive power of the difference the predicted estimation values, the case of Weibull extension distribution model than extended Weibull distribution model and Weibull model remains the helpfulness model.

The shapes of mean value function necessitate the propensity of a non-decreasing procedure. Finally, from contrast about the mean value function, Weibull extension distribution model can be realized that the closer to the true value than the extended Weibull distribution model and Weibull distribution model.

In positions of reliability, Weibull distribution model than extended Weibull distribution and Weibull extension distribution model shows the high trend. So that, in terms of reliability, Weibull distribution model than extended Weibull distribution and Weibull extension distribution model can be decided more dependable model in this arena. From this study, the software designers can be able to assistance when used as elementary material of the software produce.

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References


